Lattice Study of Nuclear Forces

Noriyoshi Ishii

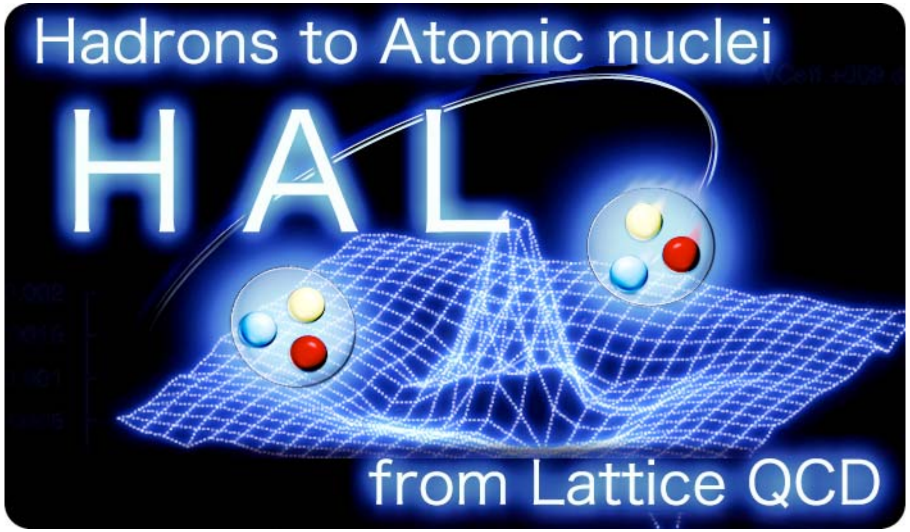
(Univ. of Tokyo)

for

PACS-CS Collaboration

and

HAL QCD Collaboration



S.Aoki (Univ. of Tsukuba),
T.Doi (Univ. of Tsukuba),
T.Hatsuda (Univ. of Tokyo),
T.Inoue (Univ. of Tsukuba),
Y.Ikeda (Univ. of Tokyo),
K.Murano (Univ. of Tokyo),
K.Nemura (Tohoku Univ.)
K.Sasaki (Univ. of Tsukuba)

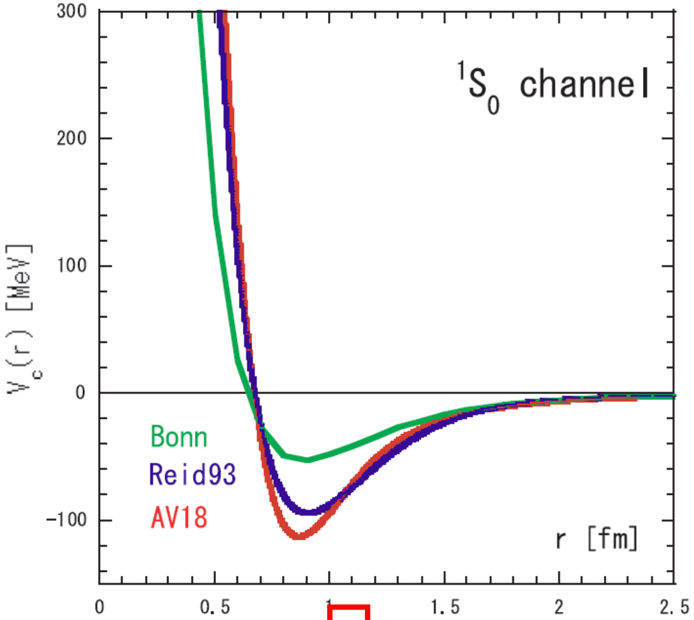
Introduction

Nuclear Force

The starting point of nuclear physics

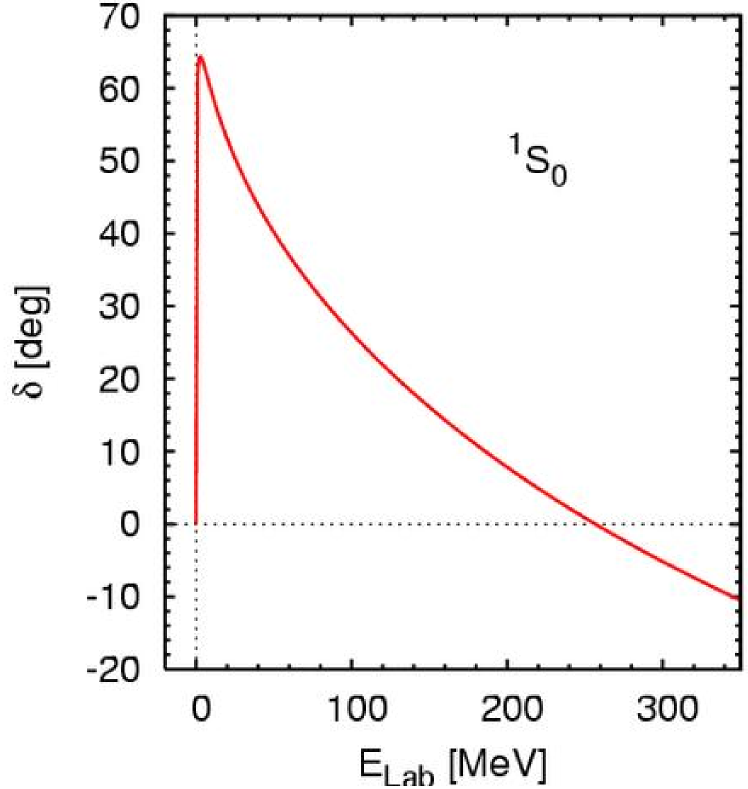
$$\left(-\frac{\vec{\nabla}_p^{\,2}}{2m_p}-\frac{\vec{\nabla}_n^{\,2}}{2m_n}+V_{NN}\right)\psi(\vec{x}_p,\vec{x}_n)=E\,\psi(\vec{x}_p,\vec{x}_n)$$

Realistic nuclear force



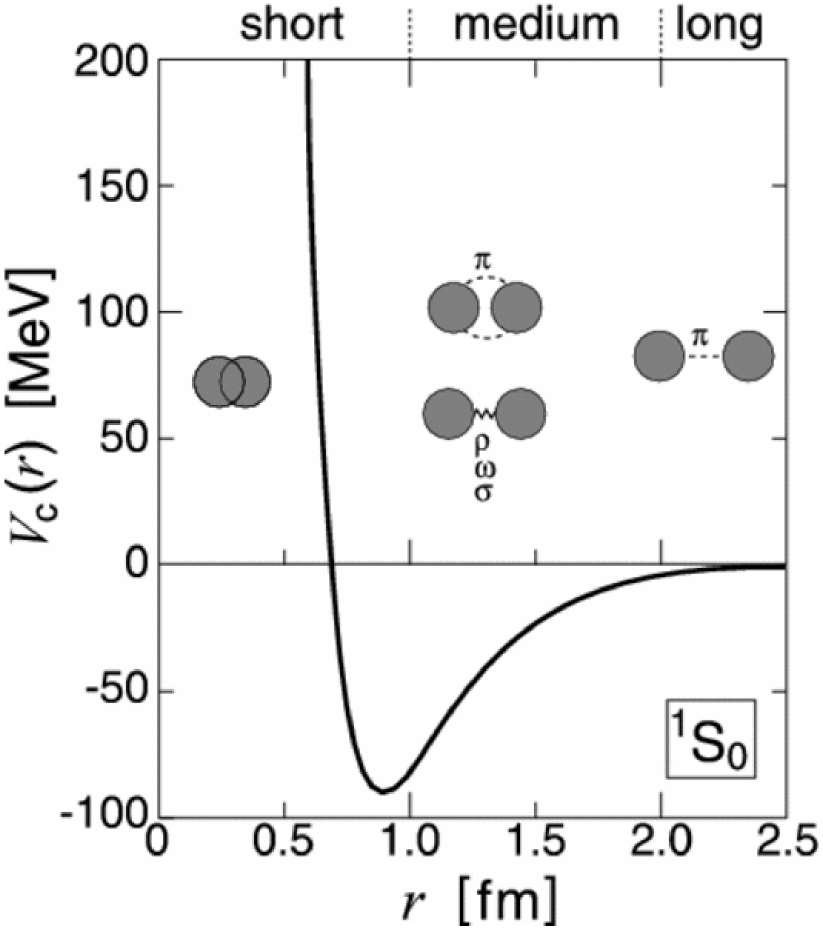
~4000 data with 18 parameters
χ^2/dof ~ 1 (AV18)

Experimental NN data:
Phase shifts (E_{Lab} < 350 MeV)
deuteron



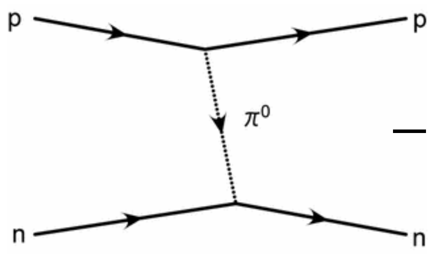
Structure and reactions of nuclei,
Nuclear matter, eq. of states,
neutron star, supernova, etc.

Nuclear Force



- **Long distance (r > 2 fm)**

  OPEP [H.Yukawa(1935)]
  (One Pion Exchange)



$$-\frac{g_{\pi NN}^{\;2}}{4\pi}\frac{e^{-m_\pi r}}{r}$$

- **Medium distance (1 fm < r < 2fm)**

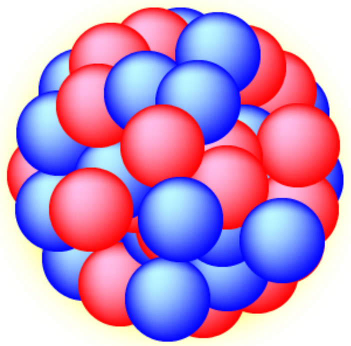
  multi-pion, $\rho, \omega, "\sigma", \cdots$
  Attraction ➔ essential for bound nuclei

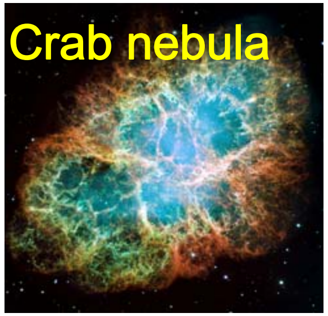
- **Short distance (r < 1 fm)**

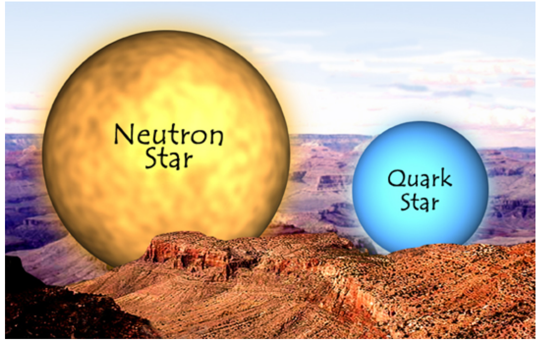
  Repulsive core [R.Jastrow(1950)]

The repulsive core

It is important for a lot of phenomena.



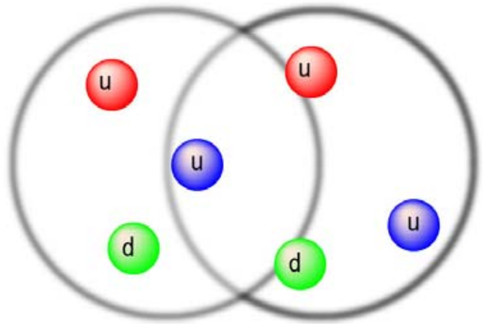




Its physical origin is still an open problem in nuclear physics

- vector meson exchange
- Pauli forbidden state + color magnetic interaction
- etc.

Two nucleons overlap at such short distance.
➔ A consequence of the structure of nucleon.



QCD is expected to play an important role.

Lattice QCD approaches to nuclear force (hadron potential)

There are (essentially) two approaches:

- Method which utilizes the static quarks

D.G.Richards et al., PRD42, 3191 (1990).
A.Mihaly et la., PRD55, 3077 (1997).
C.Stewart et al., PRD57, 5581 (1998).
C.Michael et al., PRD60, 054012 (1999).
P.Pennanen et al, NPPS83, 200 (2000),
A.M.Green et al., PRD61, 014014 (2000).
H.R Fiebig, NPPS106, 344 (2002); 109A, 207 (2002).
T.T.Takahashi et al, ACP842,246(2006),
T.Doi et al., ACP842,246(2006)
W.Detmold et al.,PRD76,114503(2007)

- Method which utilizes the Bethe-Salpeter wave function

Ishii, Aoki, Hatsuda, PRL99,022011(2007).
Nemura, Ishii, Aoki, Hatsuda, PLB673,136(2009).
Aoki, Hatsuda, Ishii, CSD1,015009(2008).

Plan of the talk

- General Strategy and Derivative expansion
- Central potential
- How good is the derivative expansion ?
- Tensor potential
- Hyperon potential
- 2+1 flavor QCD results
- Summary and Outlook

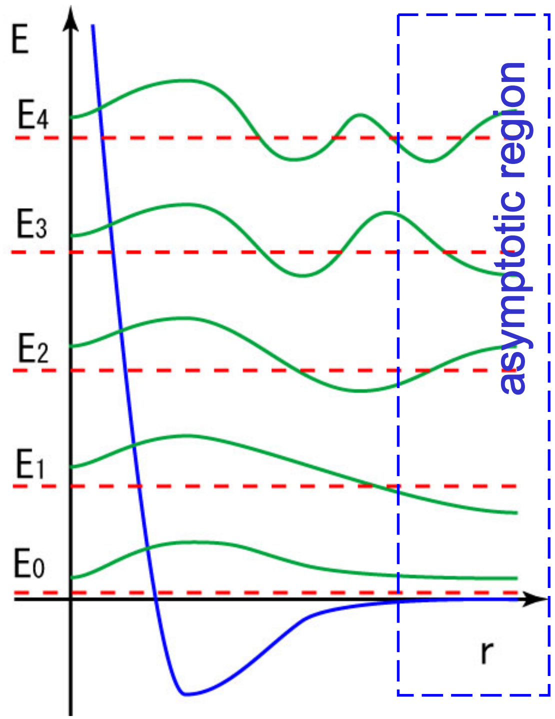
General Strategy

- **Bethe-Salpeter (BS) wave function (equal time)**

$$\psi(\vec{x}-\vec{y}) \equiv \lim_{t\to+0}\langle 0|T\left[N(x,t)N(y,0)\right]|NN\rangle$$

  - desirable asymptotic behavior as r → large.

$$\psi(\vec{r}) \approx A\frac{\sin\left(kr-\pi l/2+\delta_l(k)\right)}{kr}+\cdots$$



- **Definition of (E-independent non-local) potential**

$$(E-H_0)\psi_E(\vec{x}) \equiv \int d^3y\, U(\vec{x},\vec{y})\psi_E(\vec{y})$$

U(x,y) is defined by demanding
$\psi_E(\vec{x})$ at multiple energies E_n satisfy this equation **simultaneously**.)

