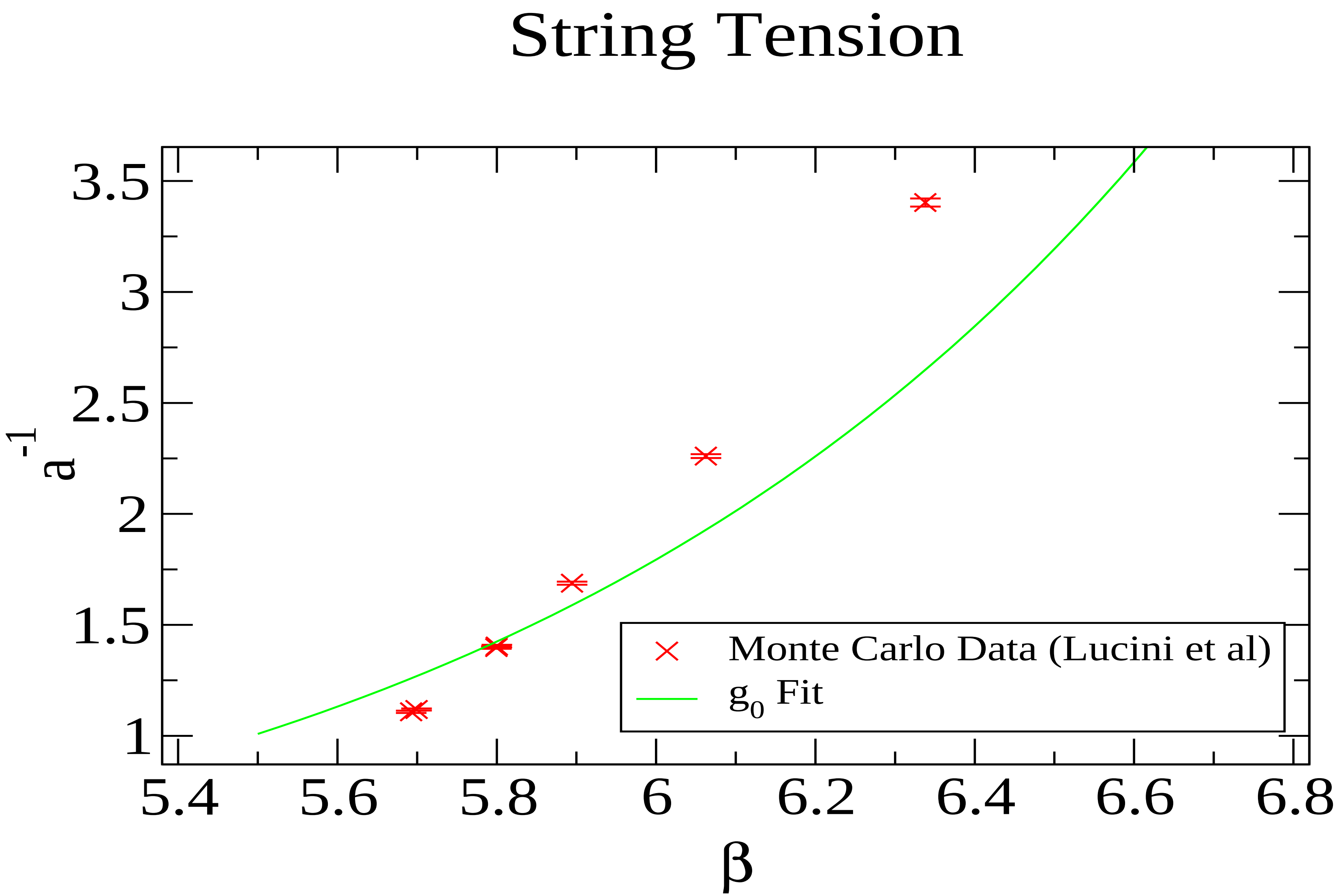
Asymptotic Scaling, Lattice Artefacts and all that.

Aurora Trivini, CRA

(Poster presented by A. Trivini at Lattice 2005, July 2005, Dublin) PoS(LAT2005)036

Mike Teper

Failure of Asymptotic Scaling (Quenched)



Outline

- Review lack of Asymptotic Scaling in g_0
- Renormalised Coupling Approach (Parisi, Lepage-Mackenzie)
- Introduce “Lattice Distorted Perturbation Theory”
- SU(3) results
- SU(N) results

What should $a^{-1}(g)$ be?

$$\beta(g^2) = -a\frac{dg^2}{da} = -2b_0 g^4 - 2b_1 g^6 - 2b_2^L g^8,$$

where $b_{0,1}$ are universal

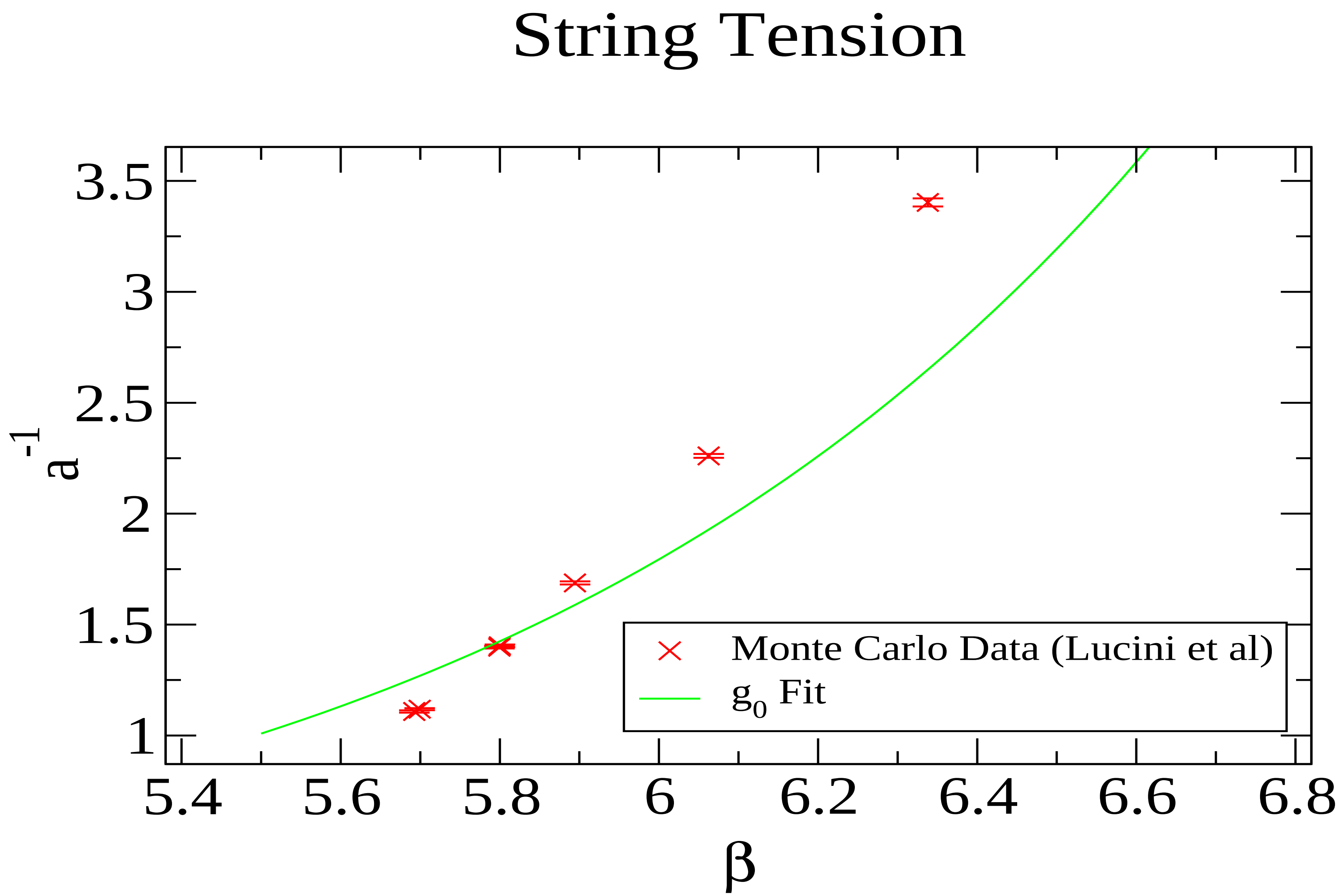
Integrating $\longrightarrow$

$$a^{-1}(g^2) = \frac{\Lambda}{f_{PT}(g^2)},$$

where Λ is the integration constant, and

$$f_{PT}(g^2) = e^{-\frac{1}{2b_0 g^2}} \; (b_0 g^2)^{\frac{-b_1}{2b_0^2}} \; (1 + \underbrace{\frac{1}{2b_0^3}(b_1^2 - b_2^L b_0)}_{d_2^L} \; g^2)$$

Failure of g_0 Asymptotic Scaling



Renormalised Coupling Approach

Parisi; Lepage, Mackenzie

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Attempt to *re-sum* the higher order terms by using a Monte Carlo quantity whose perturbative expansion is known to define a *renormalised* coupling. E.g.

$$1 - < \frac{1}{3} Tr U_{plaq} > \quad = \quad c_1 g_0^2 \, + \, c_2 g_0^4 \, + \, \ldots$$

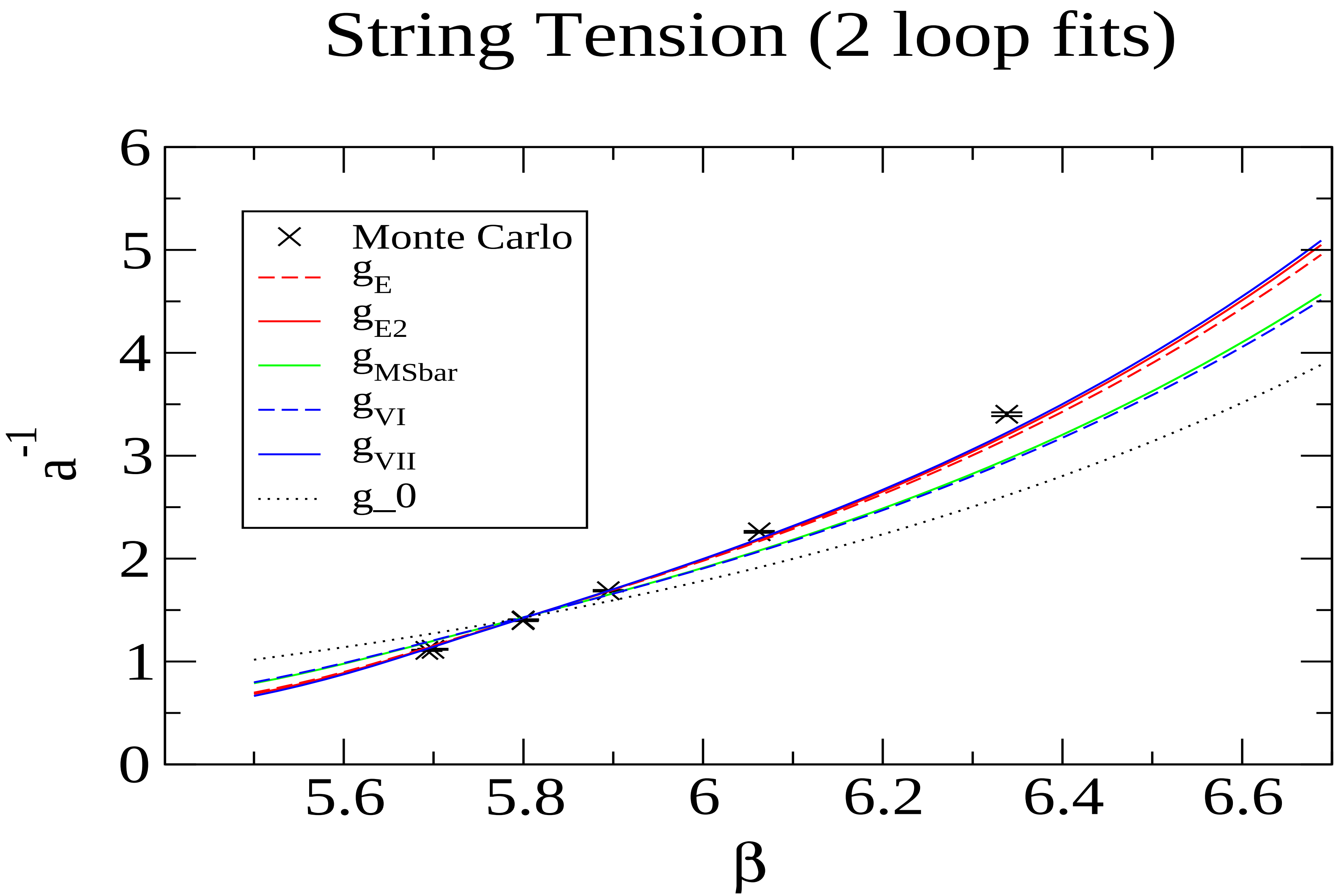
So, define g_E using:

