

# SU(2) and SU(3) chiral perturbation theory analysis of meson and baryon masses in 2+1 flavor lattice QCD

Lattice 2008 @ College of William and Mary

Center for Computational Sciences,  
University of Tsukuba  
Daisuke Kadoh for PACS-CS collaboration

Recent PACS-CS result : arXiv:0807.1661v1 [hep-lat]

## 1. Motivation and Simulation detail

### Motivation

- Examination of signals for chiral logarithms expected from ChPT
- Determination of physical points with ChPT fit
- Determination of low energy constants in chiral Lagrangian  
(Gasser Leutwyler's Li)

### Simulation detail

- O( $a$ )-improved Wilson quark and Iwasaki gauge action

$$C_{SW}^{NP} = 1.715 \quad \beta = 1.9 \quad a = 0.0907(13) \text{ fm} = (2.176(31) \text{ GeV})^{-1}$$

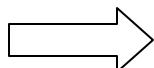
- Lattice size  $32^3 \times 64$
- Domain-decomposed HMC algorithm and algorithmic improvements
- PACS-CS computer @ CCS, University of Tsukuba  
(14.3 T flops peak)

– Simulation parameters

$\kappa_{ud}$	$\kappa_s$	$m_\pi [MeV]$	$m_{ud} [MeV]$	$m_s [MeV]$	# of conf [MD time]
0.13781	0.13640	156	3.6	88	990
0.13770	0.13640	296	12	91	2000
0.13754	0.13660	385	21	79	2000
0.13754	0.13640	411	24	94	2250
0.13727	0.13640	570	46	99	2000
0.13700	0.13640	702	68	104	2000

$$m_\pi : 156 \sim 702 \text{ MeV} \quad m_{ud} : 3.6 \sim 68 \text{ MeV}$$

Lightest point  $m_\pi \simeq 156 \text{ MeV}$ ,  $m_{ud} \simeq 3.6 \text{ MeV}$ ,  $m_\pi L \simeq 2.3$



More details were given in talks by  
 Naoya Ukita (Mon, 6:20–), Ken-Ichi Ishikawa (Wed, 11:20– )

## 2. ChPT

Gasser and Leutwyler, 1984

### Wilson chiral perturbation theory

S.Aoki, O.bär, T.Ishikawa and S.Takeda, 2006

pseudoscalar meson mass and decay constants in terms of AWI quark mass

$$\frac{m_{ps}^2}{m_q^1 + m_q^2} = B_0 \underbrace{\left\{ 1 + \text{chiral logs} + m_q - \text{linear term} - 2H''/f_0^2 \right\}}_{\text{continuum-part}}$$

$$f_{ps} = f_0 \underbrace{\left\{ 1 + \text{chiral logs} + m_q - \text{linear term} - 2H'/f_0^2 \right\}}_{\text{continuum-part}}$$

$$\text{chiral logarithm : } \mu_P = \frac{\tilde{M}_P^2}{32\pi^2} \ln \frac{\tilde{M}_P^2}{\mu^2}, \quad H'', H' : \mathcal{O}(a^2)$$

- “ WChPT(NLO, AWI quark mass) is equivalent to continuum ChPT(NLO) ”

$$B'_0 = B_0 \left( 1 - 2 \frac{H''}{f_0^2} \right), \quad f'_0 = f_0 \left( 1 - 2 \frac{H'}{f_0^2} \right)$$

$H''$  and  $H'$  can be absorbed into redefinitions of  $B_0$  and  $f_0$ .  
 Therefore, hereafter we concentrate on the continuum ChPT up to NLO.