Comments:

(1) **Exact phase shifts** at $E = E_n$

(2) As number of BS wave functions increases,
the potential becomes more and more faithful to the phase shifts.

(3) U(x,y) does **NOT** depend on energy E.

(4) U(x,y) is most generally **non-local**.

General Strategy: Derivative expansion

We obtain the non-local potential U(x,y) step by step

$S_{12} \equiv 3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})/r^2 - \vec{\sigma}_1 \cdot \vec{\sigma}_2$

- Derivative expansion of the non-local potential $U(\vec{x},\vec{y}) = V(\vec{x},\vec{\nabla})\ \delta(\vec{x}-\vec{y})$

$$V(\vec{x},\vec{\nabla}) = V_C(r) + V_T(r)\cdot S_{12} + V_{LS}(r)\cdot \vec{L}\cdot\vec{S} + \{V_D(r),\vec{\nabla}^2\} + \cdots$$

- Leading Order:
  Use BS wave function of the lowest-lying state(s) to obtain:

$$V(\vec{x},\vec{\nabla}) = V_C(r) + V_T(r)\cdot S_{12} + V_{LS}(r)\cdot \vec{L}\cdot\vec{S} + O(\vec{\nabla}^2)$$

Example ($^1S_0$): Only $V_C(r)$ survives for $^1S_0$ channel:

$$(E-H_0)\ \psi_E(\vec{x}) = V_C(r)\ \psi_E(\vec{x}) \quad \Rightarrow \quad V_C(r) = \frac{(E-H_0)\psi_E(\vec{x})}{\psi_E(\vec{x})}$$

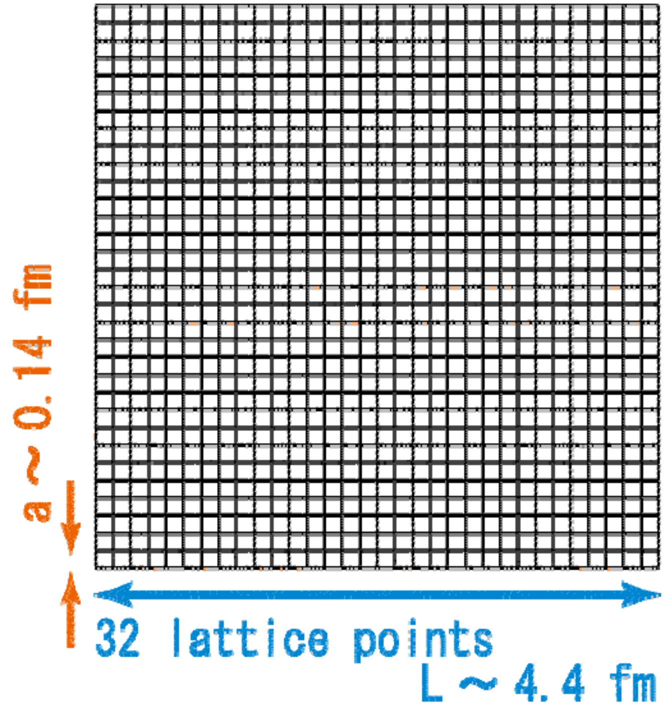
- Next to Leading Order:
  Include BS wave function of excited states to obtain $O(\vec{\nabla}^2)$ terms:

$$V(\vec{x},\vec{\nabla}) = V_C(r) + V_T(r)\cdot S_{12} + V_{LS}(r)\cdot \vec{L}\cdot\vec{S} + \{V_D(r),\vec{\nabla}^2\} + \cdots + O(\vec{\nabla}^3)$$

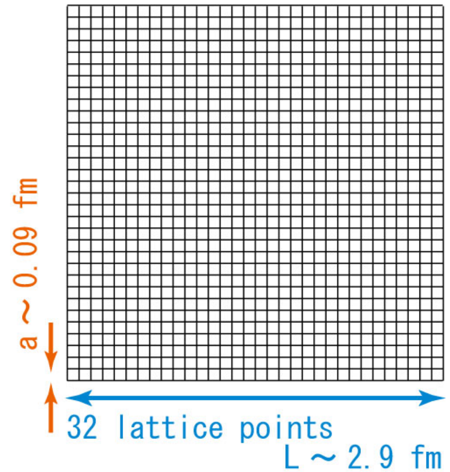
- Repeat this procedure to obtain higher derivative terms.

Numerical Setups

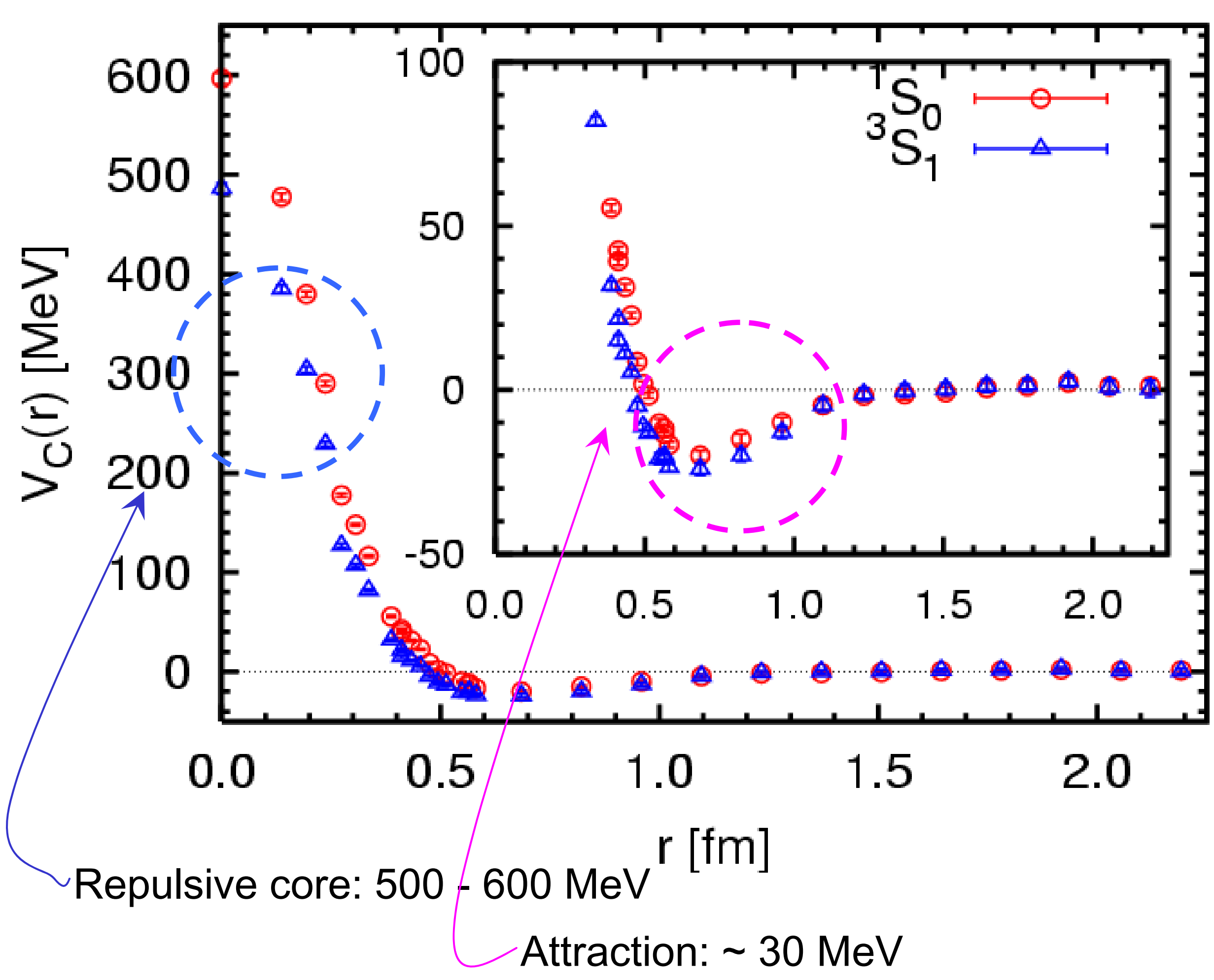
- Quenched QCD
  plaquette gauge + Wilson quark action
  m_{pi} = 380 − 730 MeV
  a=0.137 fm, L=32a=4.4fm

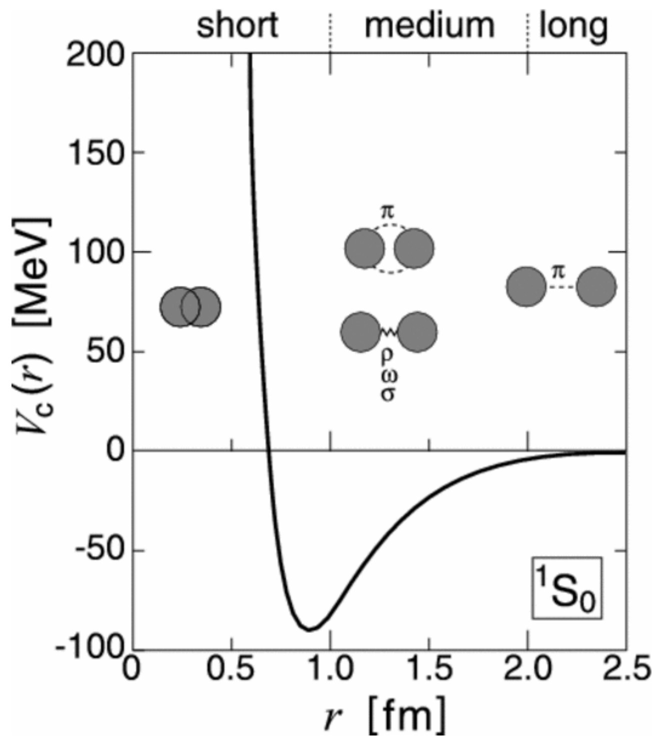


- 2+1 flavor QCD (by PACS-CS)
  Iwasaki gauge + clover quark action
  m_{pi} = 411 – 700 MeV
  a=0.091 fm, L=32a=2.9 fm



Central potential (leading order) by quenched QCD



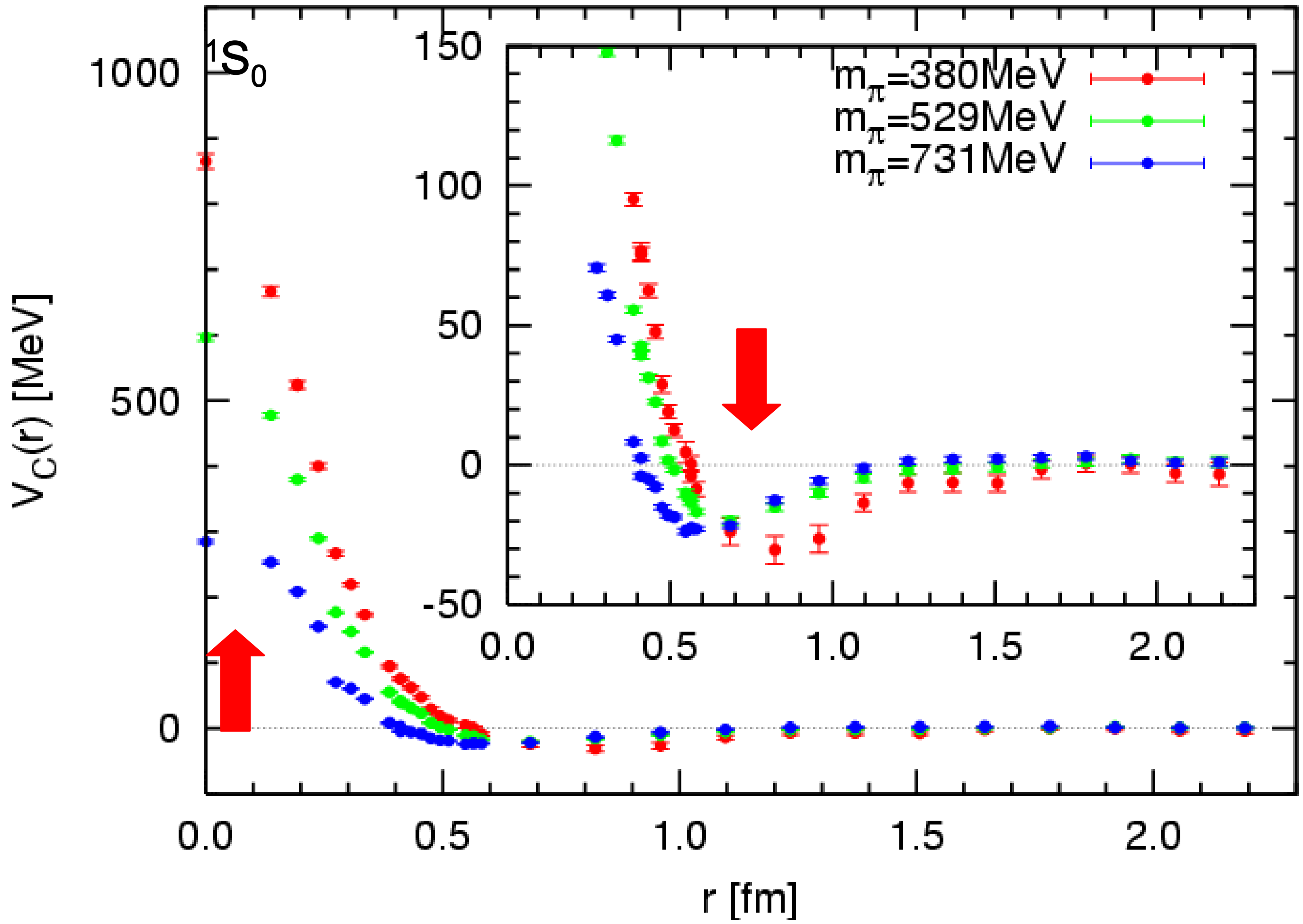


Qualitative features of the nuclear force are reproduced.

Ishii, Aoki, Hatsuda,
PRL99,022001(2007).

Quark mass dependence

Aoki,Hatsuda,Ishii,
CSD1,015009(2008).



In the light quark mass region,

✓ The repulsive core grows rapidly.

✓ Attraction gets stronger.

How good is the derivative expansion ?