$$1 - < \frac{1}{3} Tr U_{plaq} > \quad \equiv \quad c_1 g_E^2 \, .$$

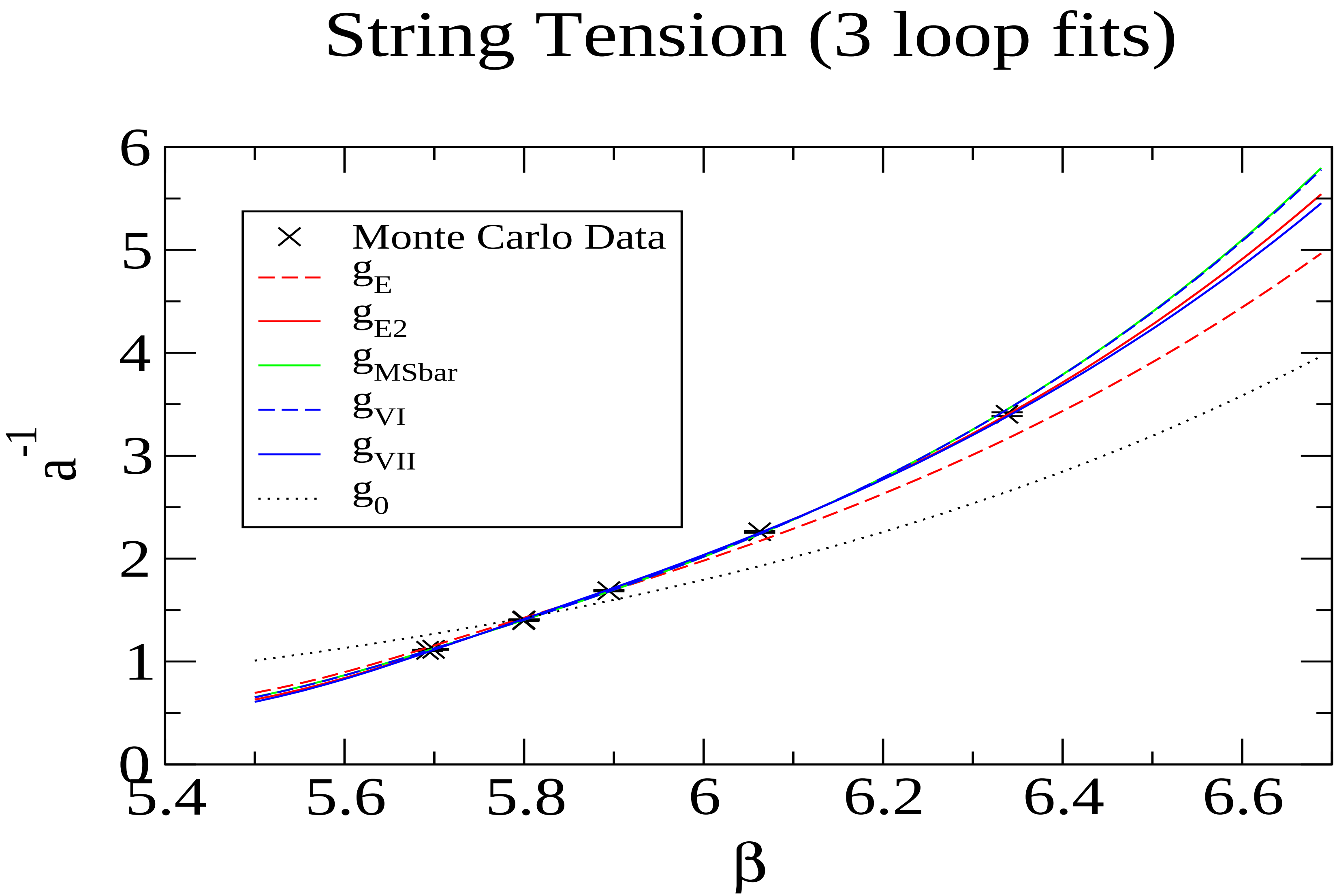
Many other possibilities have been tried:

g_E Parisi 1980, g_{E2} Bali & Schilling 1993, $g_{\overline{MS}}$ El Khadra et al 1992, g_{VI}, g_{VII} Lepage & Mackenzie 1993

Renormalised Fits - 2 Loops



Renormalised Fits - 3 Loops



Renormalised Fits - 3 Loops (contd)

Scheme	d_2^L	$\chi^2/d.o.f.$
g_0	$\equiv 0.189615162$	900
g_E	$\equiv 0.01161$	68
g_{E2}	1.01 ± 0.13	3.4
$g_{\overline{MS}}$	12 ± 5	3.6
g_{VII}	0.151 ± 0.014	5.3
g_{VI}	4.6 ± 0.9	3.3

“Lattice Distorted Perturbation Theory”

CRA, Nucl. Phys. **B437** (1995) 641, hep-lat/9610016

- Idea is that the discrepancy btw Monte Carlo Data and Asymptotic Scaling is due to lattice *artefacts*
- $\longrightarrow$ introduce $\mathcal{O}(a^n)$ term(s)

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- $\longrightarrow$ introduce $\mathcal{O}(a^n)$ term(s)

$$\beta_L(g_0^2) = -a\frac{dg_0^2}{da} \quad = \quad -\left(2b_0 g_0^4 + 2b_1 g_0^6 + 2b_2^L g_0^8 + \sum_{l=4} b_{l-1}^L g_0^{2l+2}\right)$$

$$\times \quad \left(1 + \sum_{n=1} c_n(g_0^2) a^n(g_0^2)\right)$$

Integrating $\longrightarrow$

$$a^{-1}(g_0^2) = \frac{\Lambda}{f_{PT}(g_0^2)} \times \left(1 + \sum_{l=4} d_{l-1} g_0^{2l-4}\right)^{-1} \times \left(1 + \sum_{n=1} c'_n(g_0^2) f_{PT}^{\,n}(g_0^2)\right.$$

“Lattice Distorted Perturbation Theory” (contd)

Convenient to write this as

$$a^{-1}(g_0^2) = \frac{\Lambda}{f_{PT}(g_0^2)} \times \left(1 - X\,\frac{g_0^{\nu} f_{PT}^{n}(g_0^2)}{G_0^{\nu} f_{PT}^{n}(G_0^2)} - Y\,\frac{g_0^{\nu'} f_{PT}^{n'}(g_0^2)}{G_0^{\nu'} f_{PT}^{n'}(G_0^2)}\right)$$

where G_0 is some convenient standard
(e.g. for Wilson $G_0 = 1 \;\leftrightarrow\; \beta = 6.0$)

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	Gauge Action		
	Wilson	Iwasaki	DBW2
a_{r_0,r_c}	$\mathcal{O}(g^2a^2) + \mathcal{O}(g^2a^4)$	$\mathcal{O}(g^2a^2) + \mathcal{O}(g^2a^4)$	$\mathcal{O}(g^2a^2) + \mathcal{O}(g^2a^4)$
a_{σ}	$\mathcal{O}(a^2) + \mathcal{O}(a^4)$	$\mathcal{O}(a^2) + \mathcal{O}(a^4)$	$\mathcal{O}(a^2) + \mathcal{O}(a^4)$
a_{T_c}	$\mathcal{O}(a^2) + \mathcal{O}(a^4)$	$\mathcal{O}(a^2) + \mathcal{O}(a^4)$	$\mathcal{O}(a^2) + \mathcal{O}(a^4)$
	Fermionic action		
a_{K-K^*}		$\mathcal{O}(g^2a) + \mathcal{O}(a^2)$	

Monte Carlo Data Used (Wilson)

	a^{-1} [Gev] from				
β	r_C	$\sqrt{\sigma}$	T_c	Action	Ref.
5.6925			1.2000(3)	Wilson	Necco 2003
5.6925		1.108(5)		Wilson	Lucini et al 2004
5.6993		1.119(5)		Wilson	Lucini et al 2004
5.7995		1.398(5)		Wilson	Lucini et al 2004
5.8		1.404(6)		Wilson	Lucini et al 2004
5.8941			1.8000(11)	Wilson	Necco 2003
5.8945		1.688(7)		Wilson	Lucini et al 2004
5.95	1.985(8)			Wilson	Necco 2003
6.0624			2.4000(34)	Wilson	Necco 2003
6.0625		2.260(8)		Wilson	Lucini et al 2004
6.07	2.424(8)			Wilson	Necco 2003
6.2	2.973(16)			Wilson	Necco 2003
6.3380			3.6000(99)	Wilson	Necco 2003
6.3380		3.403(18)		Wilson	Lucini et al 2004
6.4	3.938(16)			Wilson	Necco 2003
6.57	4.903(31)			Wilson	Necco 2003
6.69	5.719(39)			Wilson	Necco 2003
6.81	6.661(39)			Wilson	Necco 2003
6.92	7.704(55)			Wilson	Necco 2003

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$$r_0 = 0.49 fm$$

$$r_c = 0.5133 \times r_0$$

defined via $r_c^2 F(r_c) = 0.65$

$$\sqrt{\sigma} \equiv 440 MeV$$

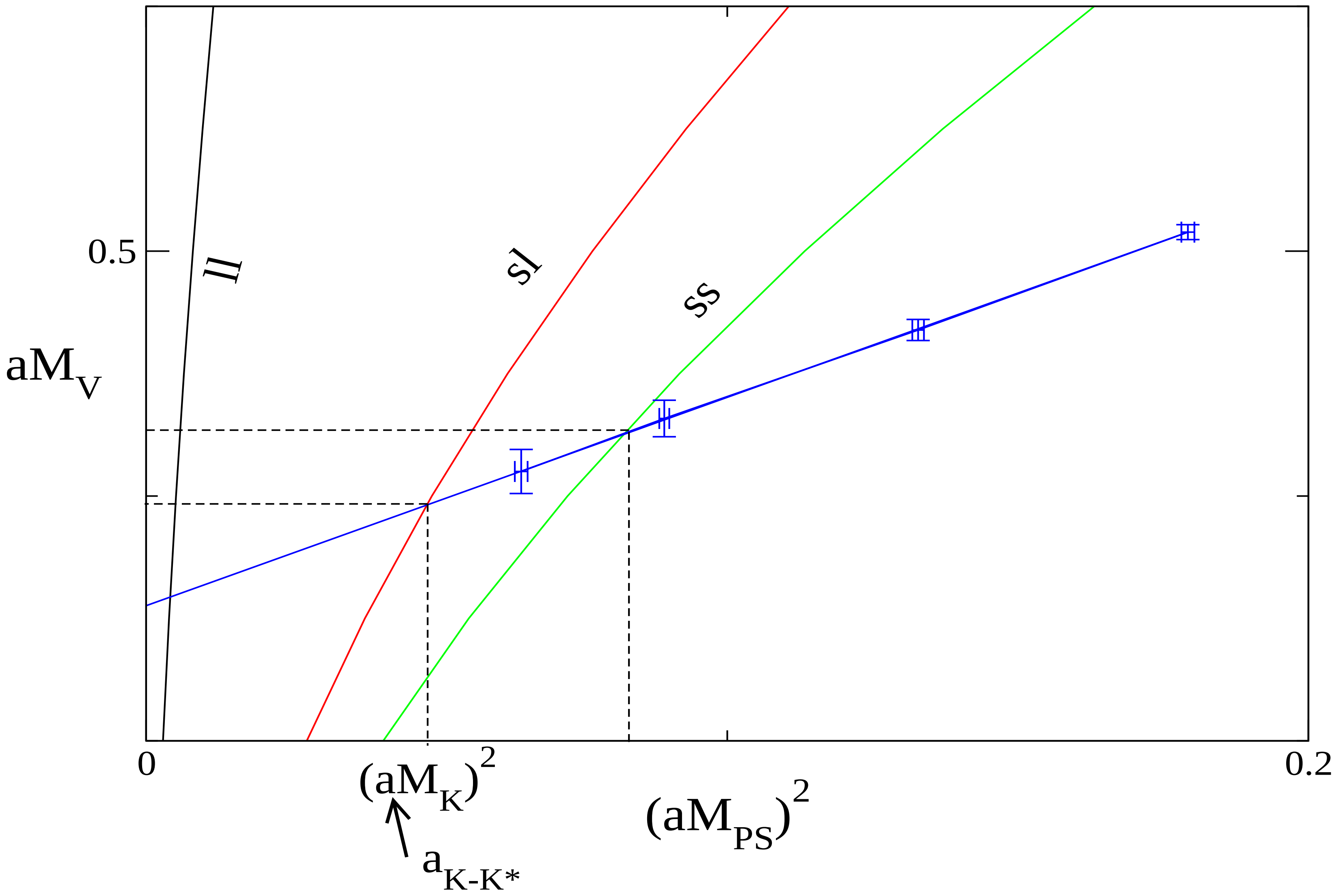
$$T_c \equiv 300 MeV = \frac{1}{N_t a(\beta_c)}$$