– Finite size effects

Colangelo, Durr and Haefeli, 2005

$$R_X = \frac{X(L) - X(\infty)}{X(\infty)}$$

$m_\pi L \simeq 2.3 > 1$  for our lightest point

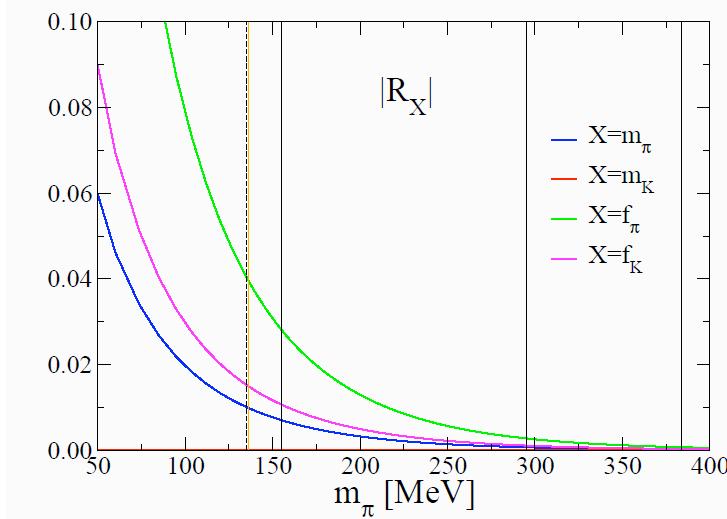
From NLO ChPT,

$$\begin{aligned} R_{m_\pi} &= \frac{1}{4}\xi_\pi\tilde{g}_1(\lambda_\pi) - \frac{1}{12}\xi_\eta\tilde{g}_1(\lambda_\eta), & R_{m_K} &= \frac{1}{6}\xi_\eta\tilde{g}_1(\lambda_\eta) \\ R_{f_\pi} &= -\xi_\pi\tilde{g}_1(\lambda_\pi) - \frac{1}{2}\xi_K\tilde{g}_1(\lambda_K), & R_{f_K} &= -\frac{3}{8}\xi_\pi\tilde{g}_1(\lambda_\pi) - \frac{3}{4}\xi_K\tilde{g}_1(\lambda_K) - \frac{3}{8}\xi_\eta\tilde{g}_1(\lambda_\eta) \end{aligned}$$

$$\xi_P \equiv \frac{M_P^2}{(4\pi F_\pi)^2}, \quad \lambda_P \equiv M_P L, \quad \tilde{g}_1(x) = \sum_{n=1}^{\infty} \frac{4m(n)}{\sqrt{nx}} K_1(\sqrt{nx}),$$

$K_1$  : Bessel function of 2nd kind  $K_1(z) \sim \sqrt{\frac{\pi}{2z}} e^{-z}$  for  $z \gg 1$

$m(n)$  : multiplicity of the partition  $n = n_x^2 + n_y^2 + n_z^2$ .

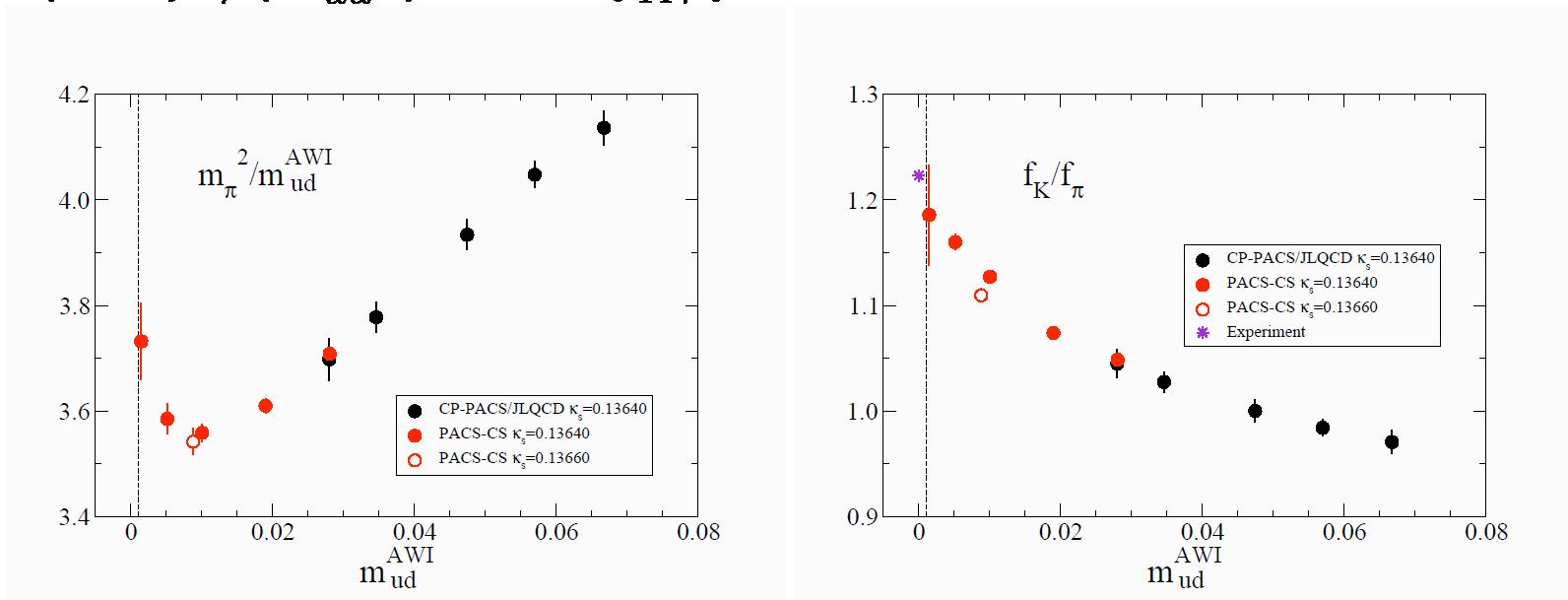


$m_\pi(L) > m_\pi(\infty)$  and  $F_\pi(L) < F_\pi(\infty)$

Contribution of FSE is 4% for  $f_\pi$   
1–2% for  $m_\pi, f_K$ , almost 0% for  $m_K$   
at physical point.