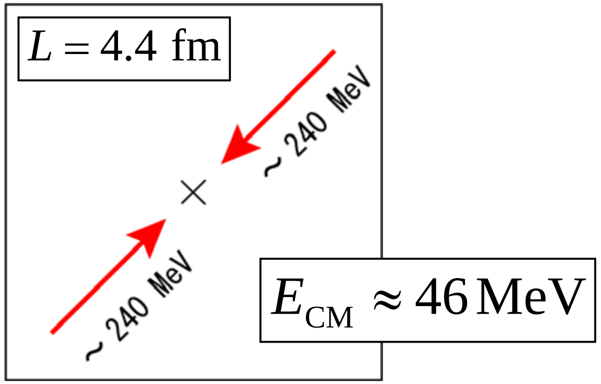
At leading order:

$$U(\vec{x},\vec{y}) = \left(V_C(r) + V_T(r)\cdot S_{12} + V_{LS}(r)\cdot \vec{L}\cdot\vec{S} + \{V_D(r),\vec{\nabla}^2\} + \cdots\right)\delta(\vec{x}-\vec{y})$$

Strategy:

- Compare two potentials at leading order at different energies

Potential at $E \neq 0$ is easily obtained by anti-periodic BC

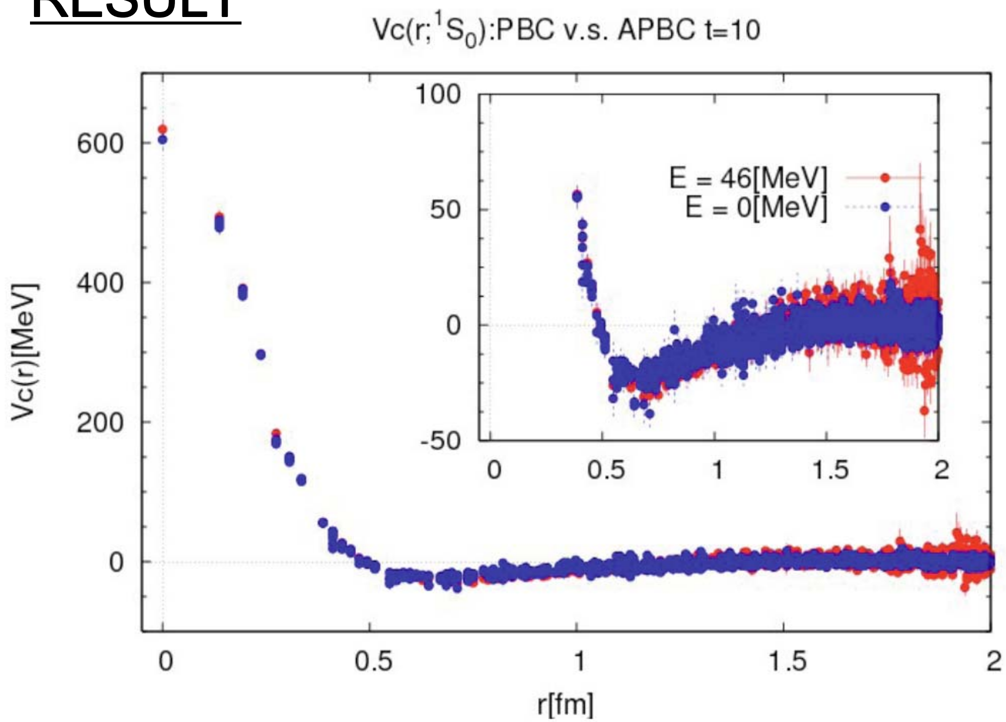


- Difference ←→ size of higher order effects
- Small difference
  ➔ small higher order effects
  ➔ leading order potential at E~0 MeV serves as a good starting point for the E-independent non-local potential U(x,y)

How good is the derivative expansion ? (cont'd)

[Murano, 26 July, 14:10]

RESULT

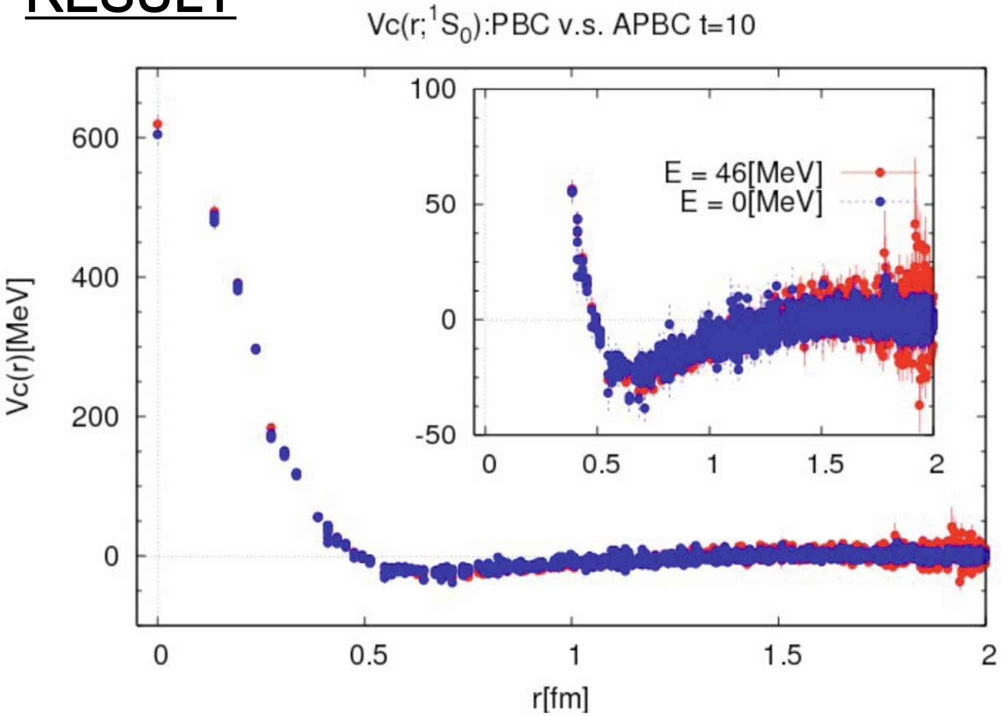


➢ Small discrepancy at short distance. (really small)

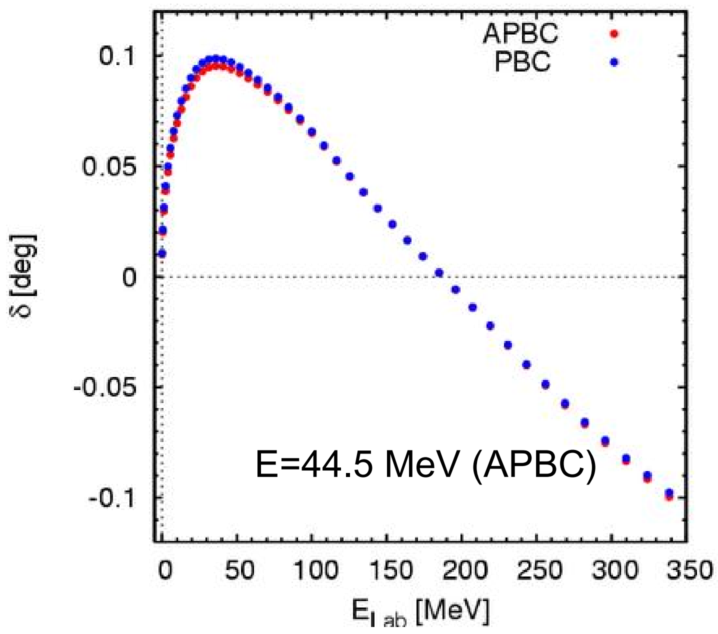
How good is the derivative expansion ? (cont'd)

[Murano, 26 July, 14:10]

RESULT



phase shifts from potentials

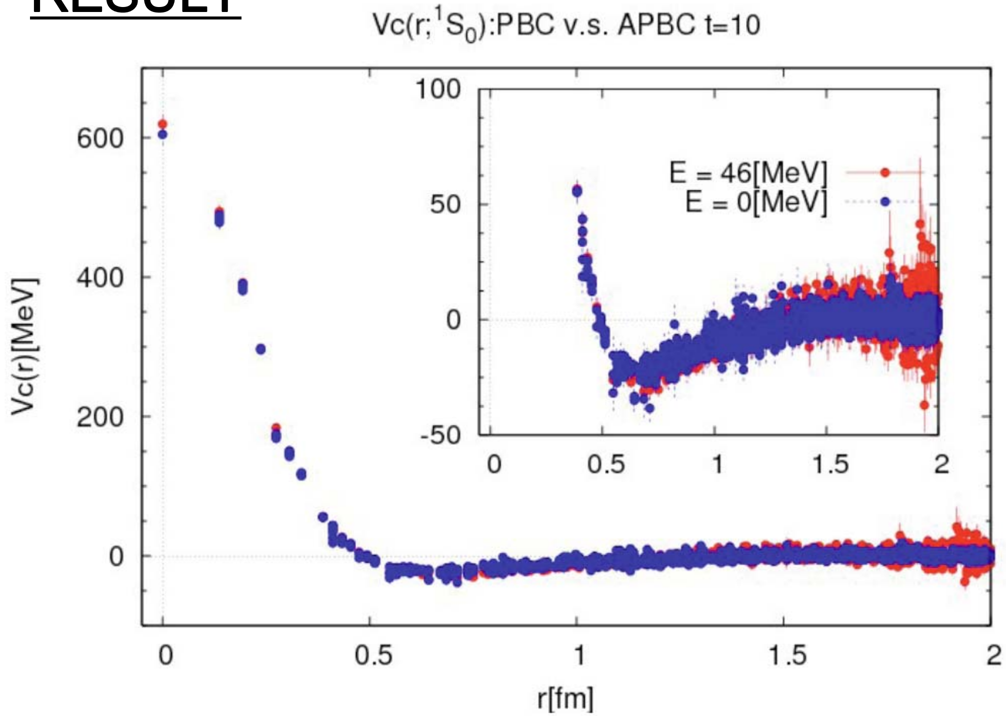


- Small discrepancy at short distance. (really small)
- With a fine tuning of E for APBC, two phase shifts agree

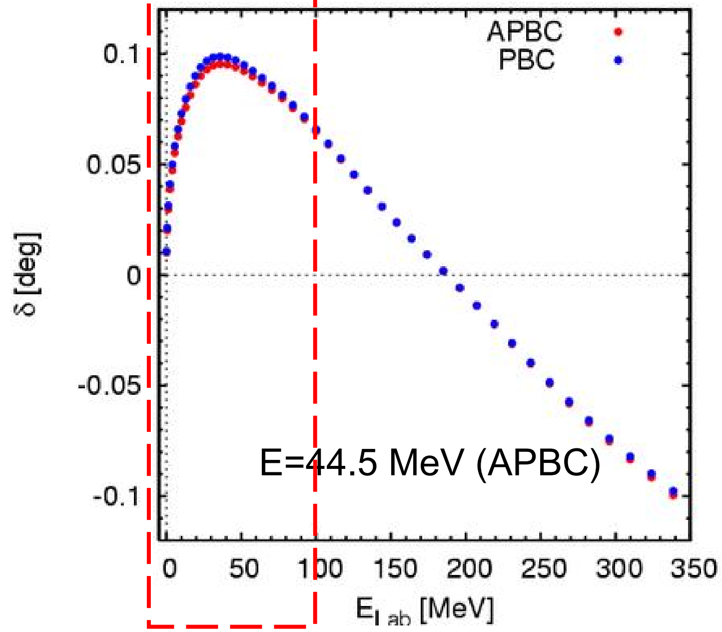
How good is the derivative expansion ? (cont'd)

[Murano, 26 July, 14:10]

RESULT



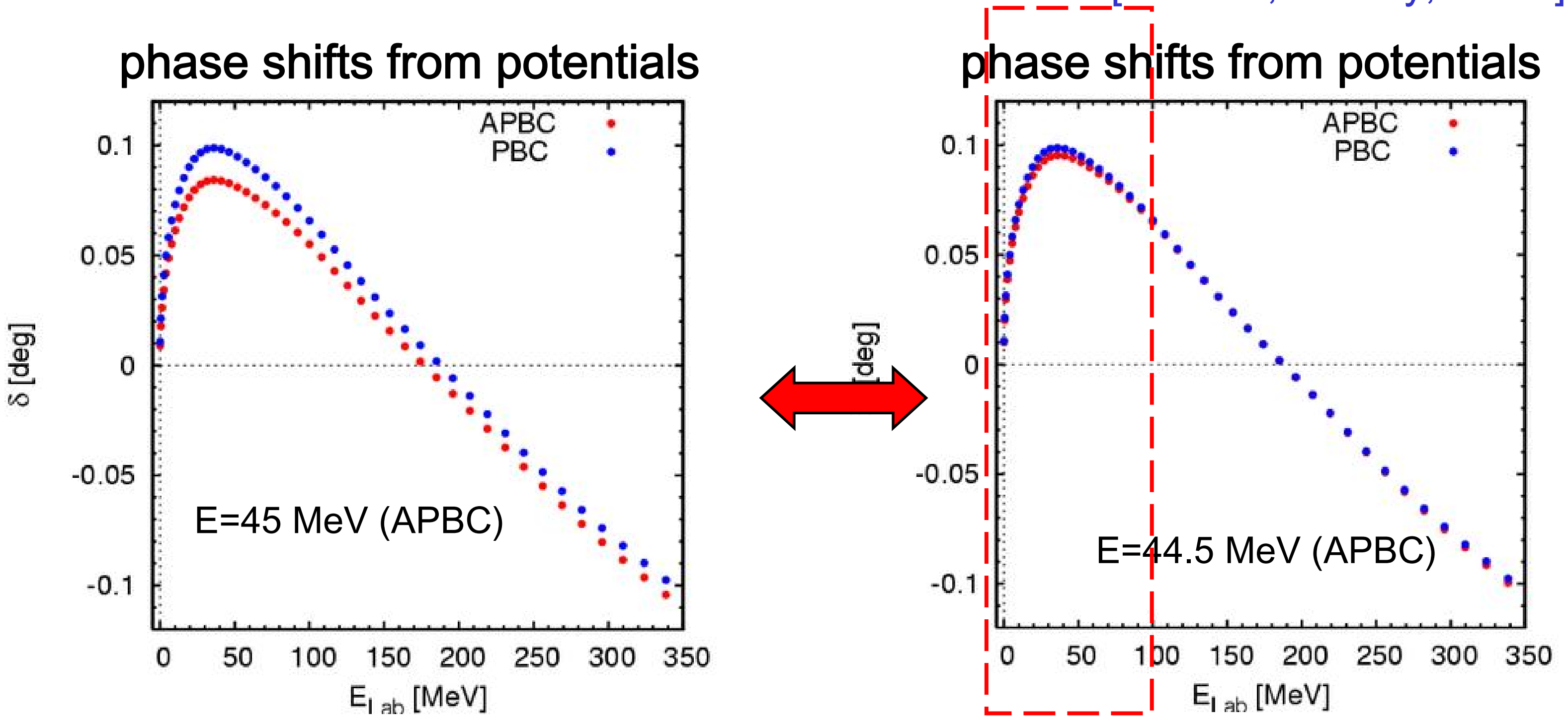
phase shifts from potentials



- Small discrepancy at short distance. (really small)
- With a fine tuning of E for APBC, two phase shifts agree
- Derivative expansion works.
- Potentials at the leading order serves as a good starting point for the E-independent non-local potential for E_{CM}=0−46 MeV.