Monte Carlo Data Used (non-Wilson)

	a^{-1} [Gev] from					
β	r_0	$\sqrt{\sigma}$	T_c	$K - K^*$	Action	Ref.
2.1551	0.934(4)		0.9000(13)		Iwasaki	Necco 2003
2.187	1.004(14)	0.947(7)		1.024(12)	Iwasaki	CP-PACS 2002
2.214	1.056(17)	0.997(6)		1.077(13)	Iwasaki	CP-PACS 2002
2.247	1.128(11)	1.063(6)		1.131(14)	Iwasaki	CP-PACS 2002
2.281	1.209(15)	1.141(7)		1.164(16)	Iwasaki	CP-PACS 2002
2.2879	1.219(2)		1.2000(16)		Iwasaki	Necco 2003
2.334	1.325(9)	1.249(7)		1.259(15)	Iwasaki	CP-PACS 2002
2.416	1.540(5)	1.450(8)		1.423(20)	Iwasaki	CP-PACS 2002
2.456	1.643(6)	1.556(16)		1.541(16)	Iwasaki	CP-PACS 2002
2.487	1.726(6)	1.634(12)		1.597(20)	Iwasaki	CP-PACS 2002
2.5206	1.817(3)		1.8000(52)		Iwasaki	Necco 2003
2.528	1.840(8)	1.743(15)		1.699(21)	Iwasaki	CP-PACS 2002
2.575	1.968(6)	1.858(12)		1.812(22)	Iwasaki	CP-PACS 2002
2.7124	2.416(10)		2.4000(98)		Iwasaki	Necco 2003
0.75696	0.896(5)		0.9000(11)		DBW2	Necco 2003
0.82430	1.223(7)		1.200(14)		DBW2	Necco 2003
0.9636	1.835(10)		1.8000(54)		DBW2	Necco 2003
1.04	2.196(11)				DBW2	Necco 2003

a^{-1} from $K - K^*$ mass point - Method of Planes

CRA, Gimènez, Giusti & Rapuano, Nucl. Phys. **B489** (1997) 427



Fit Results: Wilson

$\mathcal{O}(a)$ Order	PT Order	Quantity	Λ_L[Mev]	X	Y	χ^2/dof	$\Lambda_{\overline{MS}}$[Mev]
LO	2 Loop	r_c	6.41(2)	0.210(4)	-	2.2	184.8(6)
LO	2 Loop	T_c	6.163(7)	0.1776(4)	-	55	177.6(2)
LO	2 Loop	σ	5.94(2)	0.194(2)	-	2.1	171.1(7)
LO	3 Loop	r_c	7.48(2)	0.193(4)	-	1.2	215.5(7)
LO	3 Loop	T_c	7.250(8)	0.1683(4)	-	41	208.9(2)
LO	3 Loop	σ	6.97(3)	0.184(2)	-	1.5	200.9(8)
NLO	2 Loop	r_c	6.50(3)	0.27(2)	-0.047(16)	0.87	187(1)
NLO	2 Loop	T_c	6.44(3)	0.231(5)	-0.020(2)	0.44	185.6(8)
NLO	2 Loop	σ	6.09(6)	0.23(1)	-0.016(5)	0.40	175(2)
NLO	3 Loop	r_c	7.54(4)	0.23(2)	-0.031(16)	0.69	217(1)
NLO	3 Loop	T_c	7.53(3)	0.213(5)	-0.016(2)	0.87	216.8(9)
NLO	3 Loop	σ	7.12(7)	0.21(1)	-0.012(5)	0.42	205(2)

$$a^{-1}(g_0^2) = \frac{\Lambda}{f_{2PT}(g_0^2)(1 + d_2^L g_0^2)} \times (1 - X\mathcal{O}a^n - Y\mathcal{O}a^{n'})$$

Fit Results: Iwasaki

$\mathcal{O}(a)$ Order	PT Order	Quantity	Λ_L[Mev]	X	Y	d_2^L	χ^2/dof
LO	2 Loop	r_0	225.6(5)	0.0563(4)	-	-	5.4
LO	2 Loop	T_c	235.5(7)	0.1704(9)	-	-	5.1
LO	2 Loop	σ	222(1)	0.163(3)	-	-	0.63
LO	2 Loop	a_{K-K^*}	216(3)	0.073(4)	-	-	0.98
LO	3 Loop	r_0	490(80)	0.040(2)	-	0.5(2)	0.6
LO	3 Loop	T_c	290(20)	0.158(4)	-	0.10(4)	0.5
LO	3 Loop	σ	350(140)	0.13(2)	-	0.3(3)	0.4
LO	3 Loop	a_{K-K^*}	2(2)$\times 10^3$	0.006(6)	-	4(5)$\times 10^3$	0.69
NLO	2 Loop	r_0	238(2)	0.083(3)	-0.0099(12)	-	0.44
NLO	2 Loop	T_c	241(2)	0.193(7)	-0.007(2)	-	0.97
NLO	2 Loop	σ	231(7)	0.21(3)	-0.02(1)	-	0.39
NLO	2 Loop	a_{K-K^*}	300(50)	0.35(11)	-0.4(2)	-	0.59
NLO	3 Loop	r_0	260(70)	0.08(2)	-0.008(4)	0.05$\pm$0.14	0.47
NLO	3 Loop	σ	110(30)	0.4(1)	-0.06(2)	-0.27(6)	0.3
NLO	3 Loop	a_{K-K^*}	160(50)	0.50(9)	-0.57(11)	-0.28(9)	0.67

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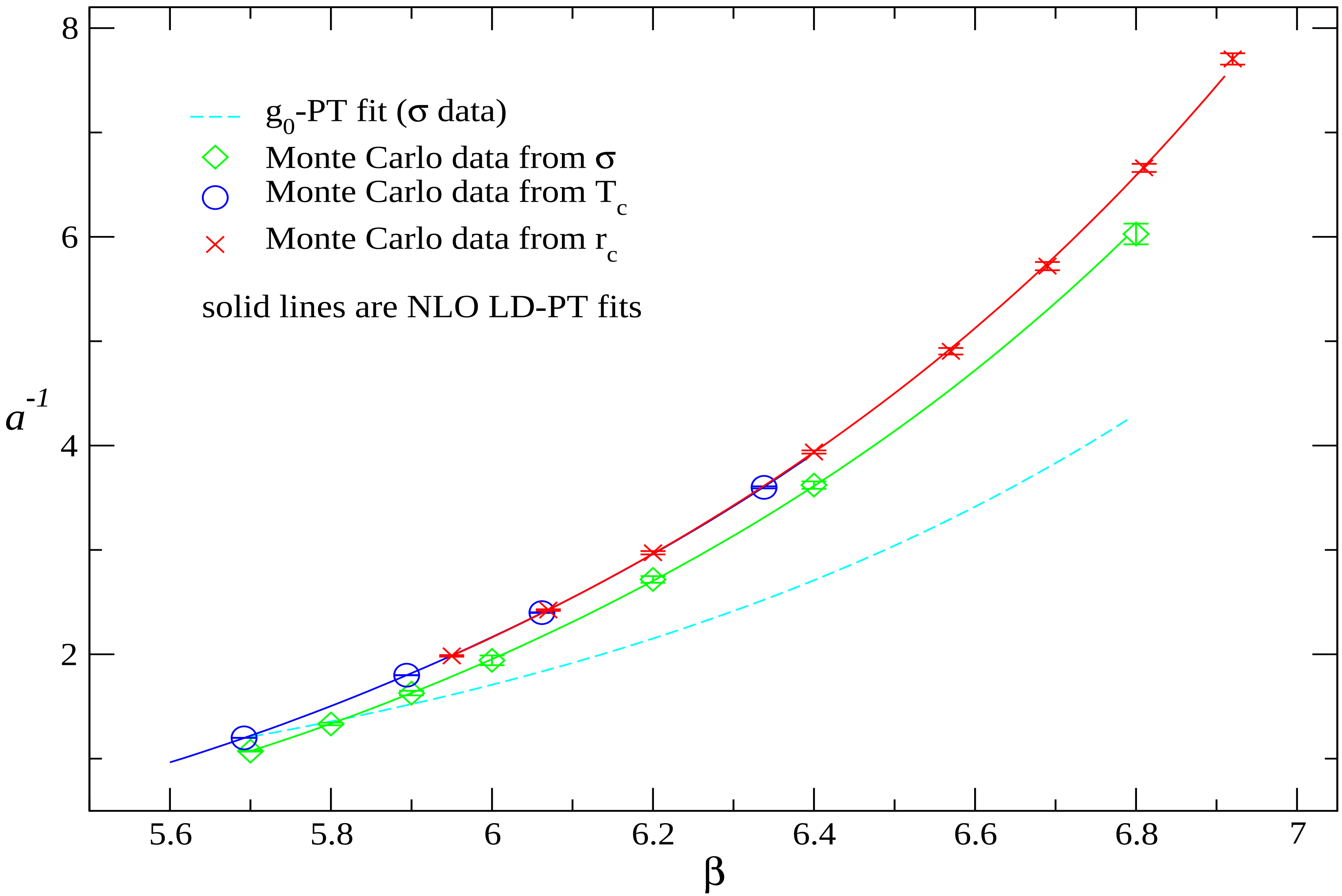
Fit Results: DBW2