### 3. Comparison with CP-PACS + JLQCD

$$(m_\pi a)^2/(m_{ud}a) \text{ and } f_K/f_\pi$$



PACS-CS (●  $\kappa_s = 0.13640$  ○  $\kappa_s = 0.13660$  )

CP-PACS/JLQCD (●  $\kappa_s = 0.13640$  )

- CP-PACS/JLQCD and PACS-CS data are connected smoothly.
- Typical behavior of chiral logarithm appears.

## 4. Fit result

– fit range

$\kappa_{ud}$	$\kappa_s$	$m_\pi [MeV]$	$m_{ud} [\text{MeV}]$	$m_s [MeV]$
0.13781	0.13640	156	3.6	88
0.13770	0.13640	296	12	91
0.13754	0.13660	385	21	79
0.13754	0.13640	411	24	94
0.13727	0.13640	570	46	99
0.13700	0.13640	702	68	104

4 data points

$m_\pi : 156 - 411 \text{ MeV}$

( $m_{ud} : 3.6 - 24 \text{ MeV}$ )

## 4.1 SU(3) ChPT fit

- simultaneous fit to  $\frac{m_\pi^2}{2m_{ud}}$ ,  $\frac{m_K^2}{m_{ud}+m_s}$ ,  $f_\pi$ ,  $f_K$  .

### (1) Low energy constants

6 unknown parameters  $B_0, f_0, L_4, L_5, L_6, L_8$  for  $m_\pi, m_K, f_\pi, f_K$   
in SU(3) ChPT up to NLO.

	w/o FSE	w FSE	phenomenology	RBC/UKQCD	MILC
$L_4$	-0.04 (10)	-0.06(10)	0.0 (0.8)	0.139 (80)	0.1 (3) $\begin{Bmatrix} +3 \\ -1 \end{Bmatrix}$
$L_5$	1.43 (7)	1.45 (7)	1.46(10)	0.872 (99)	1.4 (2) $\begin{Bmatrix} +2 \\ -1 \end{Bmatrix}$
$2L_6 - L_4$	0.10 (2)	0.10 (2)	0.0 (1.0)	-0.001 (42)	0.3 (3) $\begin{Bmatrix} +2 \\ -3 \end{Bmatrix}$
$2L_8 - L_5$	-0.21 (3)	-0.21 (3)	0.54 (43)	0.243 (45)	0.3 (1) $\begin{Bmatrix} +1 \\ -1 \end{Bmatrix}$