How good is the derivative expansion ? (cont'd)

[Murano, 26 July, 14:10]



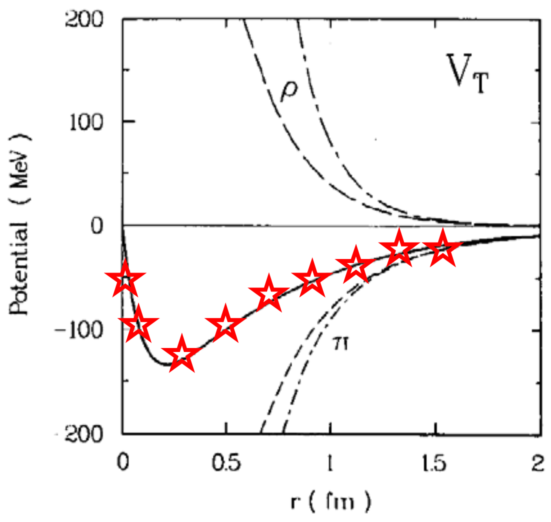
- ➢ Small discrepancy at short distance. (really small)
- ➢ With a fine tuning of E for APBC, two phase shifts agree
- ➢ Derivative expansion works.
- ➢ Potentials at the leading order serves as a good starting point for the E-independent non-local potential for E_{CM}=0−46 MeV.
- ➢ The phase shift is quite sensitive to the precise value of E for APBC. (At the moment, it is not easy to calculate E_{CM} to such a good accuracy.)

Tensor Potential

Tensor potential

Background

- Phenomenologically important for
  - Nuclear saturation density and stability of nuclei.
  - Huge influence on the structures of nuclei
  - Mixing of s-wave and d-wave ➔ deuteron
- Its form is a consequence of cancellation between π and ρ (OBEP)



from R.Machleidt, Adv.Nucl.Phys.19

Fig. 3.7. The contributions from π and ρ (dashed) to the $T = 0$ tensor potential. The solid line is the full potential. The dash-dot lines are obtained when the cutoff is omitted.

- Its experimental determination involves uncertainty at short distance due to the centrifugal barrier.

$$\frac{L(L+1)}{r^2}$$

d-wave BS wave function

BS wave function for $J^P=1^+$ (I=0) consists of two components:

s-wave (L=0) and **d-wave (L=2)**

$$\vec{J} = \vec{L} + \vec{S}$$

$$= (0 \otimes 1) \oplus (2 \otimes 1)$$

- L=1,3,5,… is not allowed due to parity ➔ L=0,2,4,…
- S=0 is not allowed due to Pauli principle. ➔ S=1

  (I=0: anti-sym)x(S=1: sym)x(parity: even)=(totally anti-sym)

On the lattice, we decompose

(1) s−wave

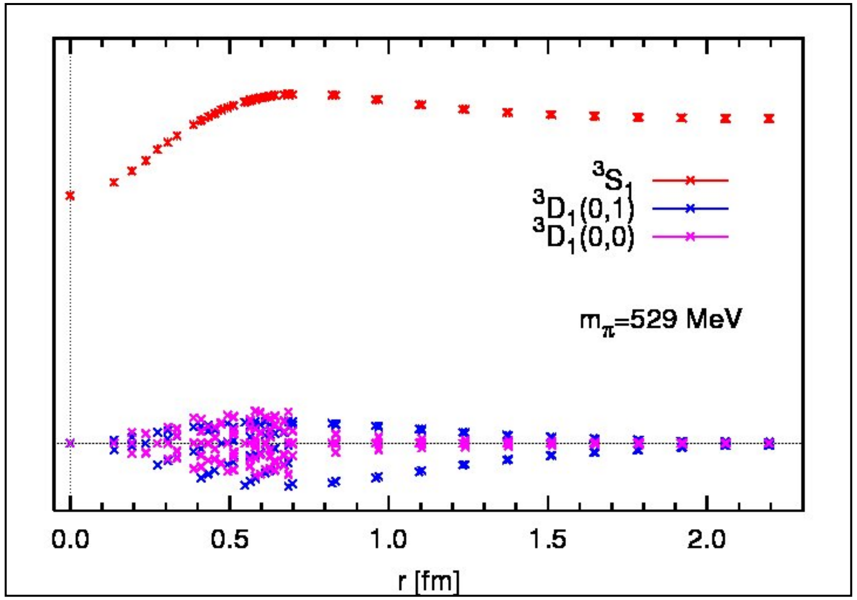
$$\psi^{(S)}_{\alpha\beta}(\vec{r}) = P[\psi](\vec{r}) \quad \equiv \frac{1}{24}\sum_{g\in O}\psi_{\alpha\beta}(g^{-1}\vec{r})$$

(2) d−wave

$$\psi^{(D)}_{\alpha\beta}(\vec{r}) = Q[\psi](\vec{r}) \quad \equiv \psi_{\alpha\beta}(\vec{r}) - \psi^{(S)}_{\alpha\beta}(\vec{r})$$

d-wave BS wave function

$J^P=1^+$, M=0



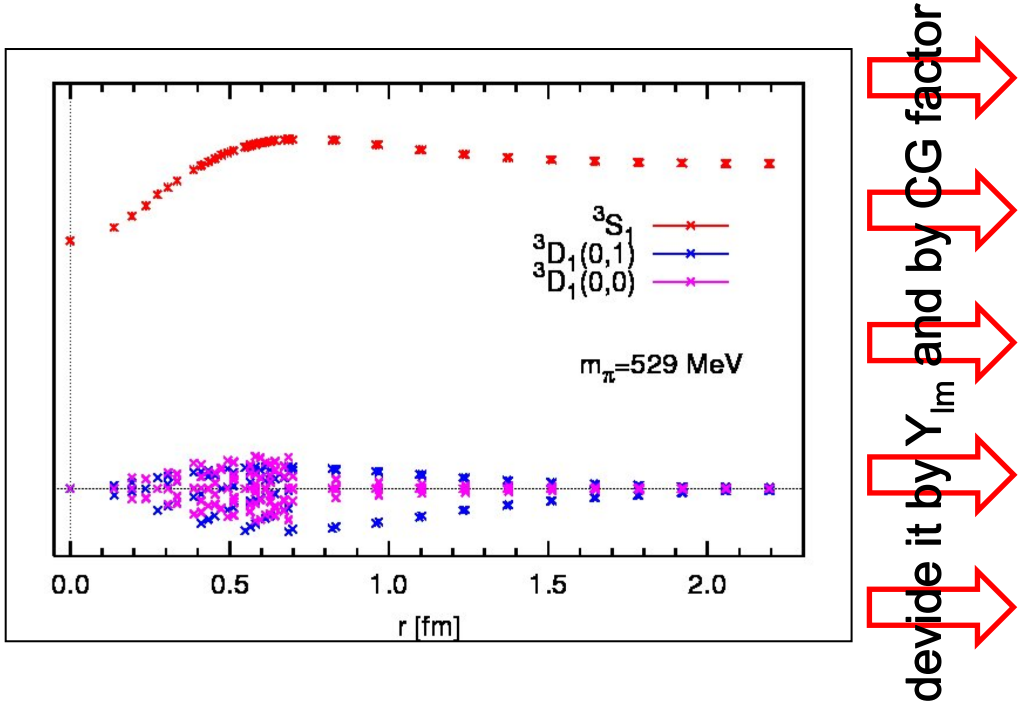
Angular dependence ➔ Multi-valued

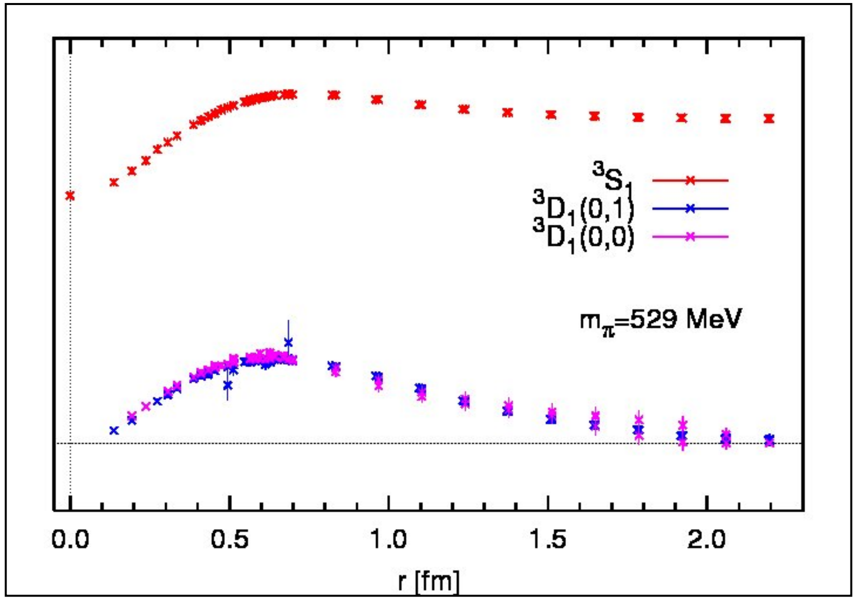
d-wave $\propto$ "spinor harmonics"

$$\begin{bmatrix} \psi^{(D)}_{\uparrow\uparrow}(\vec{r}) & \psi^{(D)}_{\uparrow\downarrow}(\vec{r}) \\ \psi^{(D)}_{\downarrow\uparrow}(\vec{r}) & \psi^{(D)}_{\downarrow\downarrow}(\vec{r}) \end{bmatrix} \propto \begin{bmatrix} Y_{2,-1}(\hat{r}) & -\frac{2}{\sqrt{6}} Y_{2,0}(\hat{r}) \\ -\frac{2}{\sqrt{6}} Y_{2,0}(\hat{r}) & Y_{2,+1}(\hat{r}) \end{bmatrix}$$

d-wave BS wave function

$J^P=1^+$, M=0





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$$\begin{bmatrix} \psi^{(D)}_{\uparrow\uparrow}(\vec{r}) & \psi^{(D)}_{\uparrow\downarrow}(\vec{r}) \\ \psi^{(D)}_{\downarrow\uparrow}(\vec{r}) & \psi^{(D)}_{\downarrow\downarrow}(\vec{r}) \end{bmatrix} \propto \begin{bmatrix} Y_{2,-1}(\hat{r}) & -\frac{2}{\sqrt{6}} Y_{2,0}(\hat{r}) \\ -\frac{2}{\sqrt{6}} Y_{2,0}(\hat{r}) & Y_{2,+1}(\hat{r}) \end{bmatrix}$$

Almost Single-valued

➔ $\psi^{(D)}$ is dominated by d-wave.

NOTE:
(0,1) [blue] ←→ E-representation
(0,0) [magenda] ←→ T_2-representation

Difference of these two →violation of SO(3)

Tensor force (cont'd)

- Derivative expansion up to local terms

$$V(\vec{x},\vec{\nabla}) = V_C(r) + V_T(r)\cdot S_{12} + V_{LS}(r)\vec{L}\cdot\vec{S} + \{V_D(r),\vec{\nabla}^2\} + \cdots$$

- Schrodinger eq for $J^P=1^+(I=0)$

$$\{H_0 + V_C(\vec{r}) + V_T(\vec{r})S_{12}\}\psi(\vec{r}) = E\,\psi(\vec{r})$$

⇩

$$V_C(\vec{r})\cdot P\psi(\vec{r}) + V_T(\vec{r})\cdot PS_{12}\psi(\vec{r}) = (E - H_0)\cdot P\psi(\vec{r}) \qquad \text{(s-wave)}$$

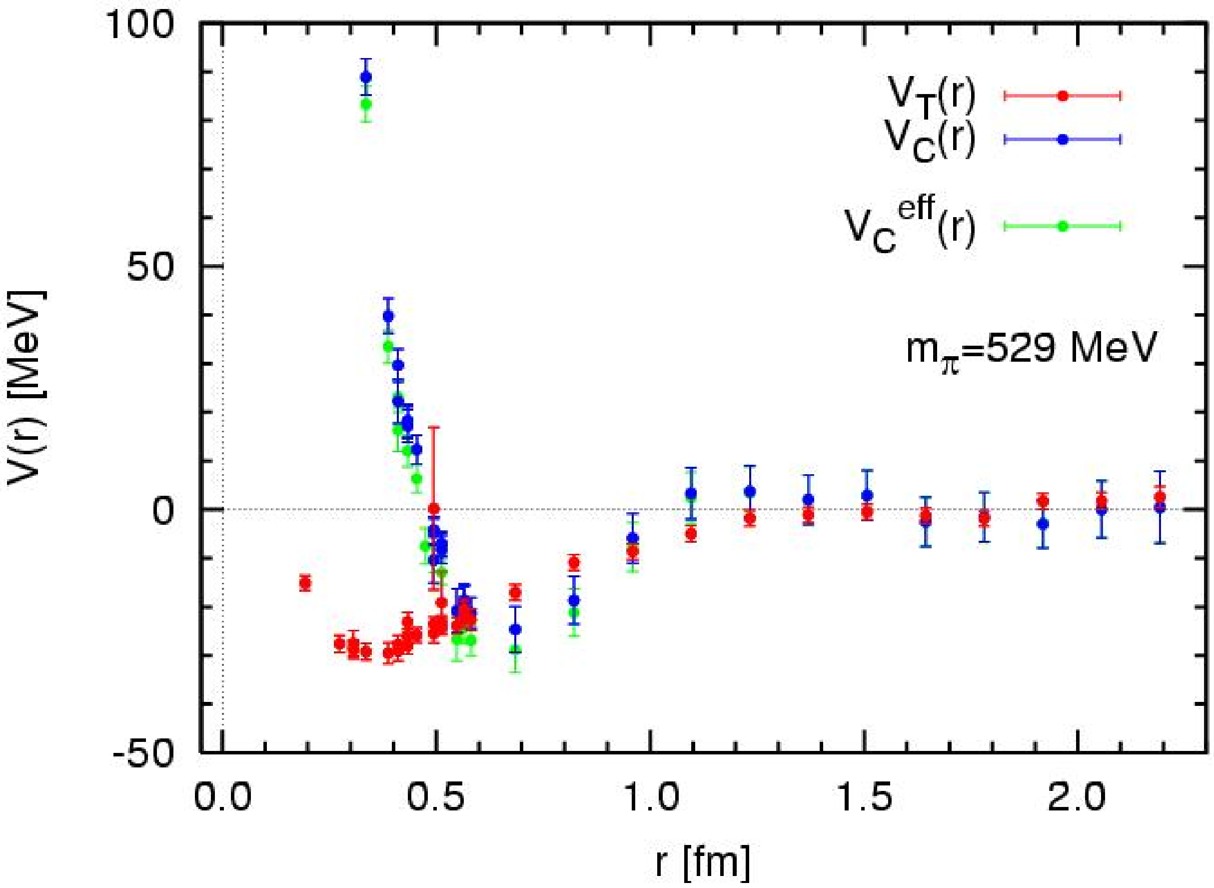
$$V_C(\vec{r})\cdot Q\psi(\vec{r}) + V_T(\vec{r})\cdot QS_{12}\psi(\vec{r}) = (E - H_0)\cdot Q\psi(\vec{r}) \qquad \text{(d-wave)}$$