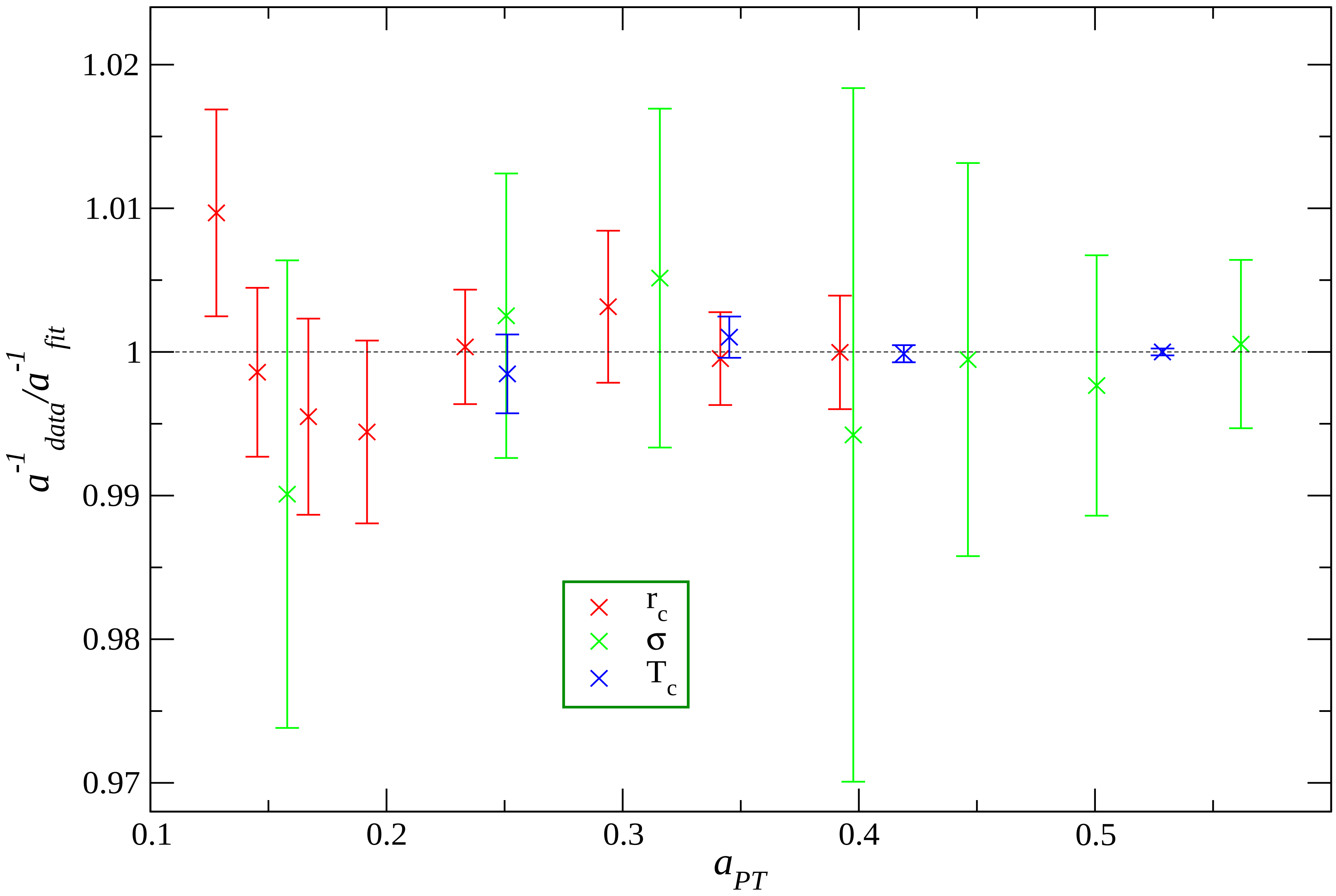
$\mathcal{O}(a)$ Order	PT Order	Quantity	Λ_L[Mev]	X	Y	d_2^L	χ^2/dof
LO	2 Loop	r_0	1352(8)	0.0550(3)	-	-	2
LO	2 Loop	T_c	1894(7)	0.4995(7)	-	-	141
LO	3 Loop	r_0	1500(200)	0.053(2)	-	0.02(2)	3
NLO	2 Loop	r_0	1420(60)	0.07(1)	-0.008(7)	-	2.9

$$a^{-1}(g_0^2) = \frac{\Lambda}{f_{2PT}(g_0^2)(1 + d_2^L g_0^2)} \times (1 - X\mathcal{O}a^n - Y\mathcal{O}a^{n'})$$

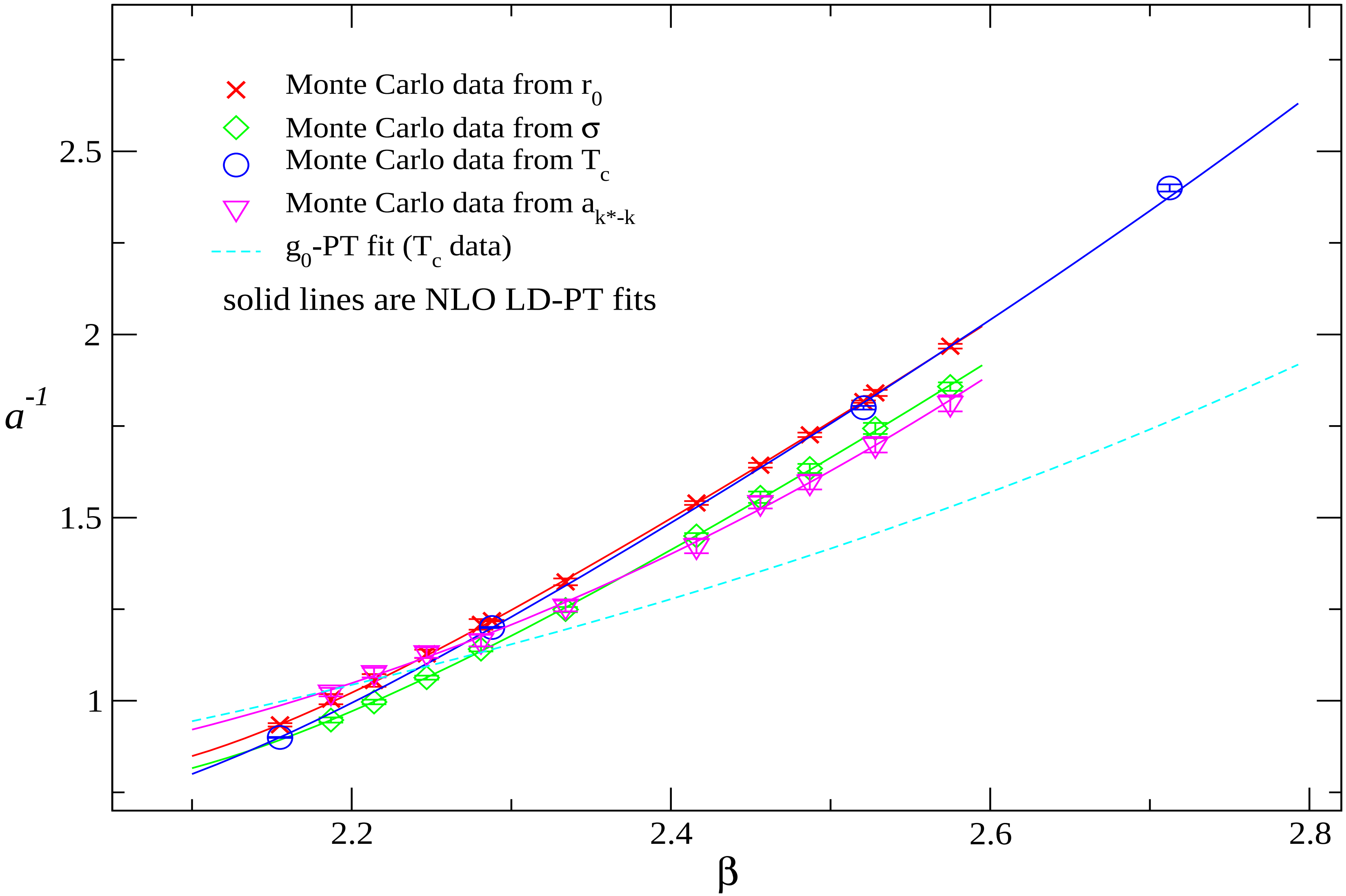
Plots: Wilson Case



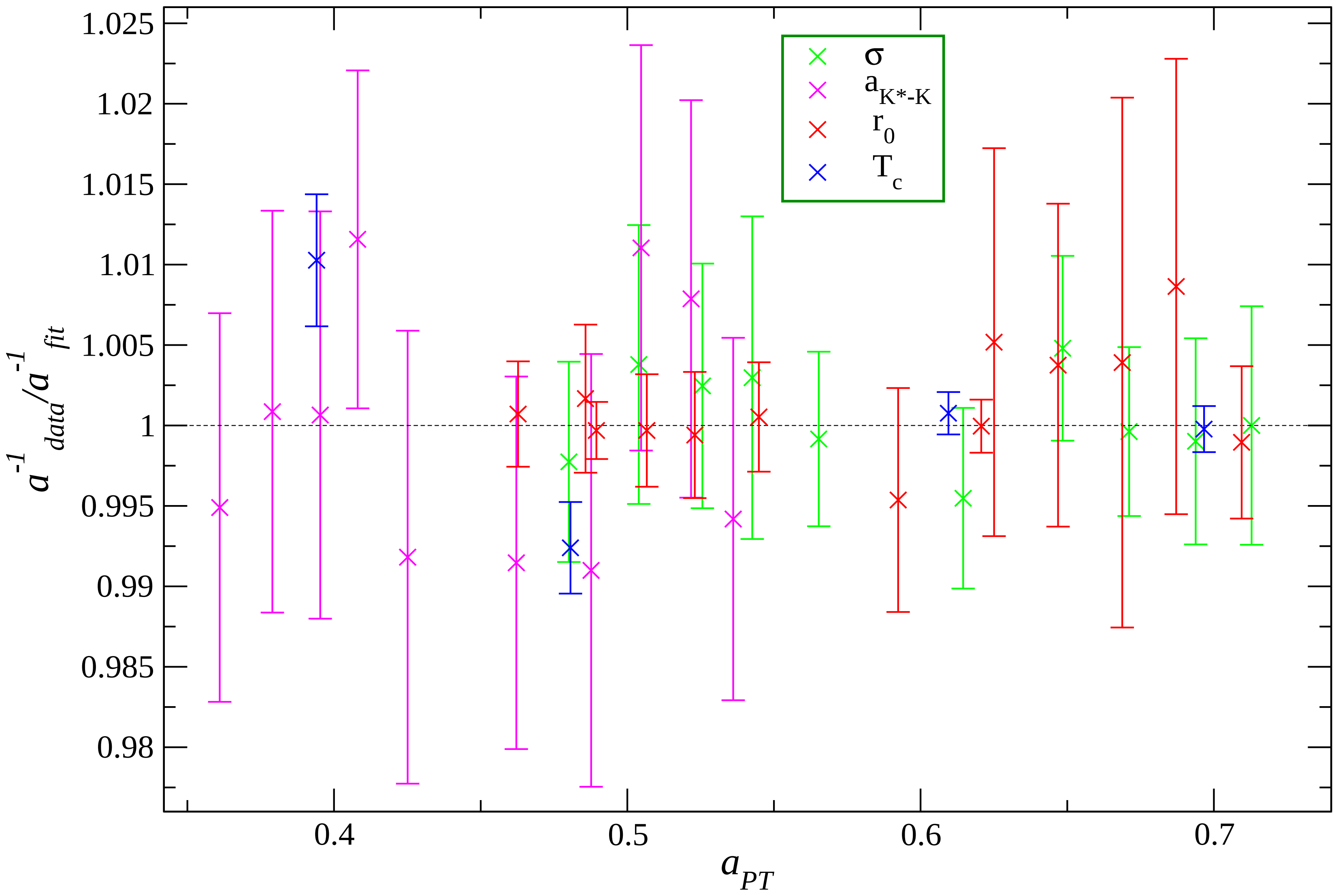
Plots: Wilson Case: Data/Fit



Plots: Iwasaki Case



Plots: Iwasaki Case: Data/Fit



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Attempt to *re-sum* the higher order terms by using a Monte Carlo quantity whose perturbative expansion is known to define a *renormalised* coupling. E.g.

$$1 - \langle \frac{1}{3} Tr U_{plaq} \rangle \quad = \quad c_1 g_0^2 + c_2 g_0^4 + \ldots$$

So, define g_E using:

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So, define g_E using:

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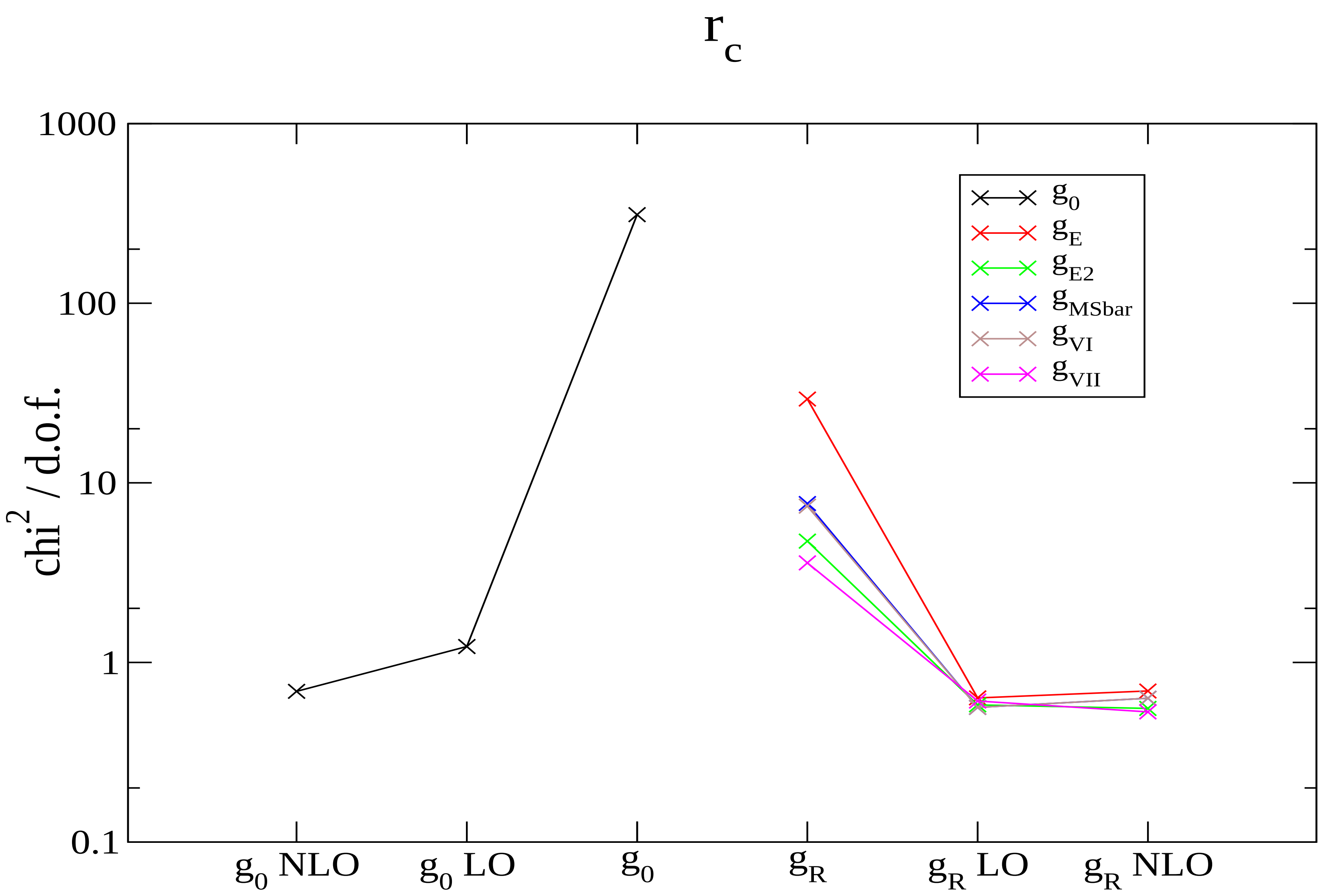
- Combine g_{renorm} with LDPT

Edwards, Heller, Klassen 1997

Renormalised Coupling Case - g_E

$\mathcal{O}(a)$ Order	PT Order	Quantity	Λ_L[Mev]	X	Y	χ^2/dof	$\Lambda_{\overline{MS}}$[Mev]
zero	2 Loop	r_c	17.42(3)			31	241.6(4)
zero	2 Loop	T_c	16.435(4)			694	227.96(5)
zero	2 Loop	σ	15.51(2)			71	215.1(3)
zero	3 Loop	r_c	17.65(3)			29	244.9(4)
zero	3 Loop	T_c	16.691(4)			632	231.50(5)
zero	3 Loop	σ	15.74(3)			68	218.3(4)
LO	2 Loop	r_c	18.08(5)	0.051(3)		0.6	250.8(7)
LO	2 Loop	T_c	17.04(2)	0.0129(3)		253	236.4(2)
LO	2 Loop	σ	16.51(6)	0.033(2)		15	229.1(8)
LO	3 Loop	r_c	18.30(5)	0.049(3)		0.6	253.8(8)
LO	3 Loop	T_c	17.27(2)	0.0122(3)		247	239.6(2)
LO	3 Loop	σ	16.73(6)	0.032(2)		14	232.1(8)
NLO	2 Loop	r_c	18.05(8)	0.04(2)	0.005(12)	0.7	250(1)
NLO	2 Loop	T_c	18.34(6)	0.094(3)	-0.0195(8)	1.7	254.3(8)
NLO	2 Loop	σ	17.33(11)	0.097(7)	-0.018(2)	0.38	240(2)
NLO	3 Loop	r_c	18.26(8)	0.04(2)	0.007(12)	0.7	253(1)
NLO	3 Loop	T_c	18.57(6)	0.093(3)	-0.0192(8)	1.8	257.5(8)
NLO	3 Loop	σ	17.6(1)	0.095(7)	-.018(2)	0.39	243(2)

Plots of other g_{renorm}: r_c



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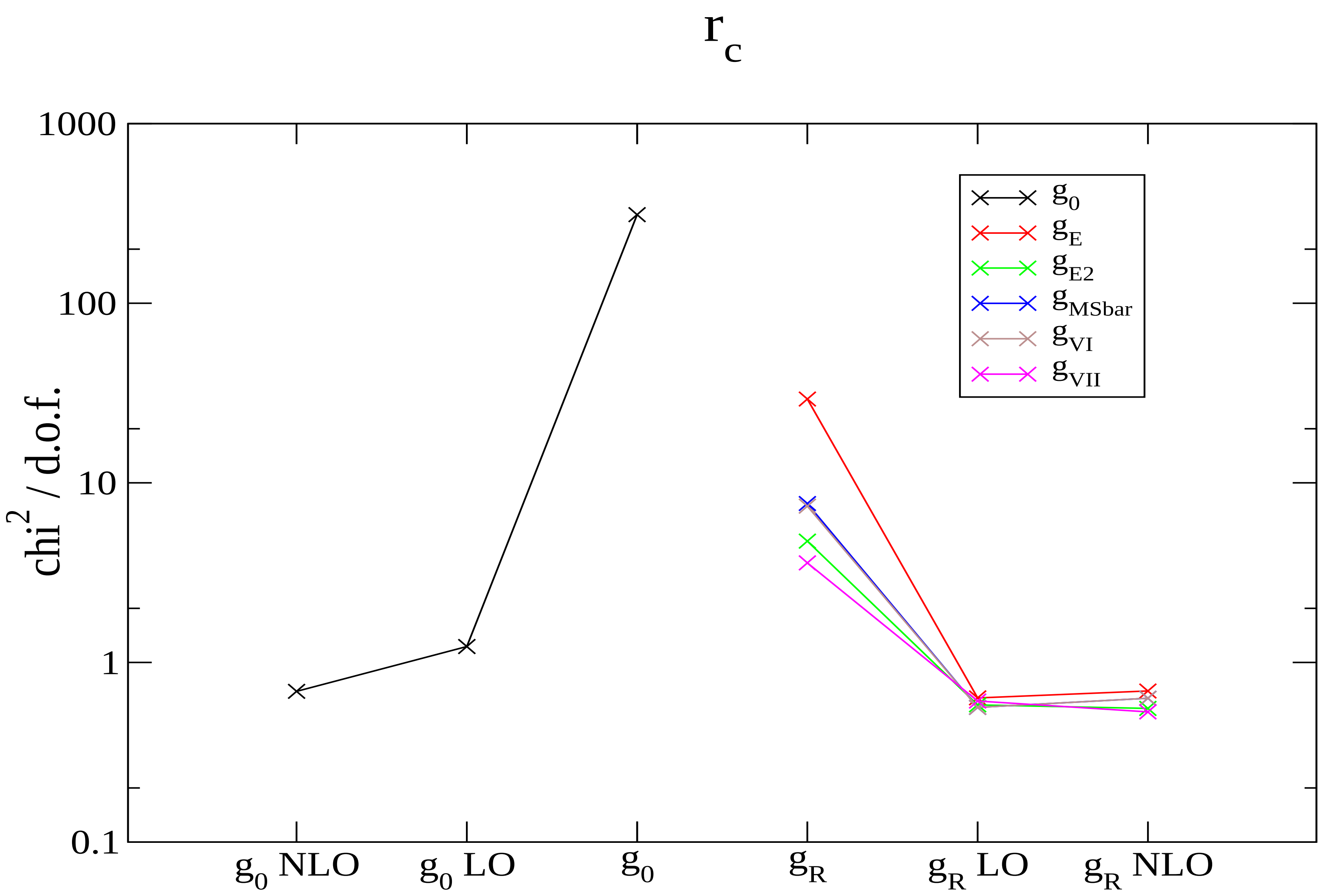
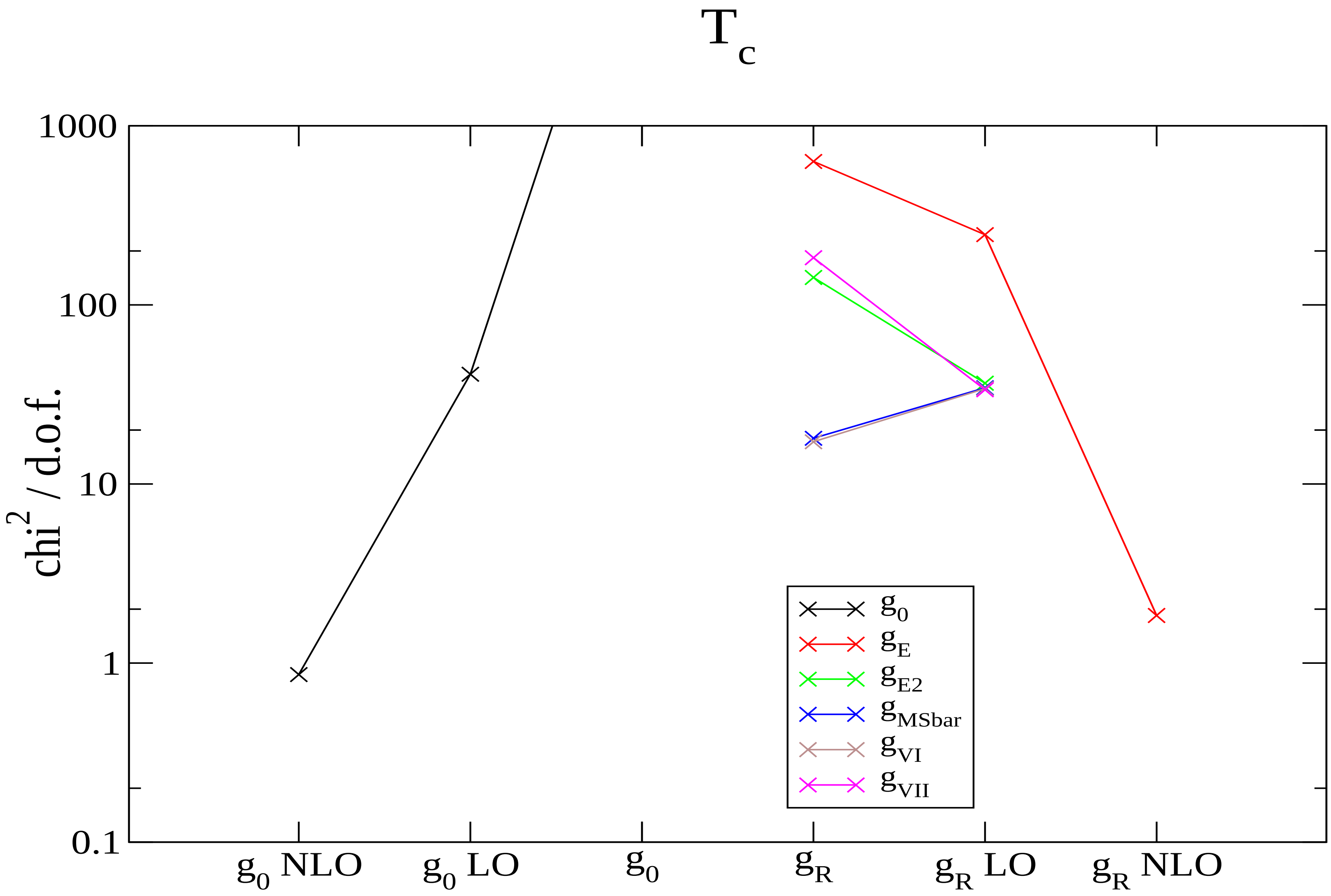
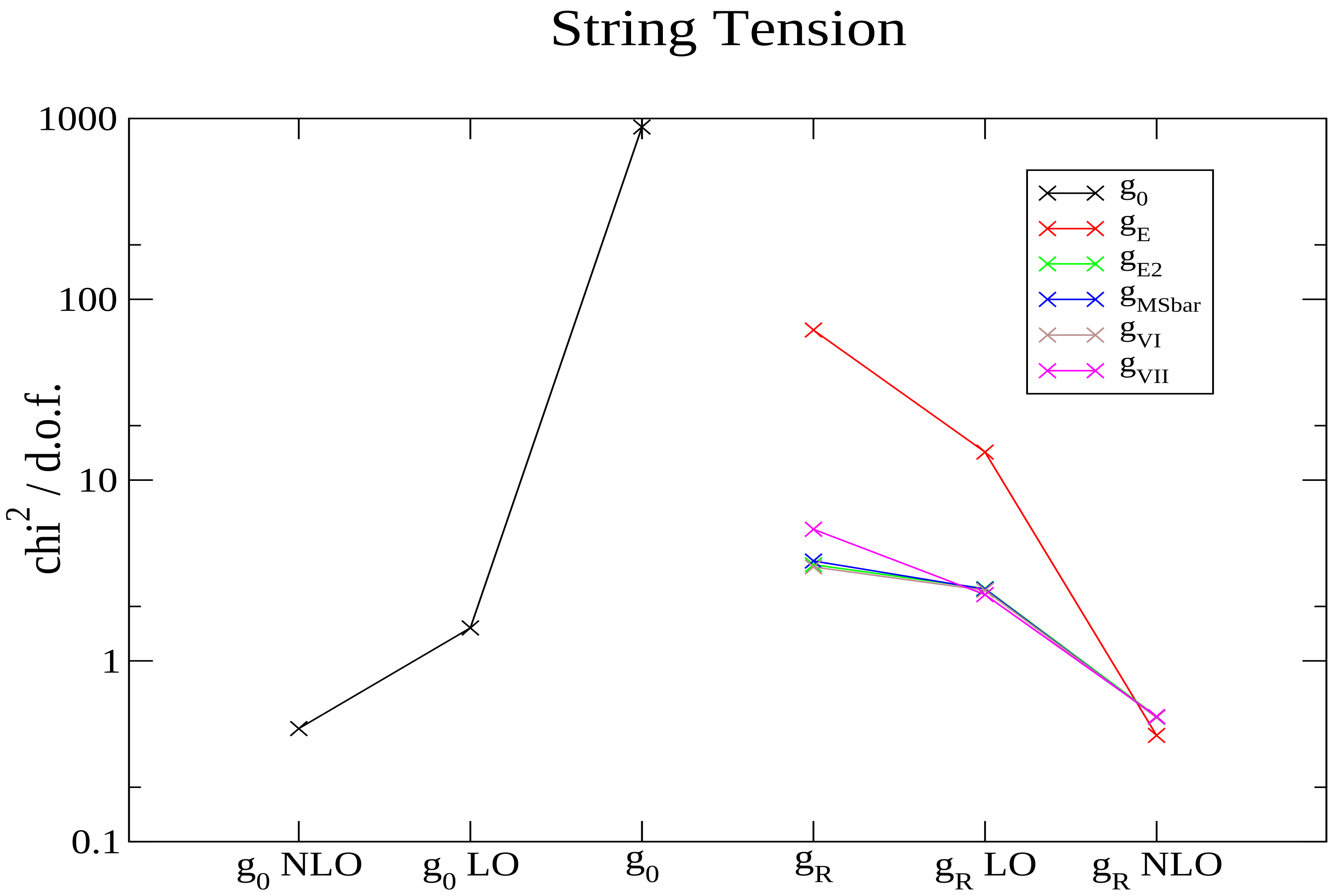