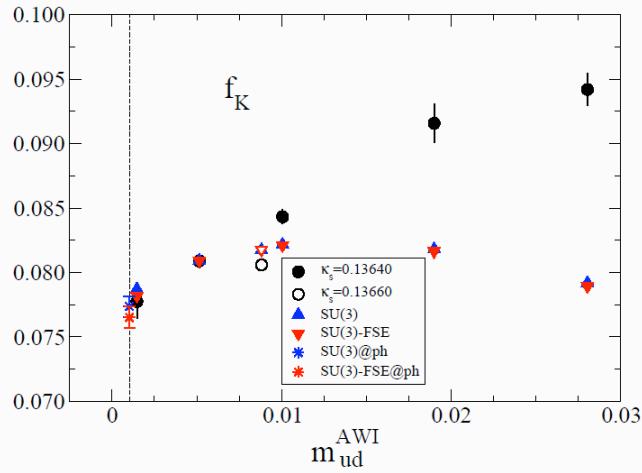
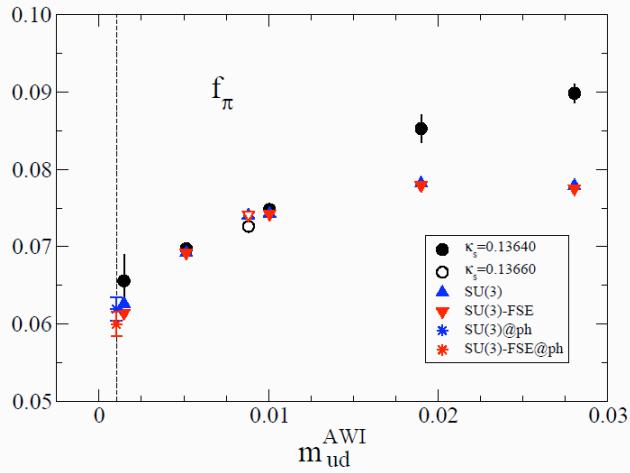
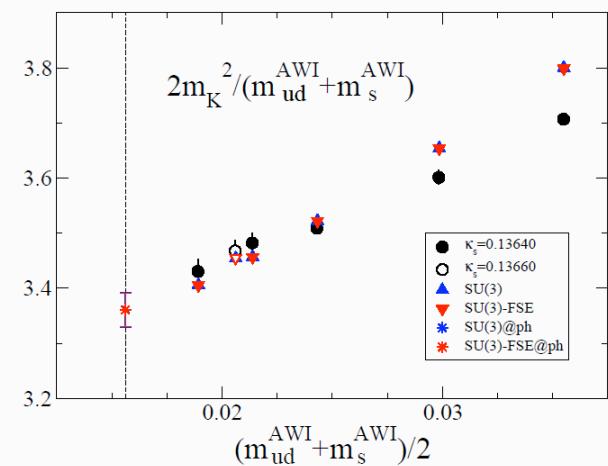
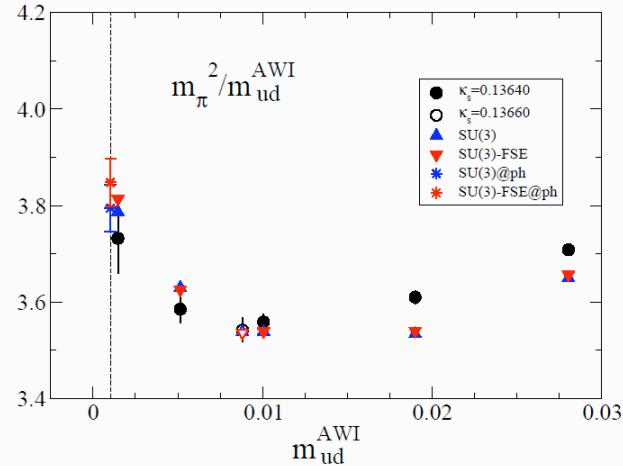
All  $L_4$  and  $L_5$  are compatible.

On the other hand,  $2L_6 - L_4$  and  $2L_8 - L_5$  show discrepancies.

Phenomenological values for LECs are taken from

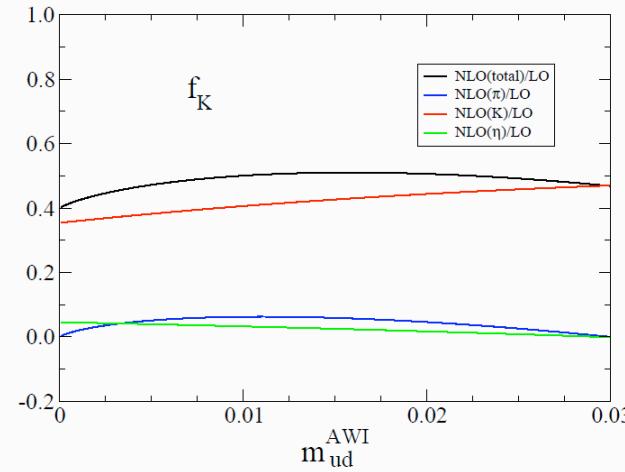
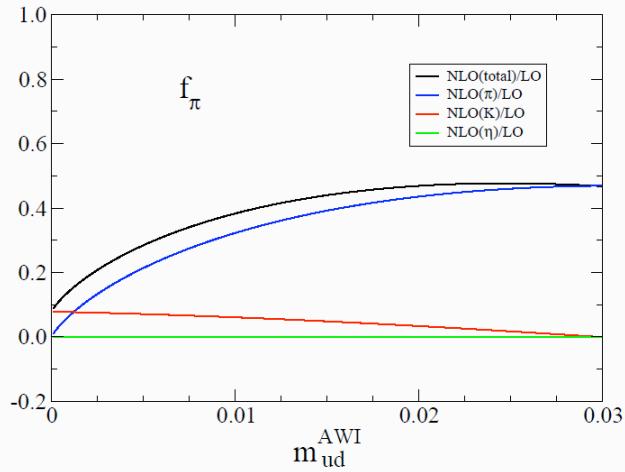
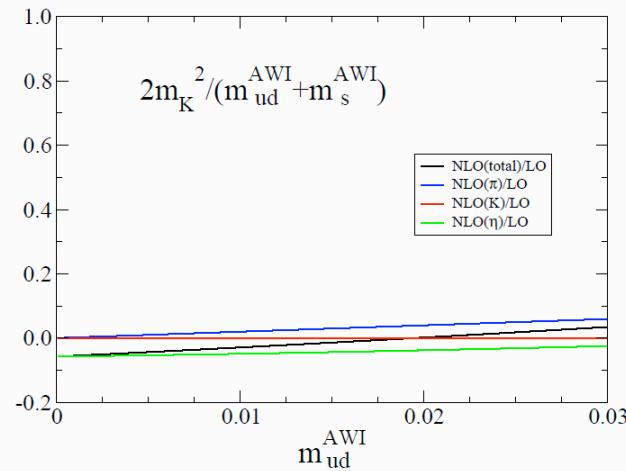
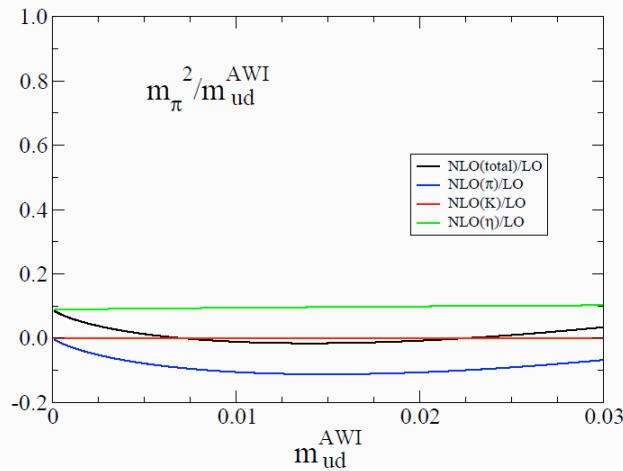
G. Amoros, J. Bijnens and P. Talavera, Nucl. Phys. B 602, 87 (2001)