- Solve them for $V_C(r)$ and $V_T(r)$ point by point

$$\begin{bmatrix} P\psi(\vec{r}) & PS_{12}\psi(\vec{r}) \\ Q\psi(\vec{r}) & QS_{12}\psi(\vec{r}) \end{bmatrix}\cdot\begin{bmatrix} V_C(\vec{r}) \\ V_T(\vec{r}) \end{bmatrix} = (E - H_0)\begin{bmatrix} P\psi(\vec{r}) \\ Q\psi(\vec{r}) \end{bmatrix}$$

Tensor potential (cont'd)

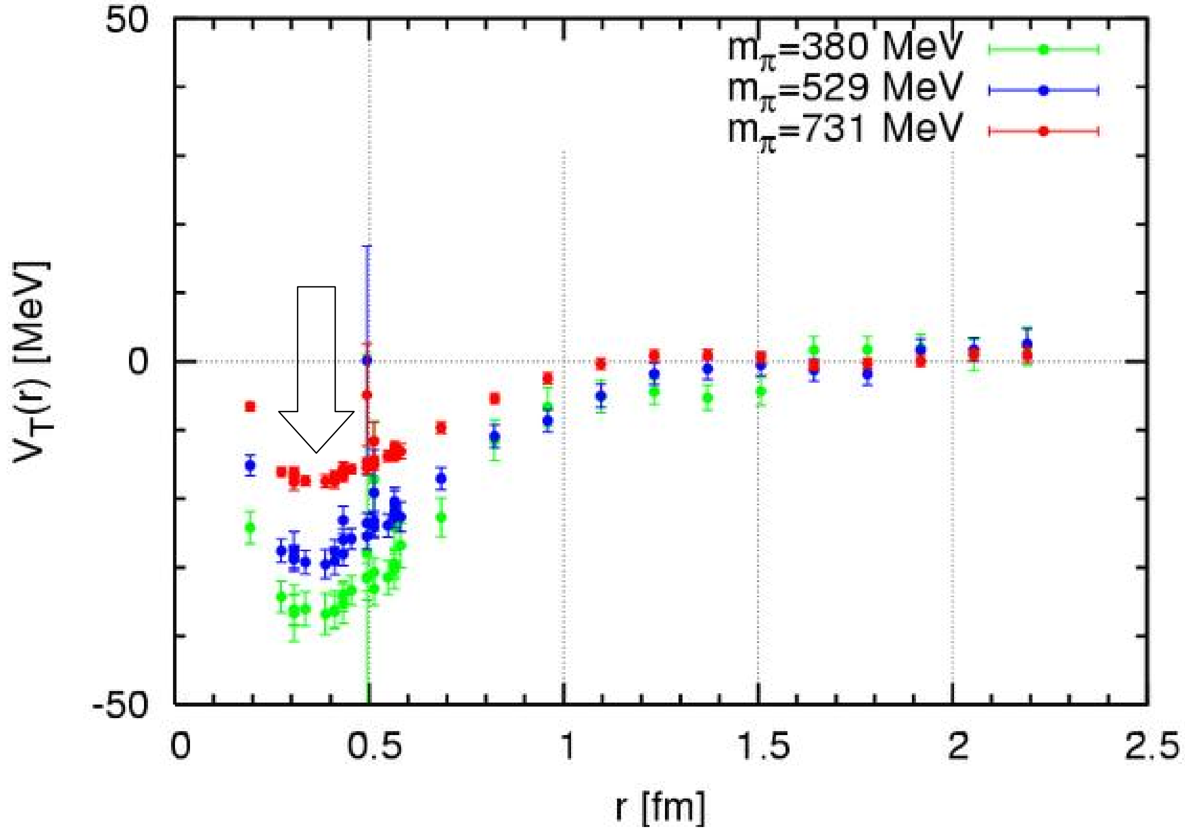
Ishii,Aoki,Hatsuda,PoS(LAT2008)155.



- No repulsive core
- A spike at r = 0.5 fm is due to zero of the spherical harmonics.

$$Y_{2,0}(\theta,\phi)=\frac{1}{4}\sqrt{\frac{5}{\pi}}\left(3\cos^2\theta-1\right) \Rightarrow$$ unobtainable points: $(\pm n, \pm n, \pm n)$

Tensor potential (quark mass dependence)

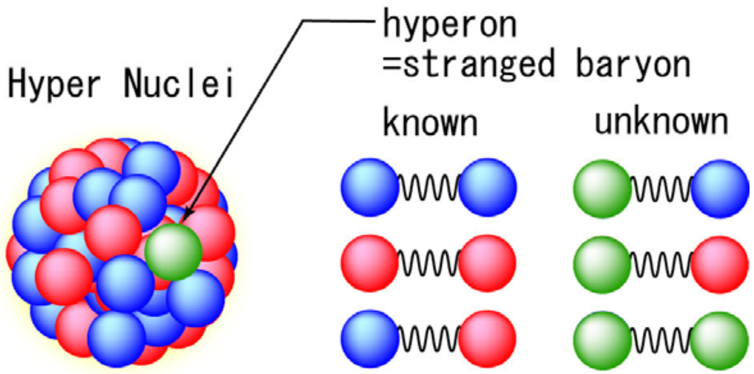


Tensor potential is enhanced
in the light quark mass region

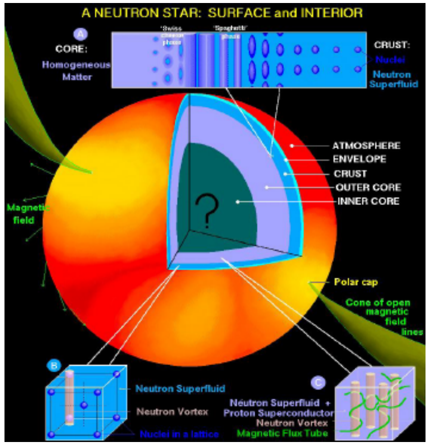
Hyperon Potentials

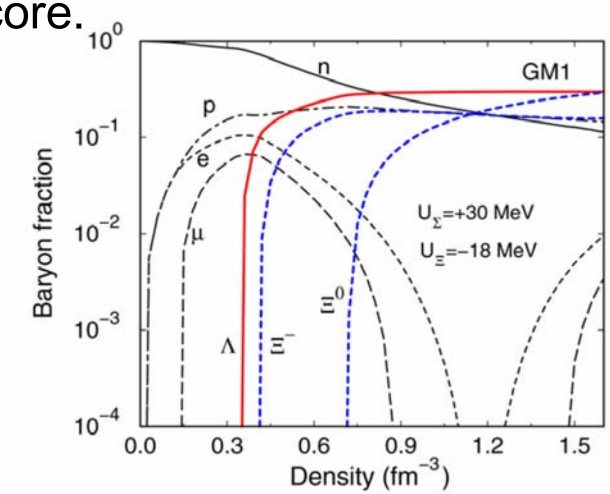
Hyperon potentials

- Important for
  - structure of hyper nuclei



  - equation of state at high density
    --- hyperon may appear in neutron star core.

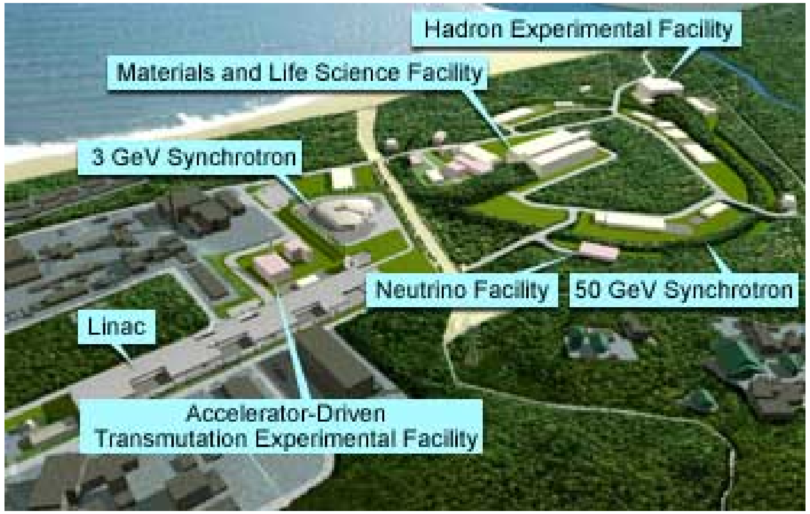




J.Schaffner-Bielich, NPA804('08)309.

J-PARC

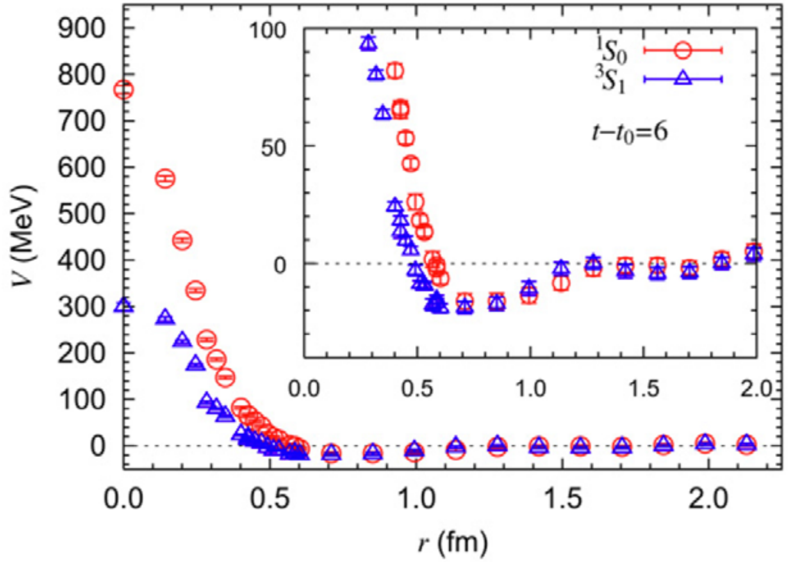
Exploration of multi-strangeness world



- Limited number of experimental information
  (No accelerator to generate direct hyperon beam)

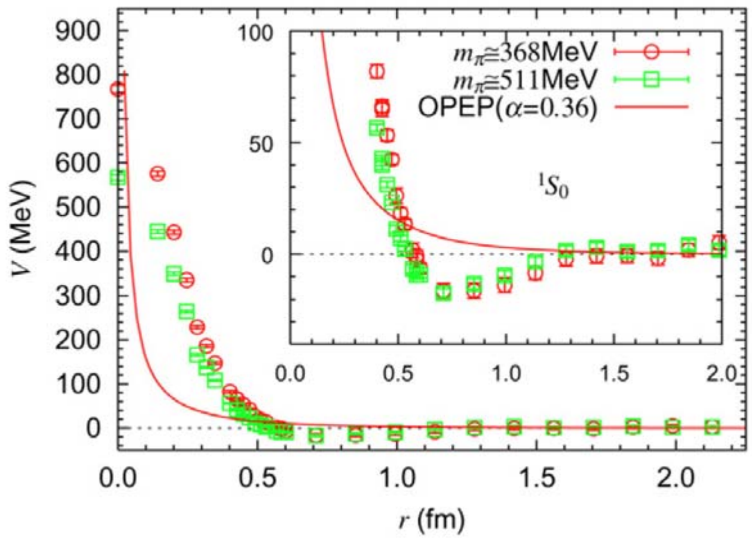
N-Xi potential (I=1)

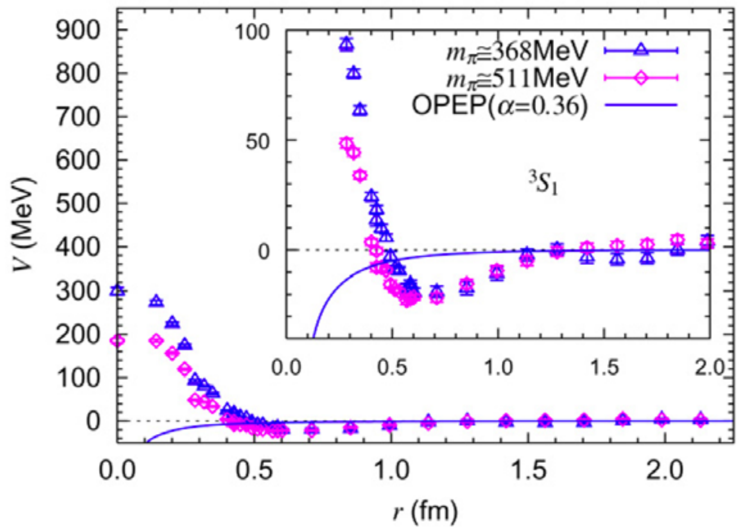
Nemura, Ishii, Aoki, Hatsuda, PLB673(2009)136.



- Repulsive core is surrounded by attraction like NN case.
- Strong spin dependence

quark mass dependence

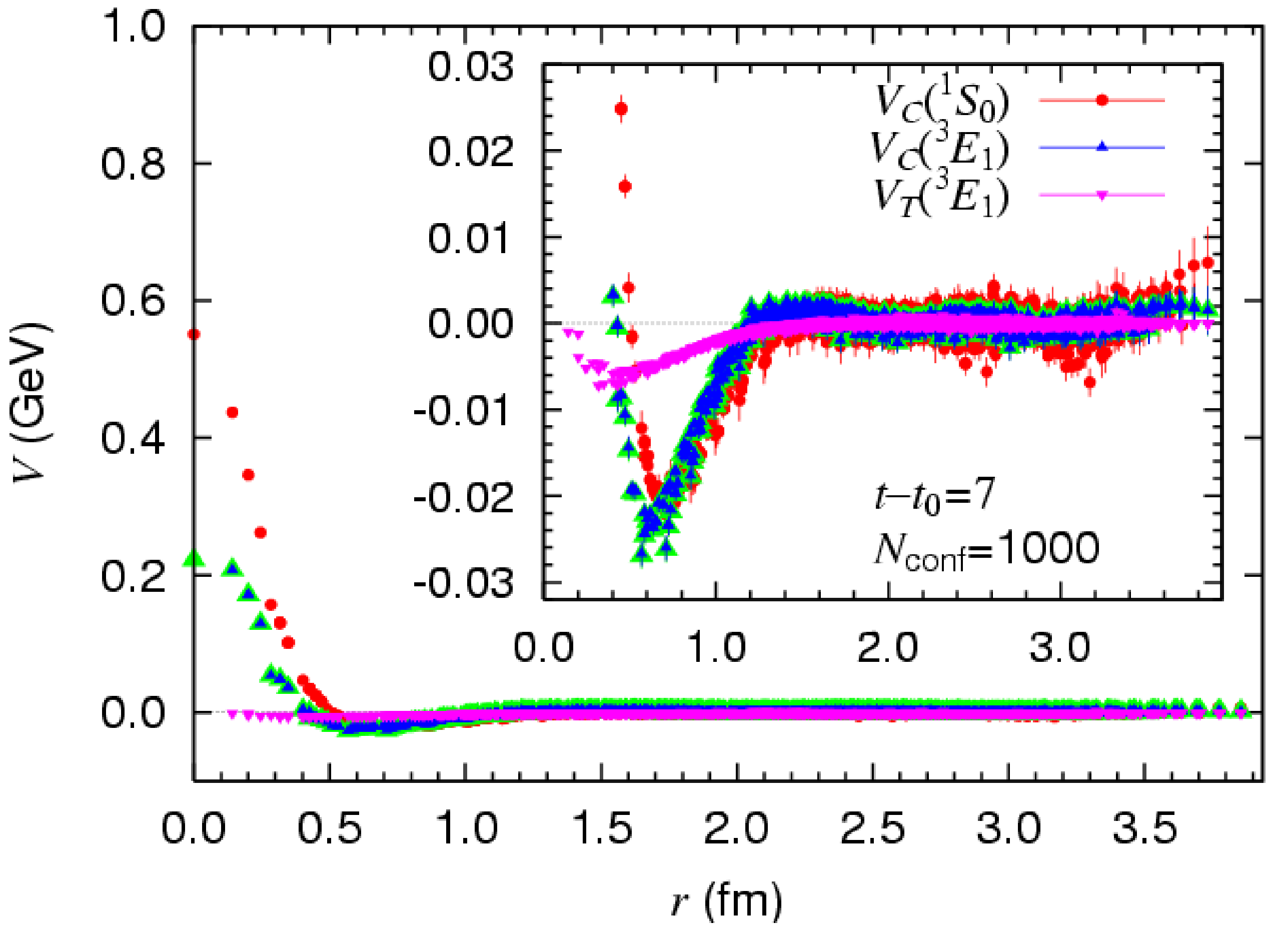




Repulsive core grows with decreasing quark mass.
No significant change in the attraction.