Note that g_E is only coupling known to 3-loops

Plots of other g_{renorm}: T_c



Plots of other g_{renorm}: σ



Other g_{renorm}: Summary

Requiring:

- $200MeV \leq \Lambda_{\overline{MS}} \leq 250MeV$
- $d_2^L \leq 0.20$ (= g_0 value for d_2^L)
- considering g_{renorm} zeroth order fits only

leaves g_E and g_{VII} only

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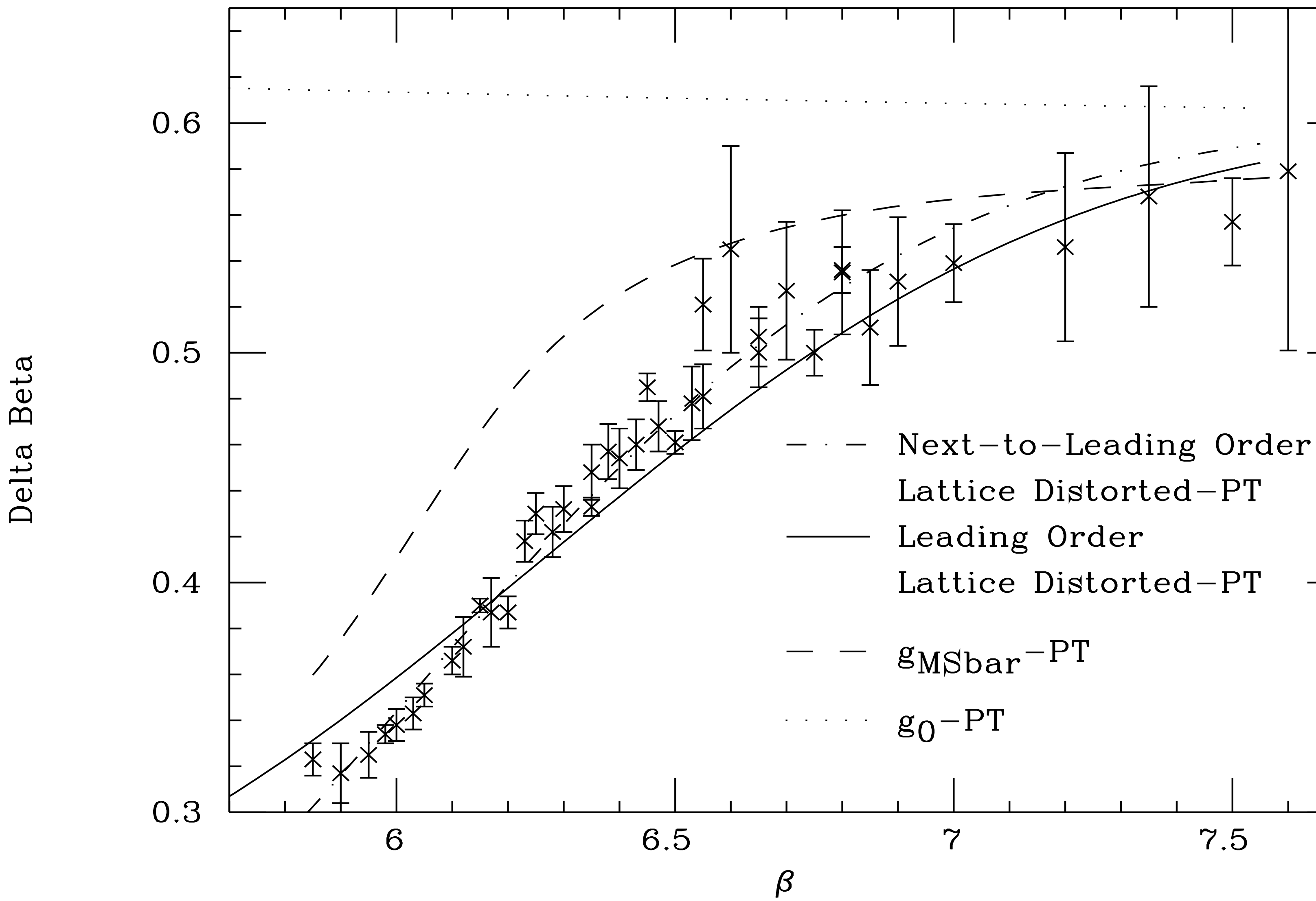
leaves g_E only

- demanding $\chi^2/dof < 10$

leaves none $\longrightarrow$ $g_{renorm} + (N)LO$

(Old) $\Delta\beta(\beta)$ Plot

$$a(\beta - \Delta\beta(\beta)) = 2a(\beta)$$



CRA hep-lat/9610016

SU(N_{col}) Case

String Tension (and some T_c) data for SU(2), SU(4), SU(6) & SU(8) taken from:

Lucini, Teper & Wenger 2003, 2004, 2005 Lucini & Teper 2001 Lucini & Teper 2006 priv.comm.

SU(N_{col}) Case

String Tension (and some T_c) data for SU(2), SU(4), SU(6) & SU(8) taken from:

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Recall:

$$\Lambda a(g^2) = e^{\frac{-1}{2b_0 g^2}} \, (b_0 g^2)^{\frac{-b_1}{2b_0^2}} \, (1 + \frac{1}{2b_0^3}(b_1^2 - b_2^L b_0) g^2)$$

$$\begin{aligned} b_0 &\sim \mathcal{O}(N_{col}) \\ b_1 &\sim \mathcal{O}(N_{col}^2) \\ b_2^L &\sim \mathcal{O}(N_{col}^3) \end{aligned}$$