## (2) Fit graph for SU(3) ChPT fit



Large discrepancies are observed between data and fit result around  $am_{ud} = 0.01$ . This causes a large value of  $\chi^2/dof = 4.2(19)$ .

### (3) Convergence of SU(3) ChPT fit



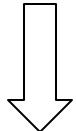
NLO contributions are almost 10% for  $m_\pi^2, m_K^2$ .

However, they are significant for decay constants,  
10–40% for  $f_\pi$  and 40–50% for  $f_K$  in our fit range.

## Motivation of SU(2) ChPT fit

SU(3) ChPT(NLO) fit gives reasonable values for the low energy constants.  
But, we do not determine the physical point from the fit,  
because

a rather large value of  $\chi^2/dof = 4.2(2.9)$   
bad convergence for  $f_\pi$  and  $f_K$ .



Alternative choices for chiral analysis

- (1) SU(3) ChPT up to NNLO
- (2) SU(2) ChPT up to NLO with analytic expansion  
of strange quark mass around the physical point

“It is difficult to determine all LECs at NNLO without increasing data points.  
On the other hand, our strange quark mass is close enough to the physical  
value. Therefore, we employ SU(2) ChPT analysis ”

## 4.2 SU(2) ChPT fit

- We apply NLO SU(2) ChPT formulae for  $m_\pi, f_\pi$  with the strange quark mass dependent parameters (a linear expansion).

$$B = B_s^{(0)} + B_s^{(1)} m_s, \quad f = f_s^{(0)} + f_s^{(1)} m_s$$

We treat kaon as a matter field in the isospin  $\frac{1}{2}$  linear representation.