N Lambda potential (quenched QCD)

[Nemura, 27 July, Poster]



$m_\pi \approx 514\,\mathrm{MeV}$

- Spin dependence of the repulsive core is large.
- Spin dependence of the attraction is small.
- Weak tensor potential

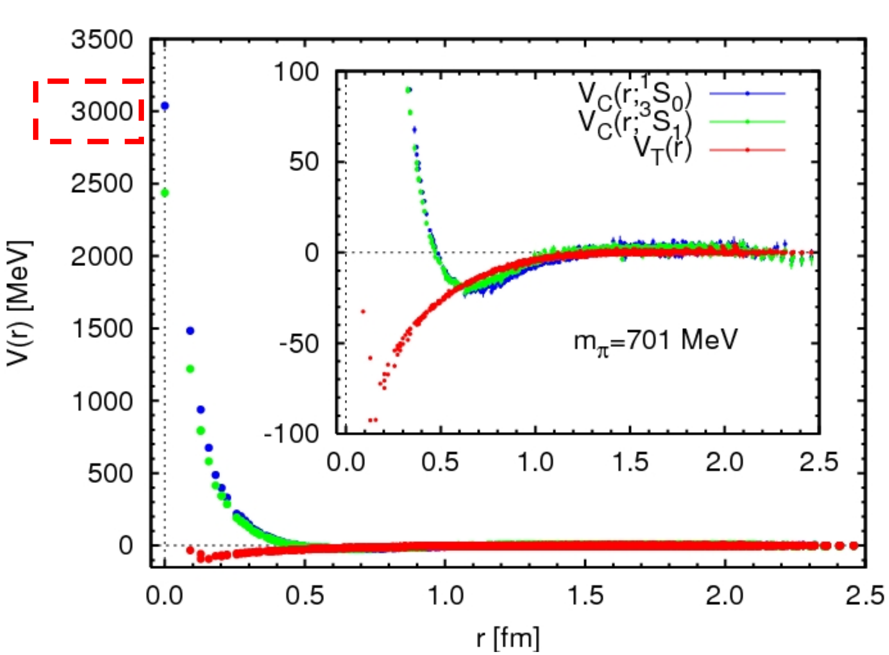
2+1 flavor full QCD

Gauge configurations by PACS-CS Collaboration:

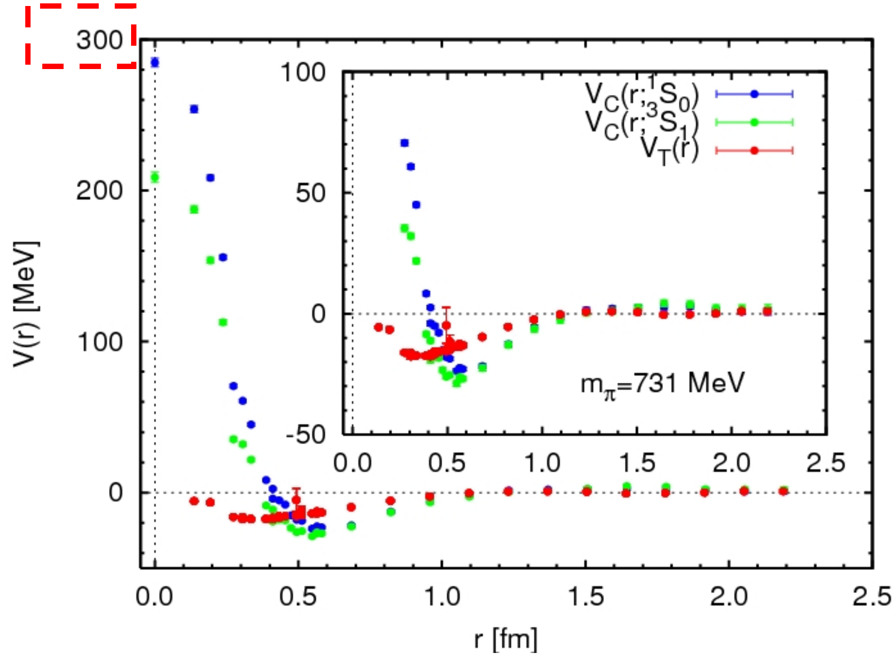
S.Aoki, K.-I.Ishikawa, N.Ishizuka, T.Izubuchi,
D.Kadoh, K.Kanaya, Y.Kuramashi, Y.Namekawa,
M.Okawa, Y.Taniguchi, A.Ukawa, N.Ukita, T.Yoshie

NN potentials

2+1 flavor results



quenched results

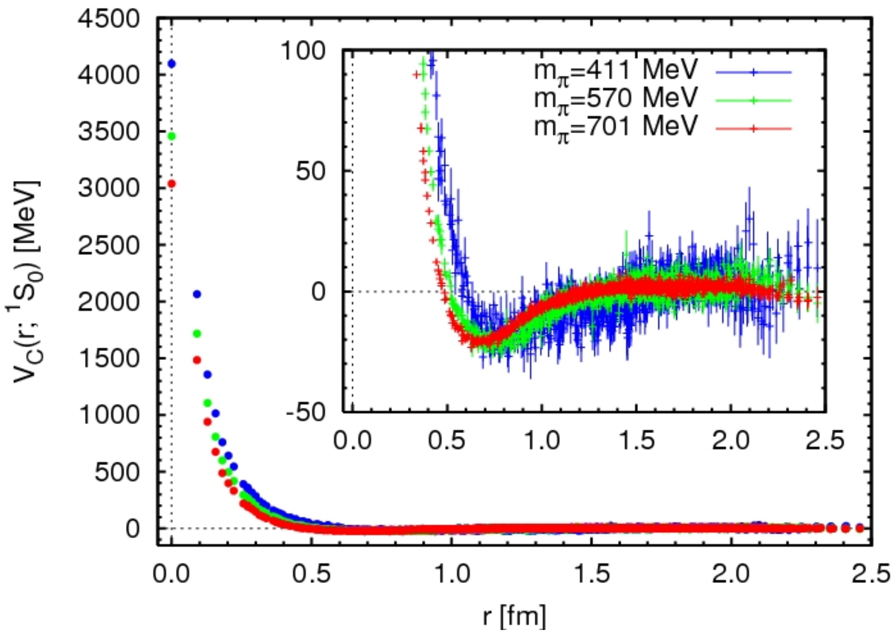


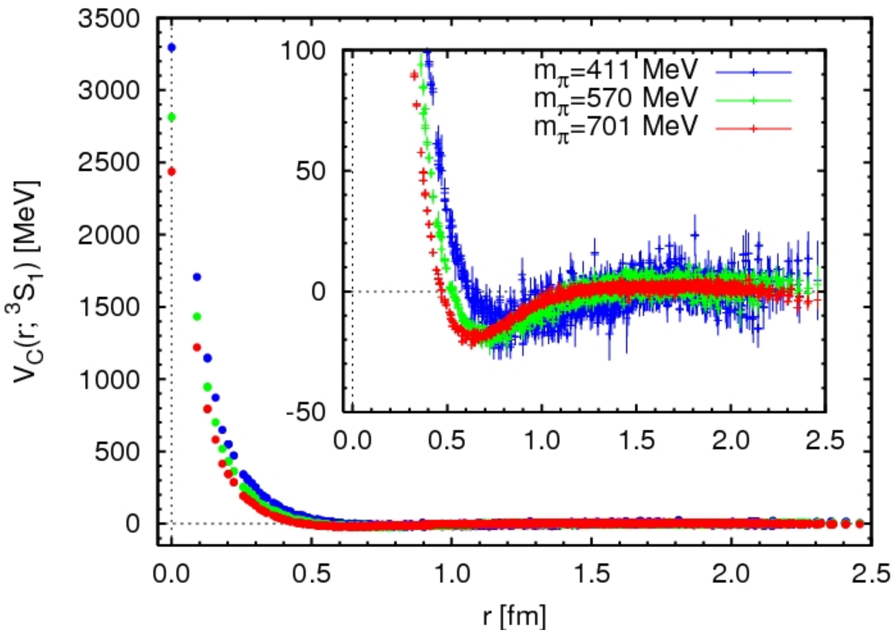
Comparing to the quenched ones,

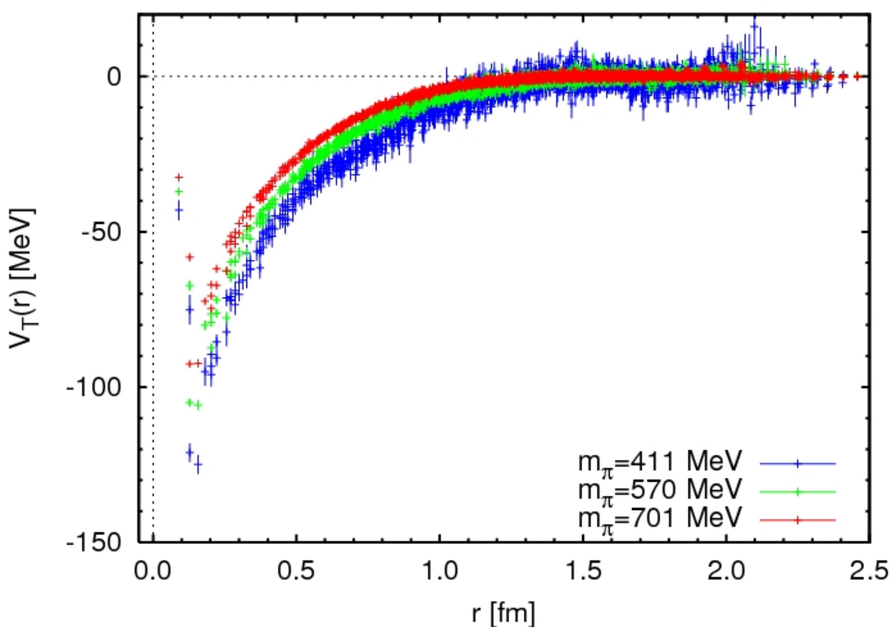
(1) Significantly stronger repulsive core and tensor force
(Reasons are under investigation)

(2) Attractions at medium distance are similar in magnitude.

NN potentials (quark mass dependence)