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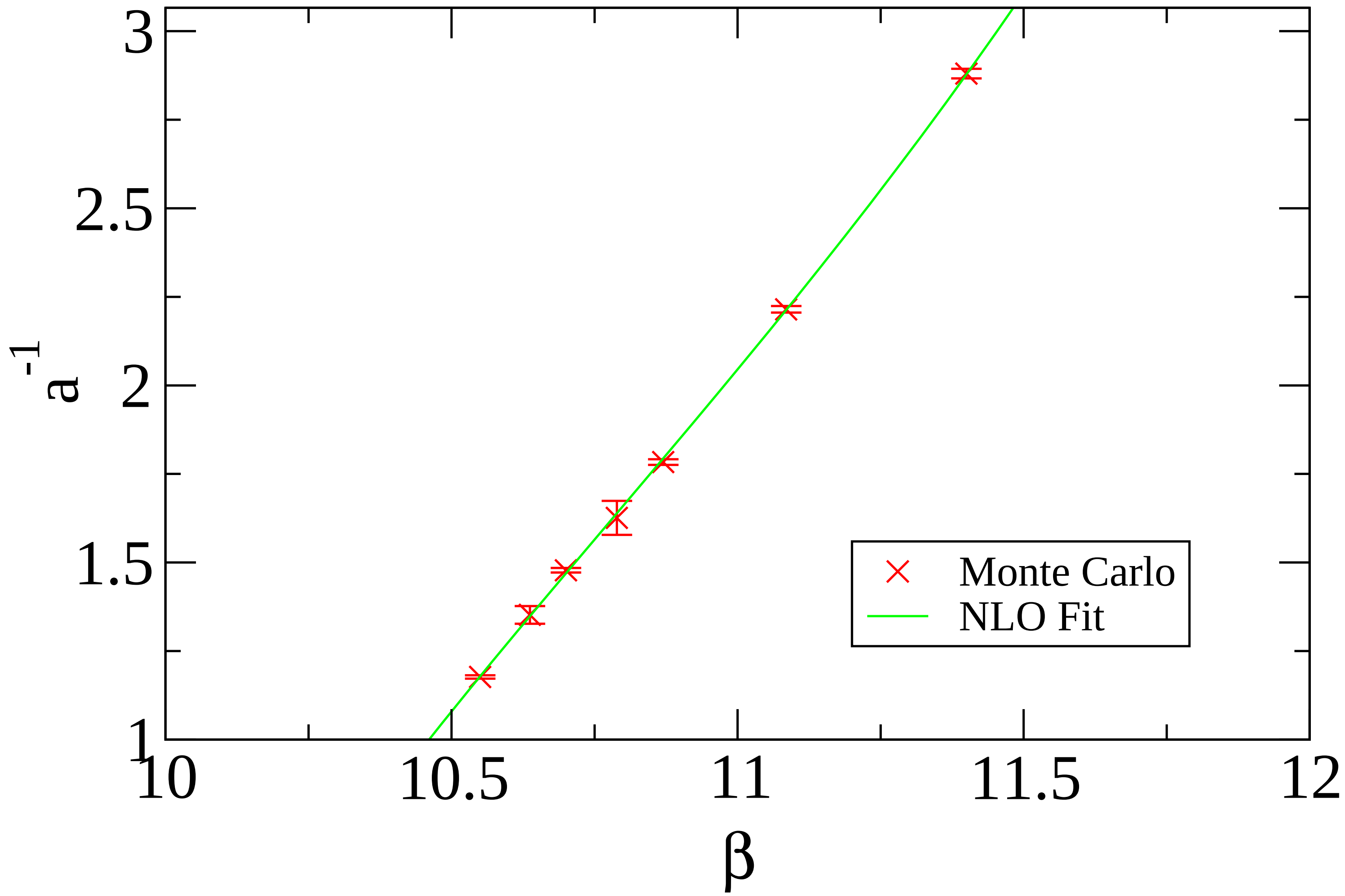
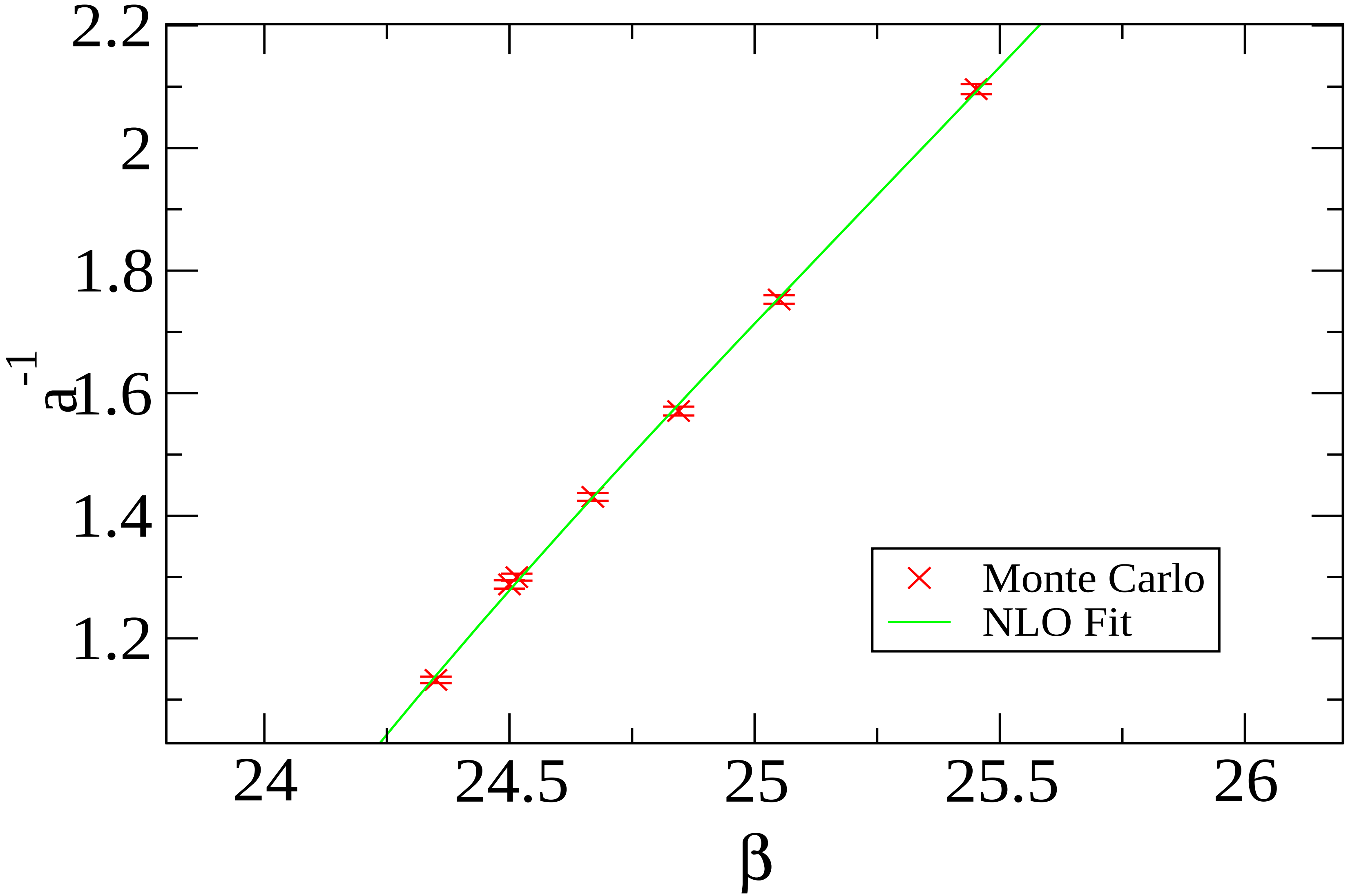
- expect g_0-PT to get worse with N_{col} (?)
- typically $\beta \nearrow$ as $N_{col} \nearrow$