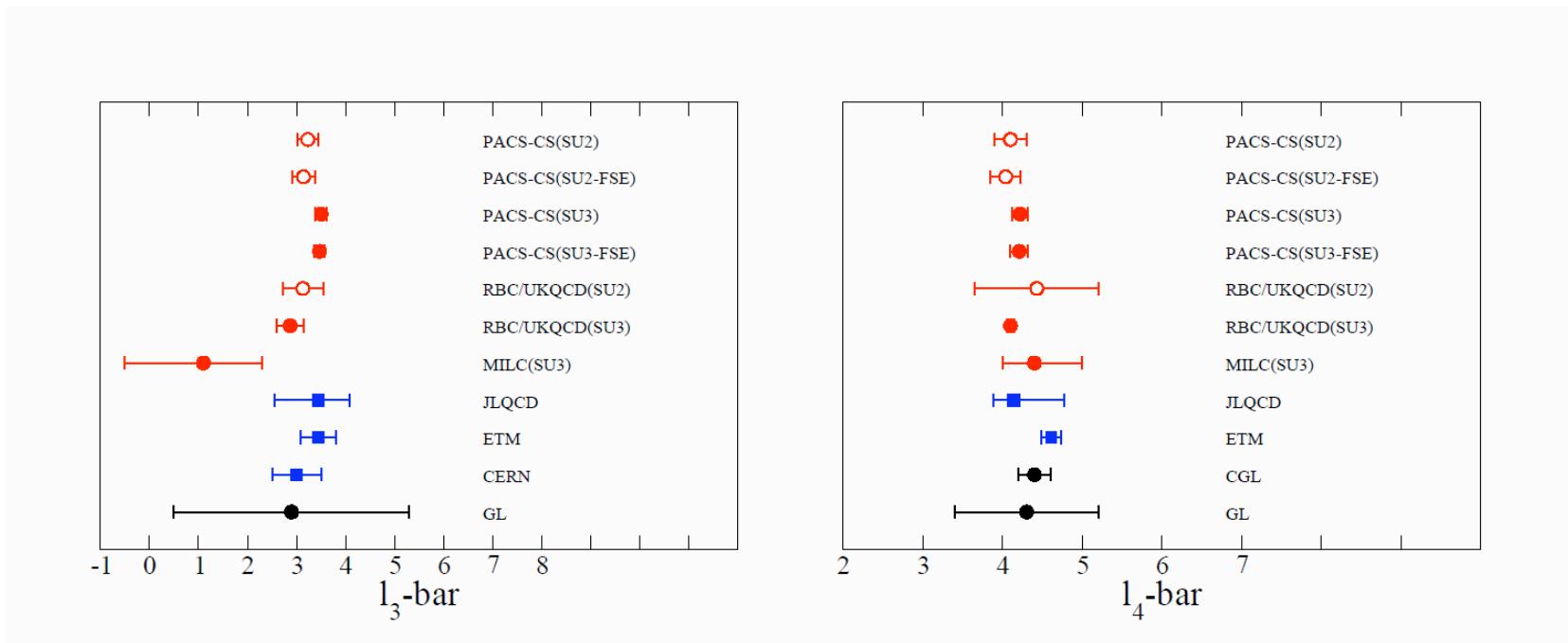
$$\begin{aligned} m_K^2 &= \alpha_m + \beta_m m_{ud} + \gamma_m m_s \\ f_K &= \bar{f} \left\{ 1 + \beta_f m_{ud} - \frac{4}{3} \frac{2m_{ud}B}{16\pi^2 f^2} \ln \left( \frac{2m_{ud}B}{\mu^2} \right) \right\} \end{aligned}$$

with  $\bar{f} = \bar{f}_s^{(0)} + \bar{f}_s^{(1)} m_s$

- simultaneous fit to  $\frac{m_\pi^2}{2m_{ud}}, f_\pi, f_K$ .  
( $m_K^2$  is independently fitted.)