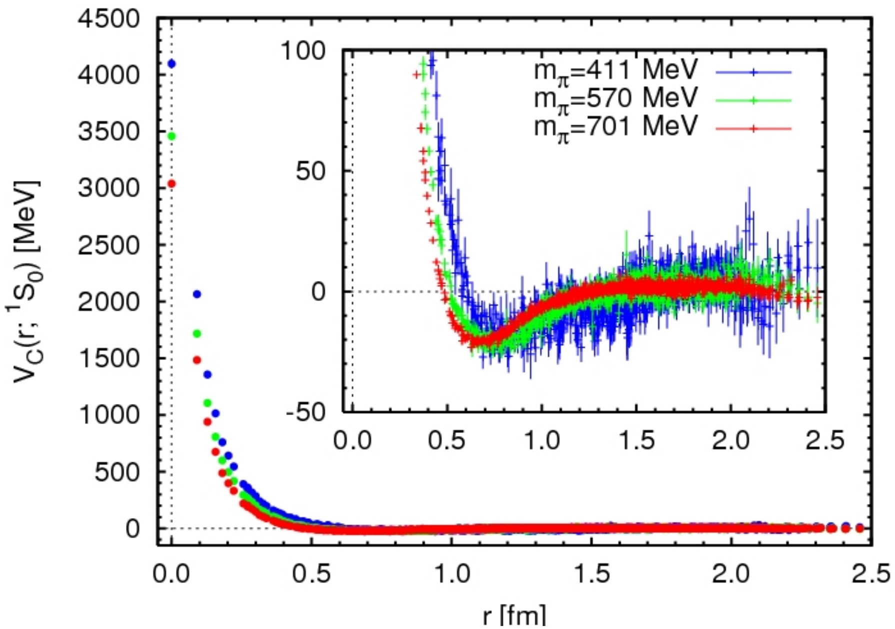


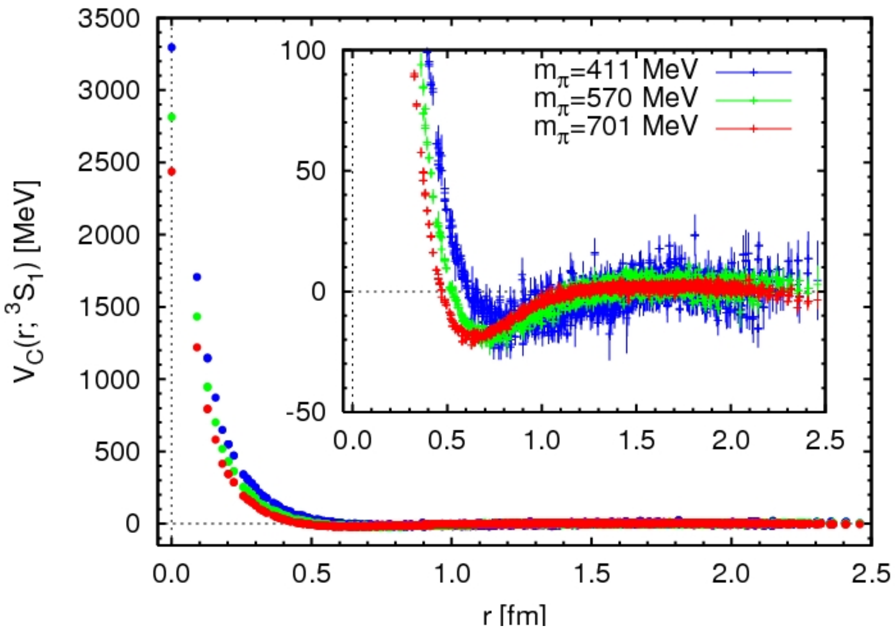


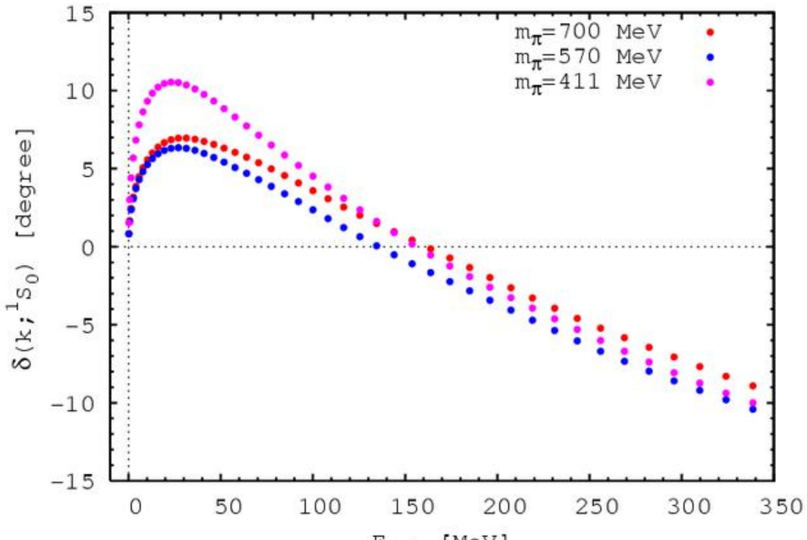
In the light quark mass region,

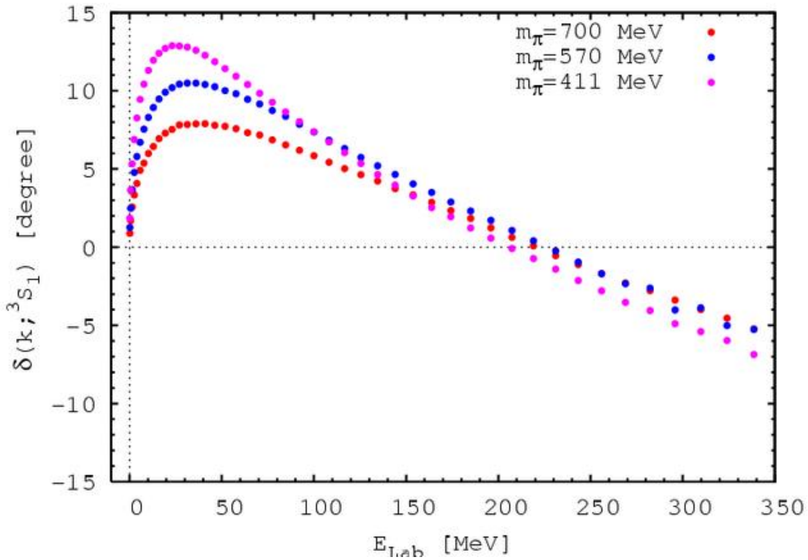
- Repulsive core grows.
- Attraction becomes stronger

NN (phase shift from potentials)



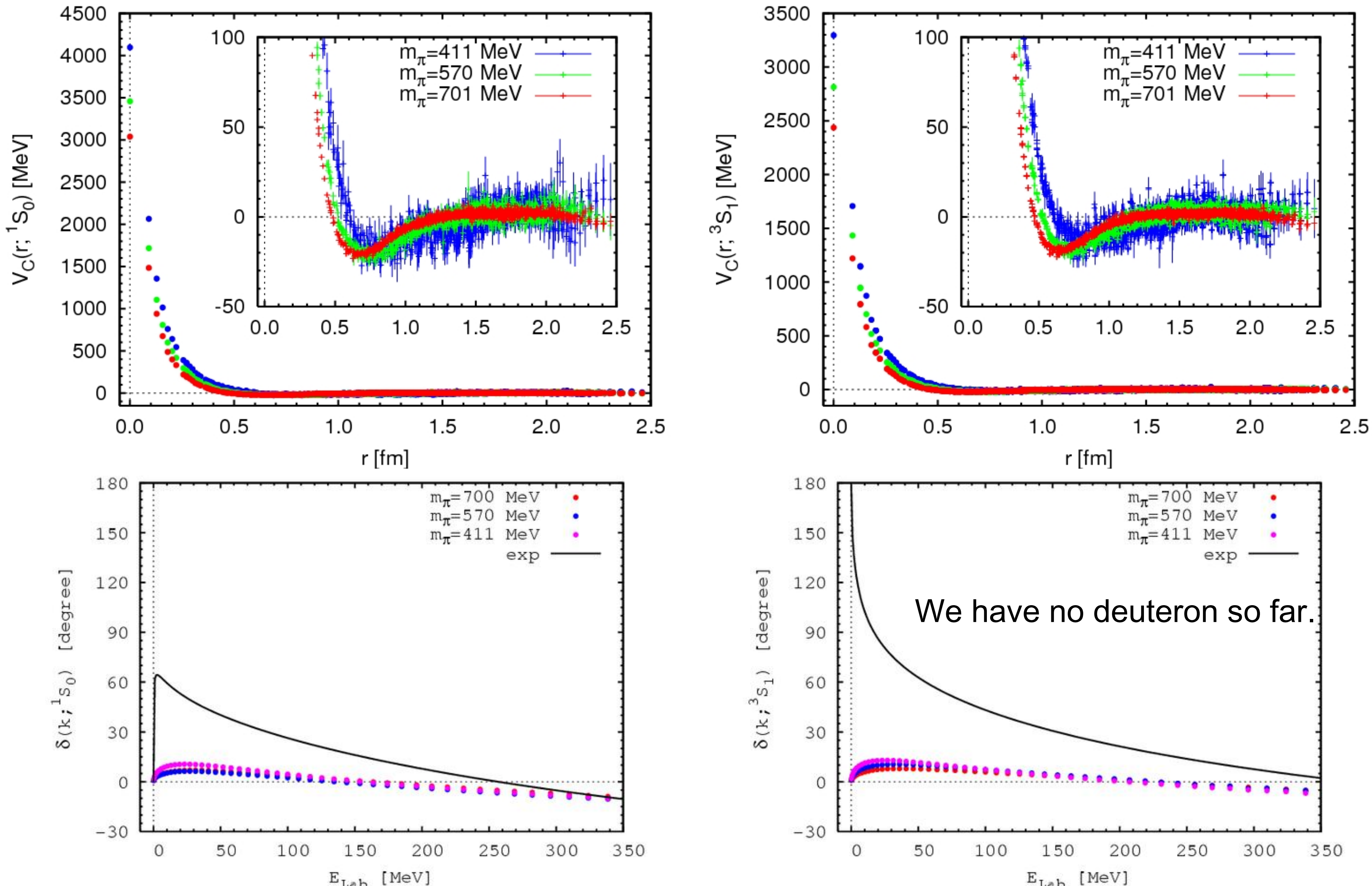






They have reasonable shapes.

NN (phase shift from potentials)

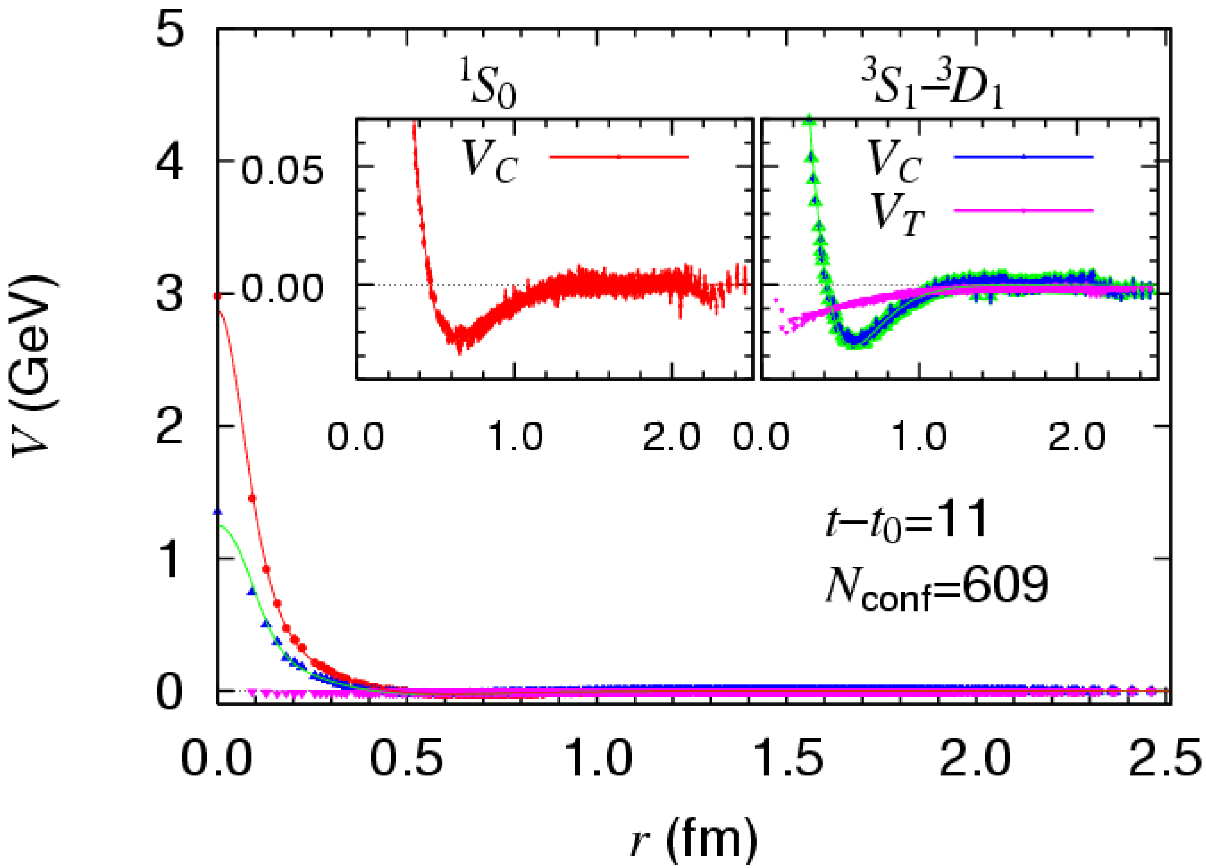


They have reasonable shapes. The strength is much weaker.

➔ Importance of physical quark mass.

N Lambda potential (2+1 flavor QCD)

[Nemura, 27 July, Poster]



$m_\pi = 701\text{MeV}$

- Large spin dependence of repulsive core
- Weak tensor force
- Net interaction is attractive.

Summary

- General strategy (NN potentials from BS wave functions)
  - These potentials are faithful to the phase shift data (by construction)

- Numerical results
  - Central potential, tensor potential, hyperon potentials (NXi [I=1] and NLambda)
  - Derivative expansion of (E-independent) non-local potential works well [E_{CM} = 0−46 MeV]
  - 2+1 flavor QCD results (by PACS-CS gauge config.)
    NN and NLambda (central and tensor potentials) [L ~ 3 fm]

Outlook:

- Realistic potentials at physical quark mass point in large spatial volume (L ~ 6 fm)
  by PACS-CS gauge configuration [planned]

- Higher derivative terms (LS force and more), p-wave, various hyperon potentials

- Three-nucleon potential

- Physical origin of the repulsive core
  flavor SU(3) limit [Inoue, 27 July, 13:30]
  short distance analysis by Operator Product Expansion [Aoki, 27 July, 16:40]

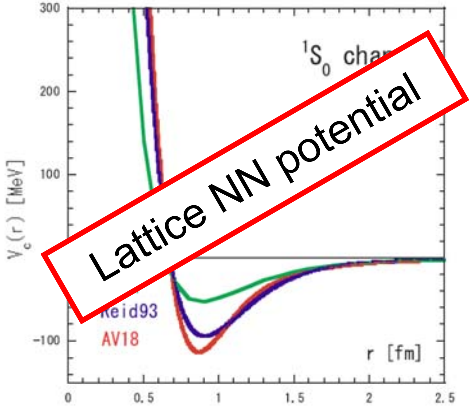
- Applications:
  - Nuclear physics based on lattice QCD
  - Eq. of states at finite density for supernovae and neutron stars

Final Remark: (Potential v.s. Phase shift)

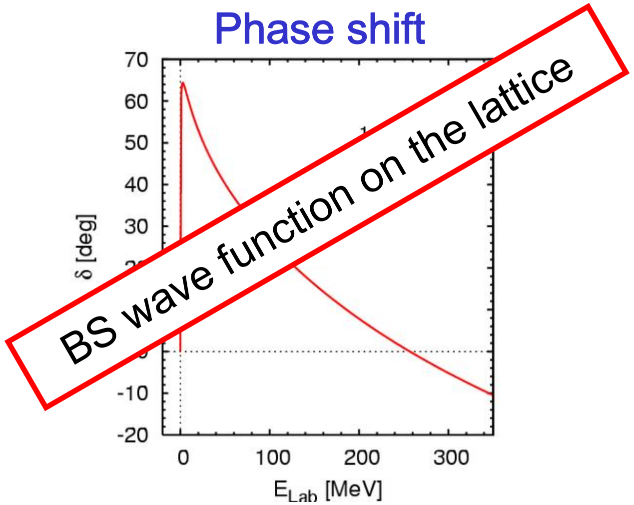
- For precise evaluation of scattering phase shift,
  - Do direct lattice calculations of phase shifts with Luscher's method.

- If you wish to study nuclei and more,
  - Convert them in the form of potential.
  - Potential itself is not a direct experimental observable.
    It is a tool designed to reproduce physical observables (the phase shifts).
  - Once it is constructed, it can be conveniently used to study a lot of phenomena.

Realistic nuclear force



Phase shift



Many nucleon,
phase shifts, …

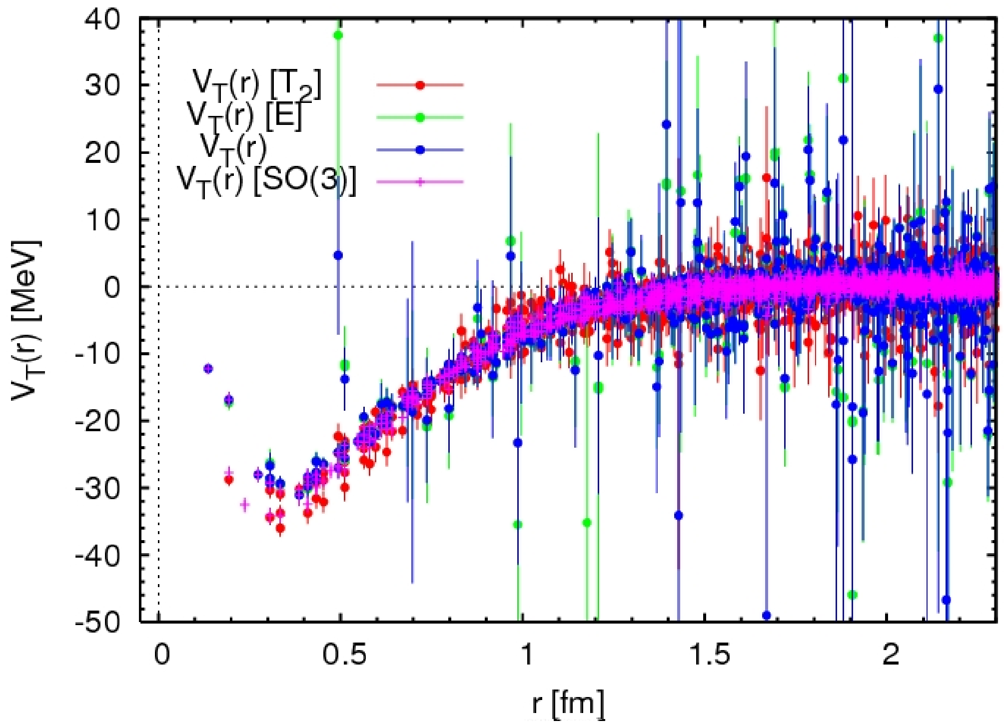
Str… eactions of nuclei,
Nuc… atter, eq. of states,neutron star, supernova, etc.