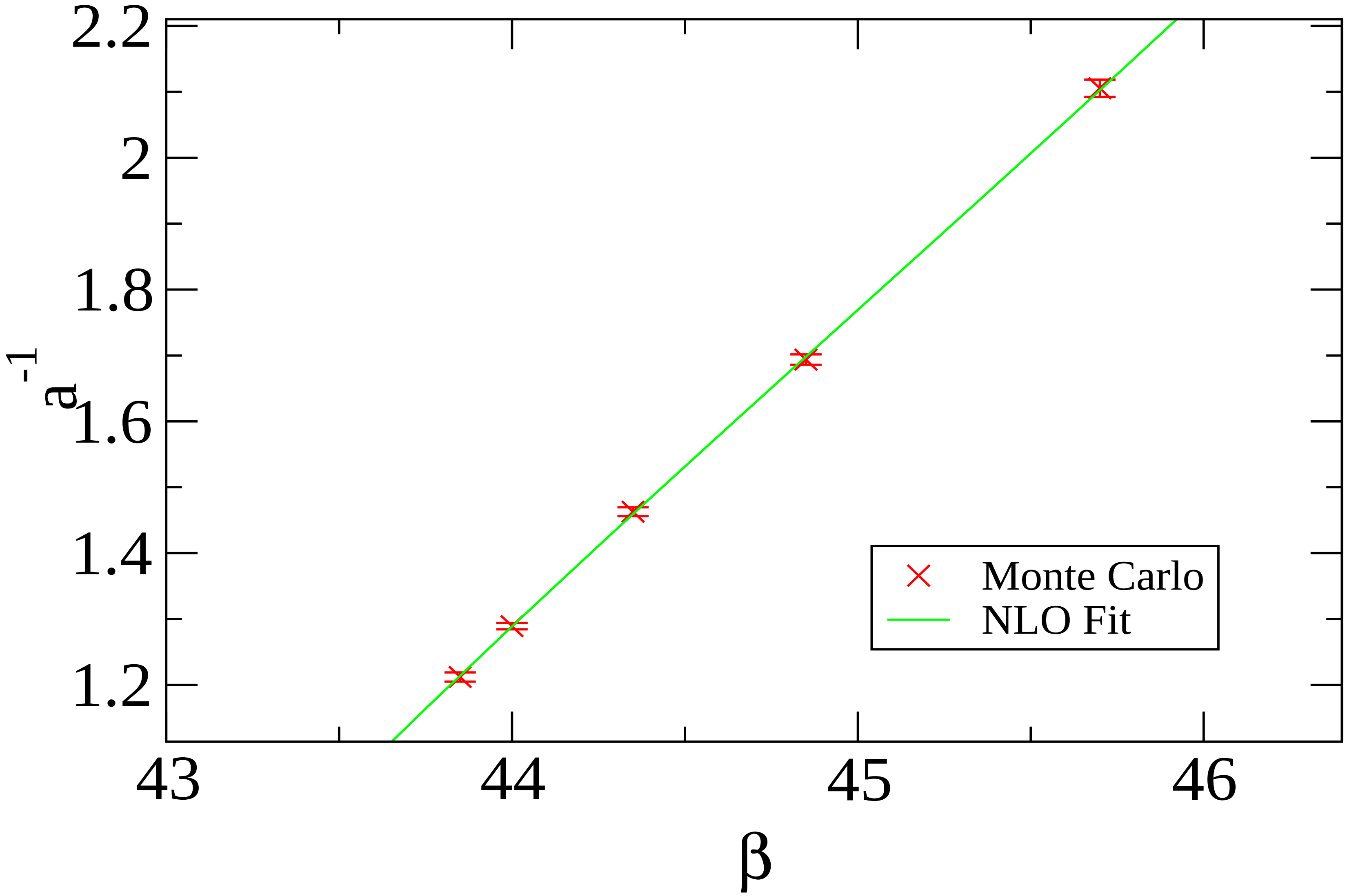
SU(4): string tension data + fit



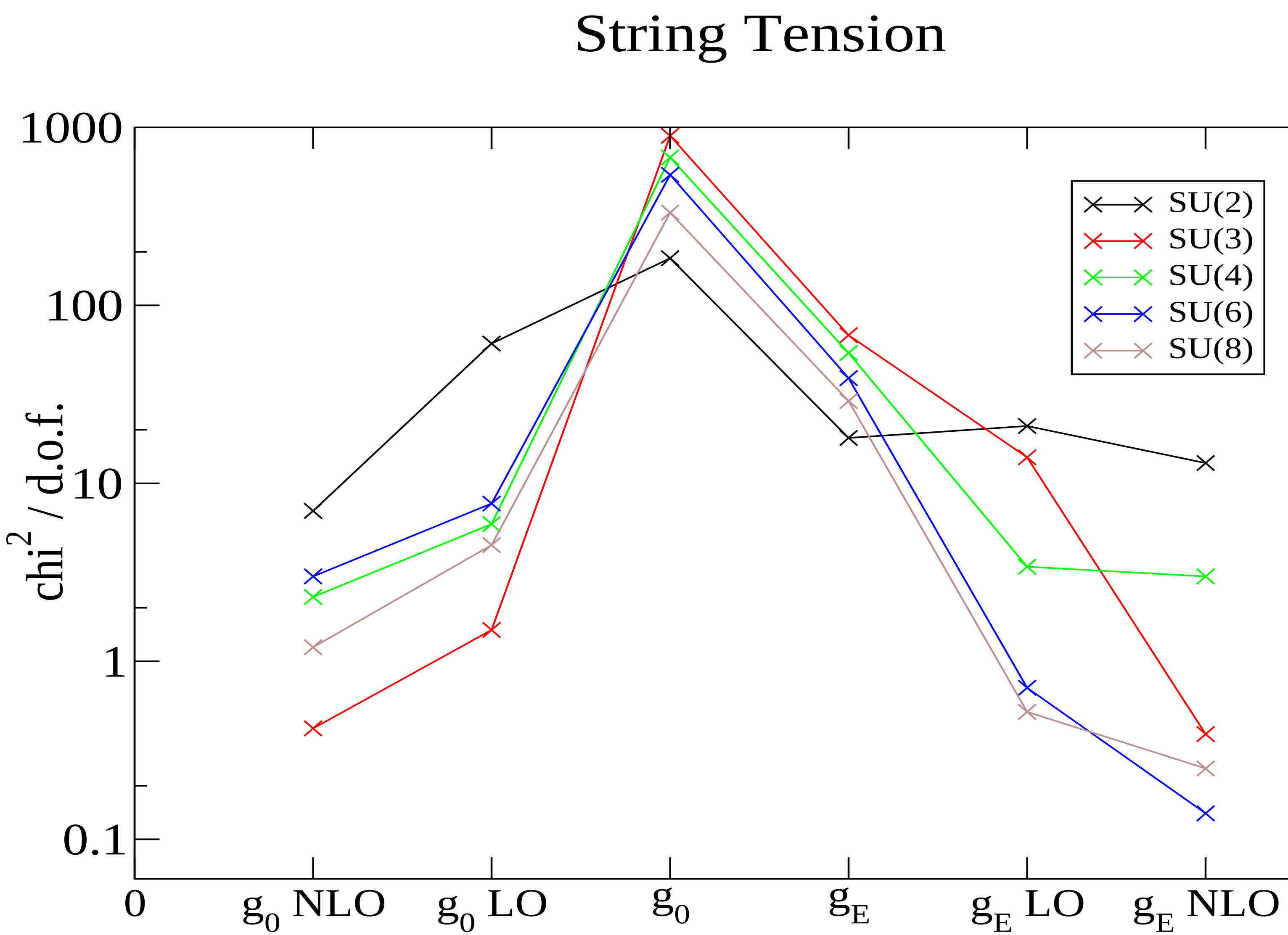
SU(6): string tension data + fit



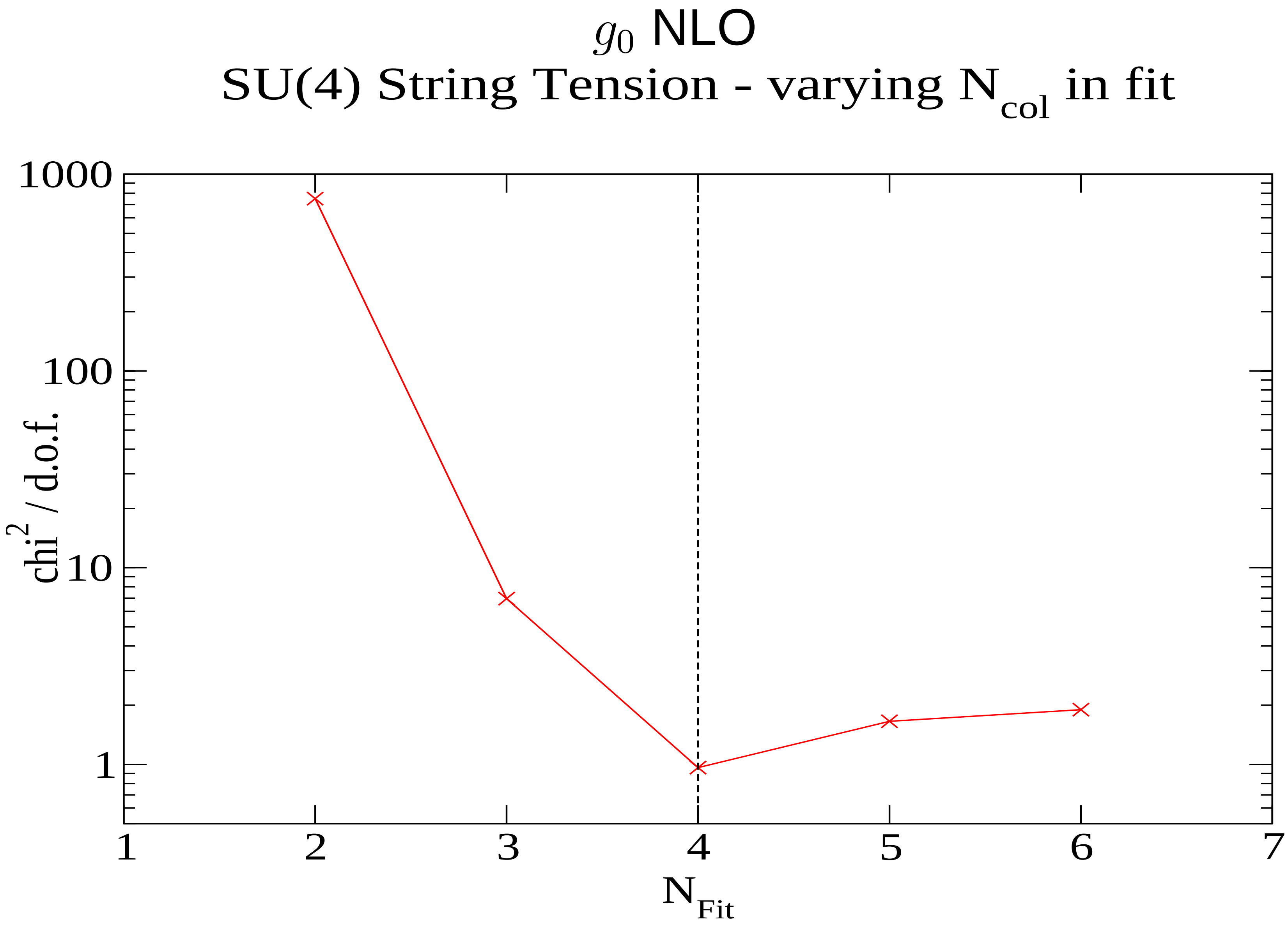
SU(8): string tension data + fit



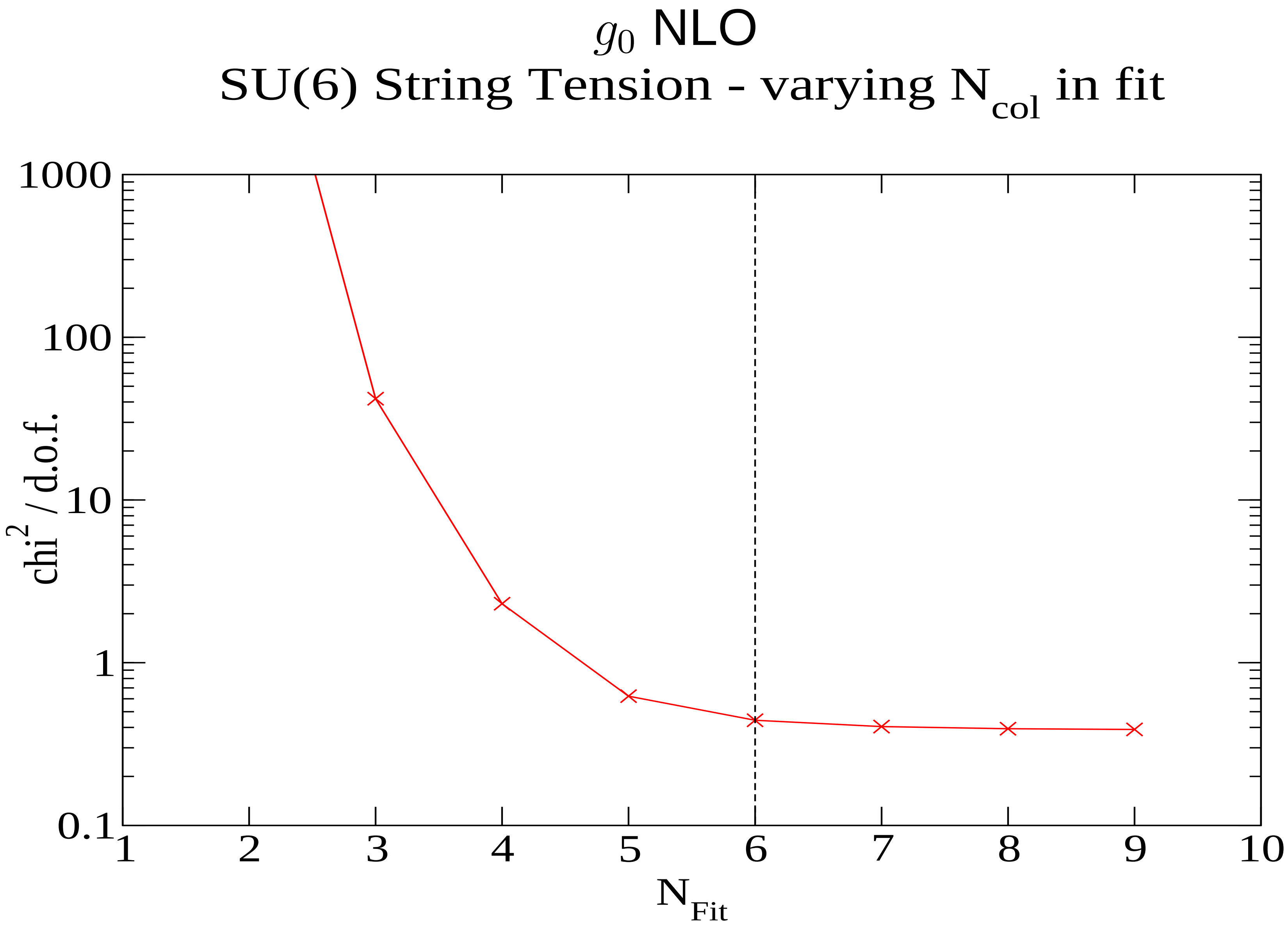
SU(N_{col}): string tension (3 Loop)



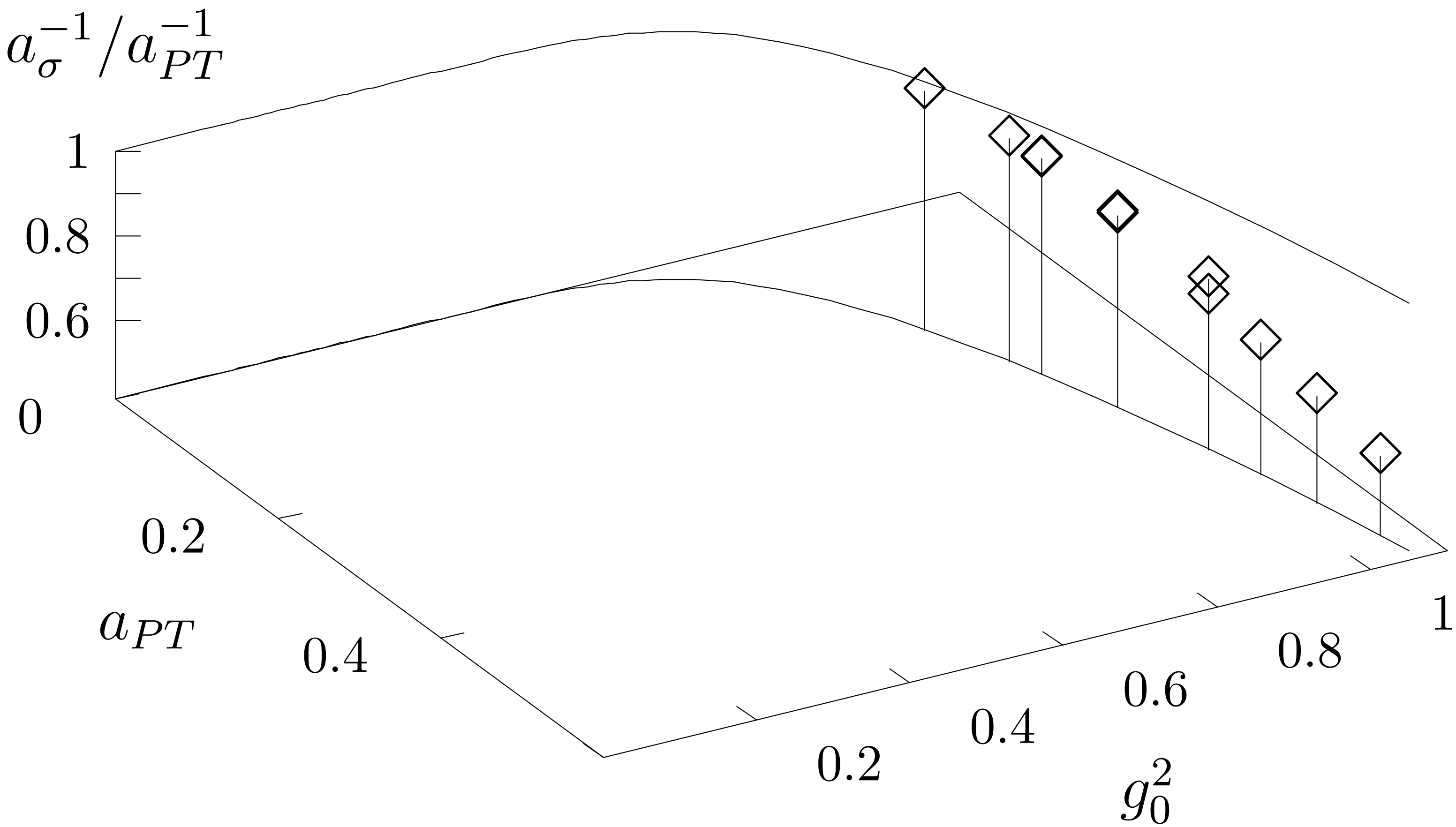
$SU(4)$ - varying N_{col} in fit



$SU(6)$ - varying N_{col} in fit



3-dimensional view: SU(3)



CRA hep-lat/9610016

Physical Predictions in Continuum Limit

Want lattice prediction of Ω in continuum:

$$\Omega = \lim_{g_0 \to 0} \left[Z^{Ren}(g_0^2) \times \Omega^{\#}(g_0^2) \times a^{-1}(g_0^2) \right]$$

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- $\Omega^{\#} a^{-1} = \Omega^{expt}$, so we can fit $1/\Omega^{\#}$ as we have a^{-1}:

$$\frac{1}{\Omega^{\#}(g_0^2)} = \frac{\lambda_\Omega}{f_{PT}(g_0^2)} \times [1 - X\mathcal{O}(a)]$$

giving

$$\Omega = \frac{\Lambda_L}{\lambda_\Omega}$$

Conclusions

- LDPT is best way of interpolating the data (i.e. smallest χ^2) - may even be correct!
- “Has” to be this way - people do continuum fits of $M(a) = M(0)(1 + Xa)$ all the time!

Note $$M(a) \equiv M(a)^{\#} a^{-1} \equiv \frac{M(a)^{\#}}{\Omega(a)^{\#}} \Omega^{expt}$$

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Future Work:

- More simulations for SU(N)
- ? Apply to Dynamical Simulations ?