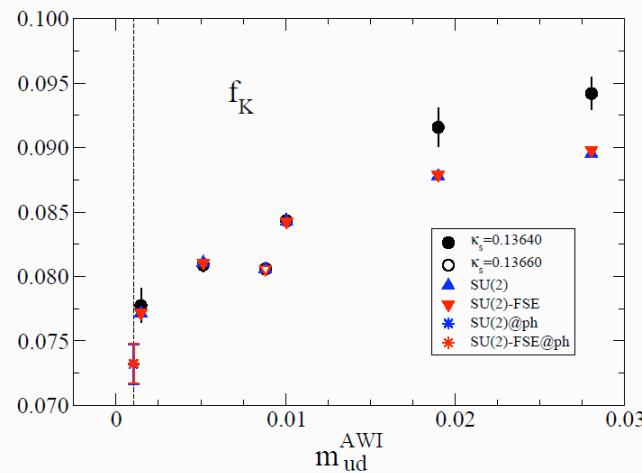
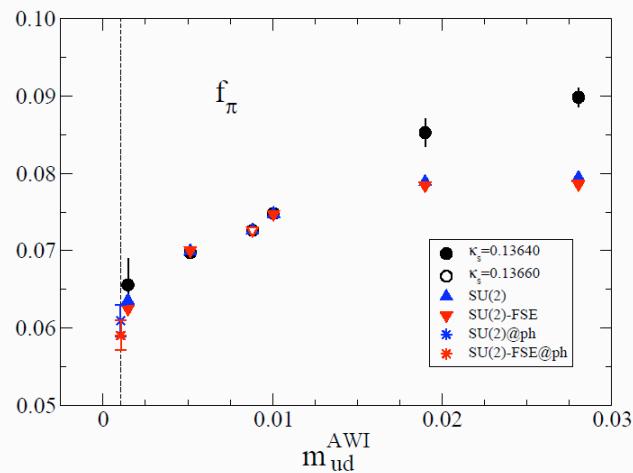
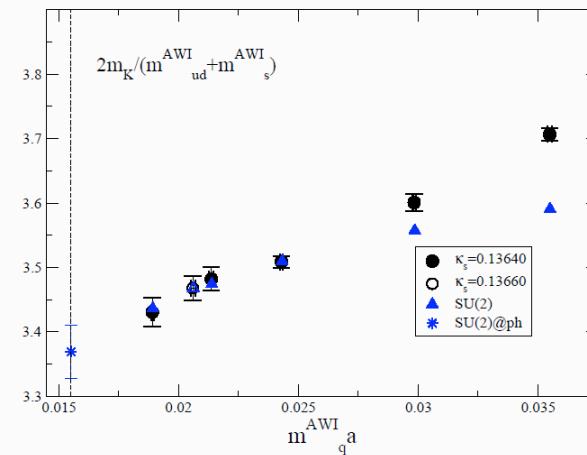
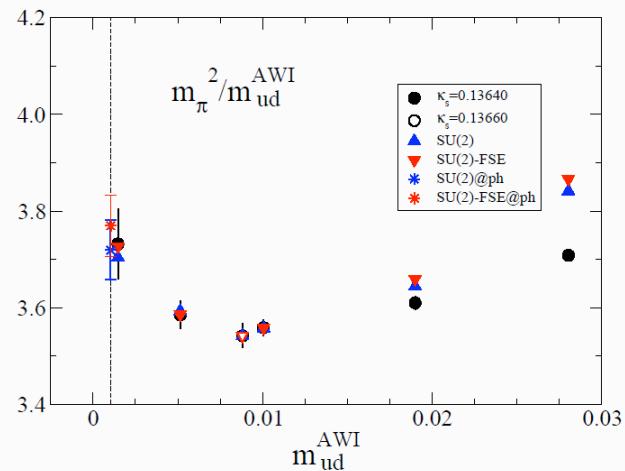
## (1) Low energy constants

8 unknown parameters  $B_s^{(0)}, B_s^{(1)}, f_s^{(0)}, f_s^{(1)}, \bar{f}_s^{(0)}, \bar{f}_s^{(1)}, l_3, l_4$  for  $\frac{m_\pi^2}{2m_{ud}}, f_\pi, f_K$  in SU(2) ChPT with ms-linear expansion.



$$\bar{l}_3 \sim 3.0 - 3.5, \quad \bar{l}_4 \sim 4.0 - 4.5$$

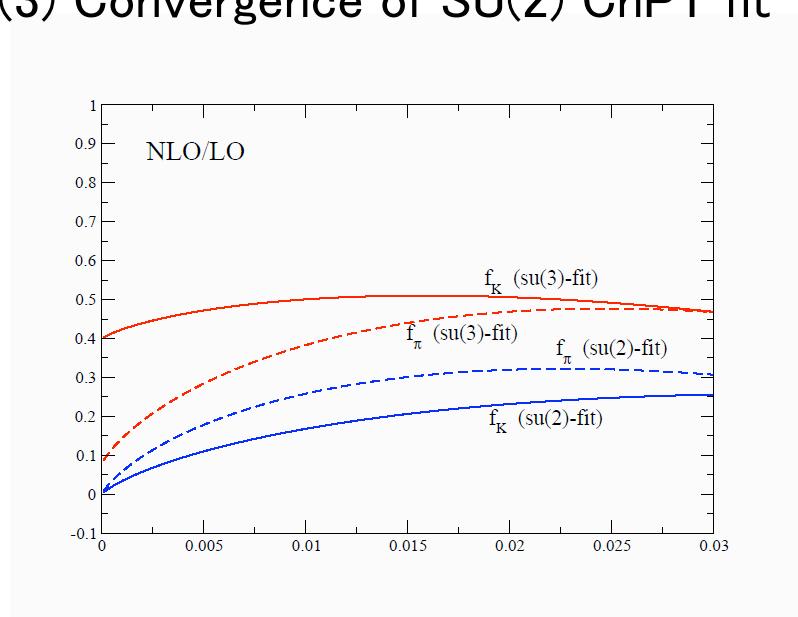
## (2) Fit graph for SU(2) ChPT fit



Fit results are in good agreement with data points.  
This corresponds reasonable values of  $\chi^2/dof$ .

$$\chi^2/dof = \begin{cases} 0.4(0.8) & \text{for } m_\pi, f_\pi, f_K - \text{fit.} \\ 0.3(1.5) & \text{for independent } m_K - \text{fit.} \end{cases}$$

### (3) Convergence of SU(2) ChPT fit



Convergence of SU(2) ChPT becomes better than one of SU(3) ChPT.