END

Backup Slides

Tensor potential (E v.s. T_2 representation)

d-wave ←→ E-rep + T_2-rep
We may play with this "1 to 2" correspondence.



No significant change
except for sizes of statistical errors

1. The simplest choice
   Regard E-rep as d-wave
   Unobtainable pt.: $(\pm n, \pm n, \pm n)$
   (pt. where Y_{lm} vanishes)

2. Cubic group friendly choice

   $$V_T(\vec{r}) \Rightarrow V_T^{(E)}(\vec{r}) \quad \& \quad V_T^{(T_2)}(\vec{r})$$

   Maximum # of unobtainable pt.
   $(\pm n, \pm n, \pm n)$, z-axis, xy-plane

3. Angle-dependent combination of E and T_2-rep. to achieve Minimum # of unobtainable pt.
   (0,0,0)
   [SO(3) sym must be good.]

General form of NN potential

★ By imposing following constraints:

- Probability (Hermiticity):
- Energy-momentum conservation:
- Galilei invariance:
- Spatial rotation:
- Spatial reflection:
- Time reversal:
- Quantum statistics:
- Isospin invariance:

The most general (off-shell) form of NN potential is given as follows:
[S.Okubo, R.E.Marshak,Ann.Phys.4,166(1958)]

$$V = V^0 + V^\tau \cdot (\vec{\tau}_1 \cdot \vec{\tau}_2)$$

$$V^i = V_0^i + V_\sigma^i \cdot (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_{LS}^i \cdot (\vec{L} \cdot \vec{S}) + \{V_T^i, S_{12}\} + \frac{1}{2}\{V_{\sigma p}^i, (\vec{\sigma}_1 \cdot \vec{p})(\vec{\sigma}_2 \cdot \vec{p})\} + \frac{1}{2}\{V_Q^i, Q_{12}\}$$

$$Q_{12} \equiv \frac{1}{2}\left[(\vec{\sigma}_1 \cdot \vec{L})(\vec{\sigma}_2 \cdot \vec{L}) + (\vec{\sigma}_2 \cdot \vec{L})(\vec{\sigma}_1 \cdot \vec{L})\right]$$

where $V_j^i = V_j^i(\vec{r}^2, \vec{p}^2, \vec{L}^2), \quad \vec{p} \equiv i\vec{\nabla}$

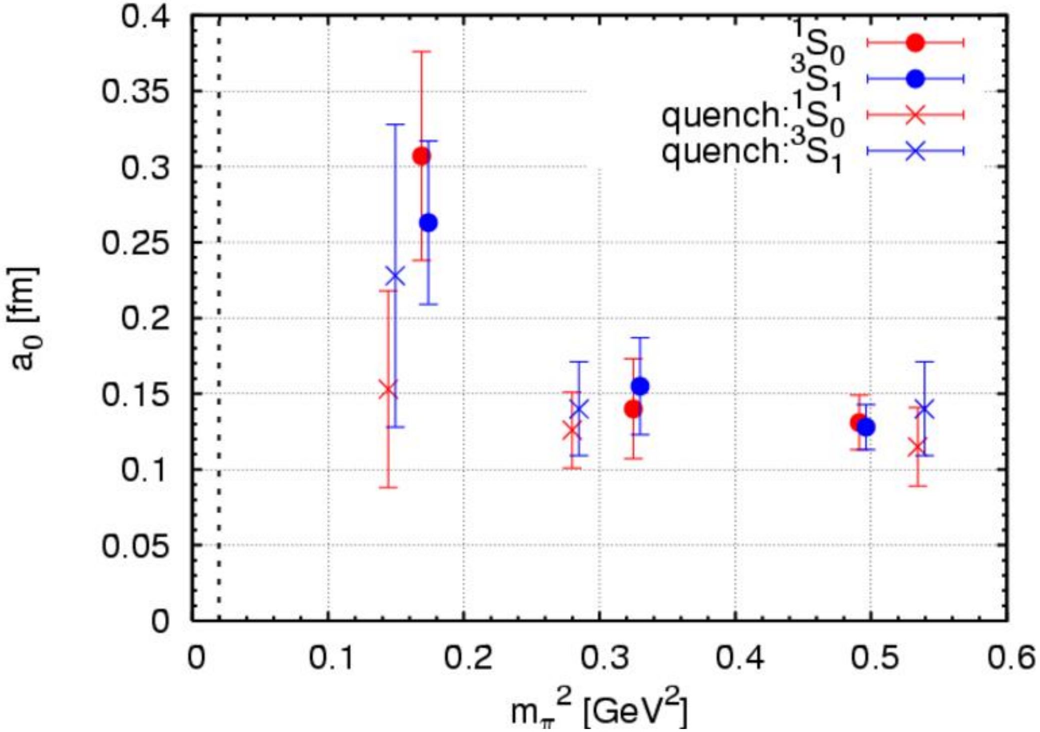
★ If we keep the terms up to O(p), we are left with the convensional form of the potential in nuclear physics:

$$V = V_0(r) + V_\sigma(r)(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_{LS}(r)\vec{L} \cdot \vec{S} + V_T(r) S_{12} + O(\vec{\nabla}^2).$$

$V_0(r) + V_\sigma(r)(\vec{\sigma}_1 \cdot \vec{\sigma}_2) \leftrightarrow V_C(r)$

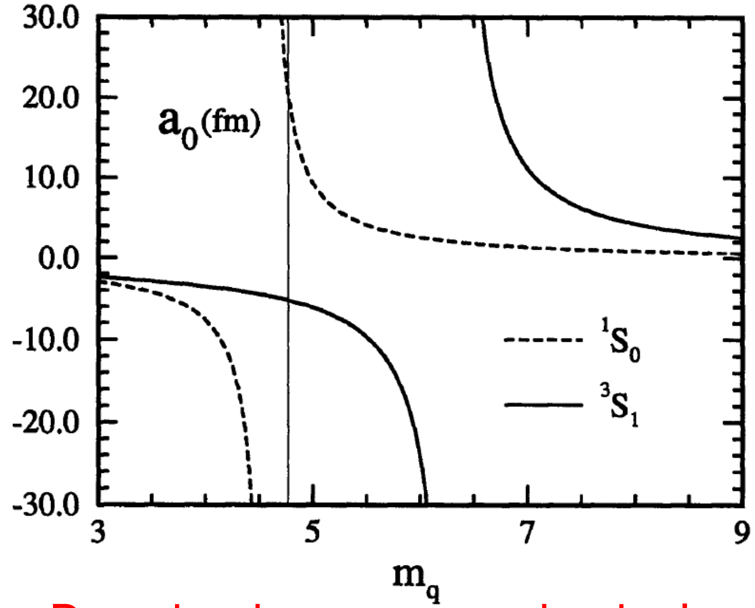
Scattering length of NN (quark mass dependence)

wave function →k^2→ Luscher's formula



- Attractive scattering length
- Attraction is enhanced as the quark mass decreases.
- The behavior is similar to the model below

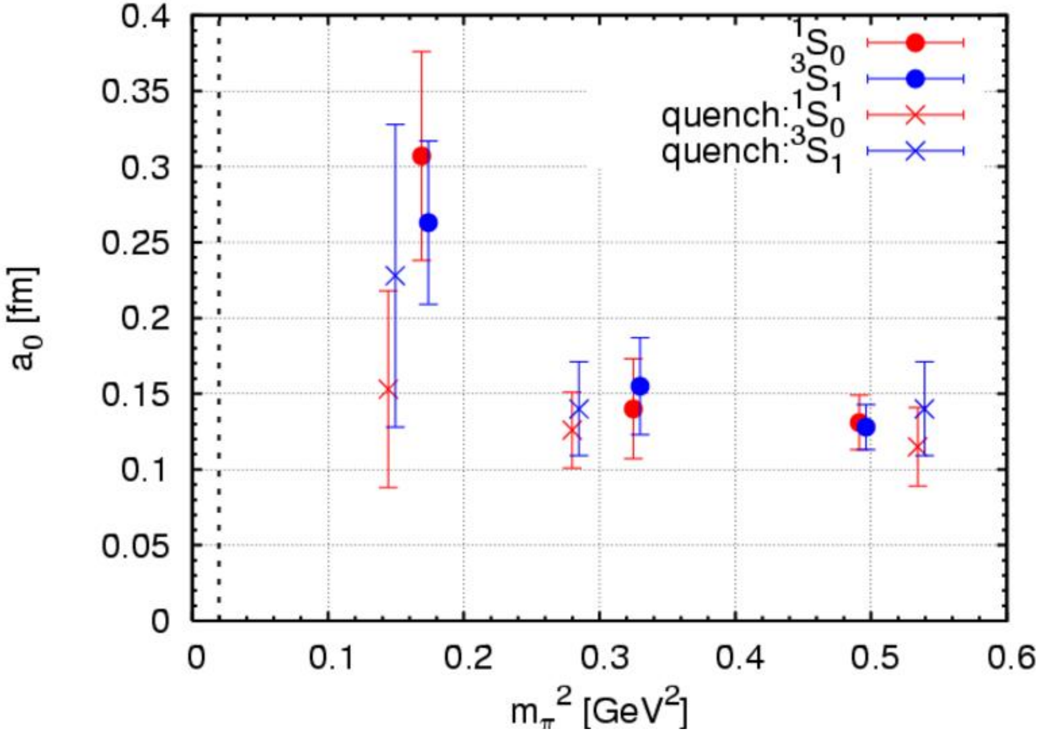
OBE potential + lattice hadron mass
Kuramashi, PTP122,153(1996)



Drastic change near physical m_q.

Scattering length of NN (quark mass dependence)

wave function → k^2 → Luscher's formula



- Attractive scattering length
- Attraction is enhanced as the quark mass decreases.
- The behavior is similar to the model below

Systematic Uncertainty:

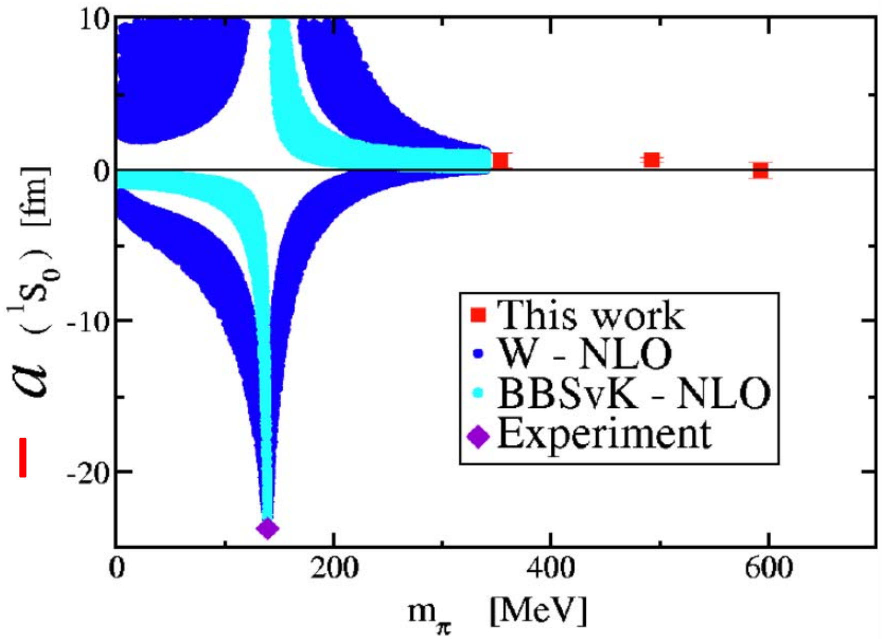
At the moment, we have uncertainty in determining scattering length from

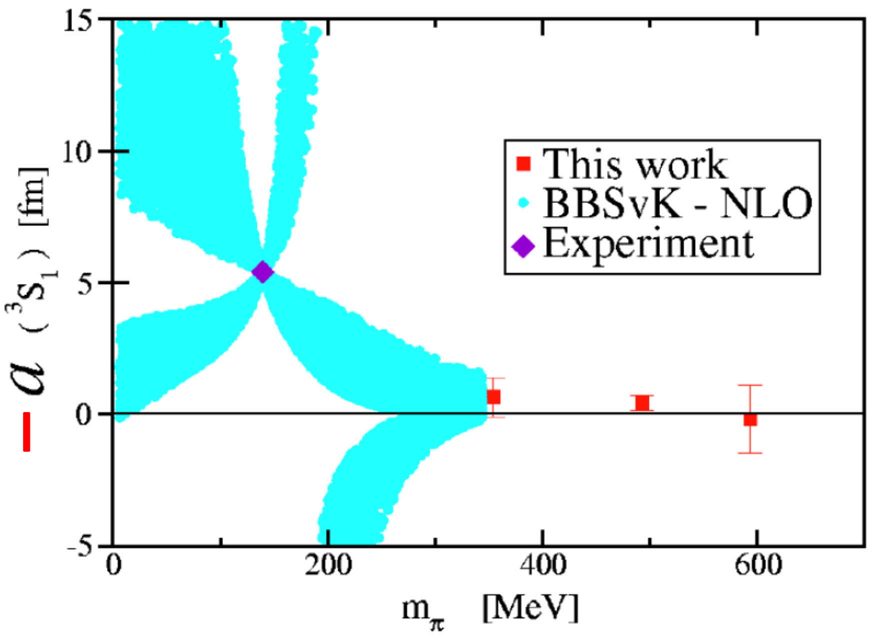
(1) spatial correlations (wave function) → $a_0(^1S_0)$ = 0.131(18) fm $\quad m_\pi = 701\,\mathrm{MeV}$

(2) temporal correlations (energy) → $a_0(^1S_0)$ = 4.8(5) fm

The inconsistency has to be resolved soon.

Scattering length (quark mass dependence II)





NPLQCD, PRL97,012001(2006).

- Nuclear effective theory with KSW coupling
- Repulsive scattering length