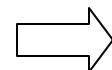
- Cut-off, renormalized quark masses with  $m_\pi, m_K, m_\Omega$  input

	w/o FSE	w FSE
$a^{-1}[\text{GeV}]$	2.176 (31)	2.176 (31)
$m_{ud}(\overline{MS}, 2\text{GeV})[\text{MeV}]$	2.509 (46)	2.527 (47)
$m_s(\overline{MS}, 2\text{GeV})[\text{MeV}]$	72.74 (78)	72.72 (82)

- Pseudoscalar meson decay constants

	w/o FSE	w FSE	experiment
$f_\pi$	132.6 (4.5)	134.0 (4.2)	$130.7 \pm 0.1 \pm 0.36$
$f_K$	159.2 (3.2)	159.4 (3.1)	$159.8 \pm 1.4 \pm 0.44$
$f_K/f_\pi$	1.201 (22)	1.189 (20)	1.223 (22)

(\*) We use perturbative renormalization factors, ZA and ZP  
 to calculate renormalized quark mass and pseudoscalar decay constants  
 Non-perturbative renormalization factors are now calculating.



Talk by Yusuke Taniguchi (Fri, 5:40–)

## 5. SU(2) BChPT for nucleon

Nucleon mass up to  $\mathcal{O}(p^4)$

$$\begin{aligned} m_N = m_0 - 4c_1 m_\pi^2 - \frac{6g_A^2}{32\pi^2 f_\pi^2} m_\pi^3 \\ + \left[ e_1(\mu) - \frac{6}{64\pi^2 f_\pi^2} \left( \frac{g_A^2}{m_0} - \frac{c_2}{2} \right) - \frac{6}{32\pi^2 f_\pi^2} \left( \frac{g_A^2}{m_0} - 8c_1 + c_2 + 4c_3 \right) \ln \frac{m_\pi}{\mu} \right] m_\pi^4 \\ + \frac{6g_A^2}{256\pi f_\pi^2 m_0^2} m_\pi^5 + O(m_\pi^6) \end{aligned}$$

We set some LECs to physical values,

$$g_A = 1.267, \quad c_2 = 3.2 \text{ GeV}^{-1}$$

and two possibilities for  $c_3$ ,

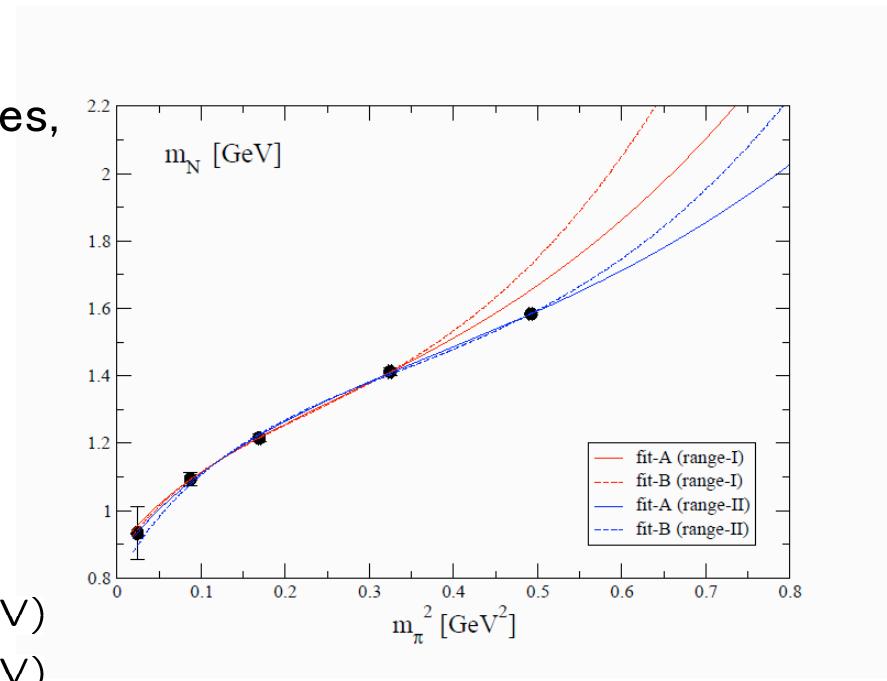
$$\text{fit-A: } c_3 = -3.4 \text{ GeV}^{-1}$$

$$\text{fit-B: } c_3 = -4.7 \text{ GeV}^{-1}$$

We choose two data region,

range-I : 4 points ( $m_\pi$  : 156 – 702 MeV)

range-II : 5 points ( $m_\pi$  : 156 – 702 MeV)



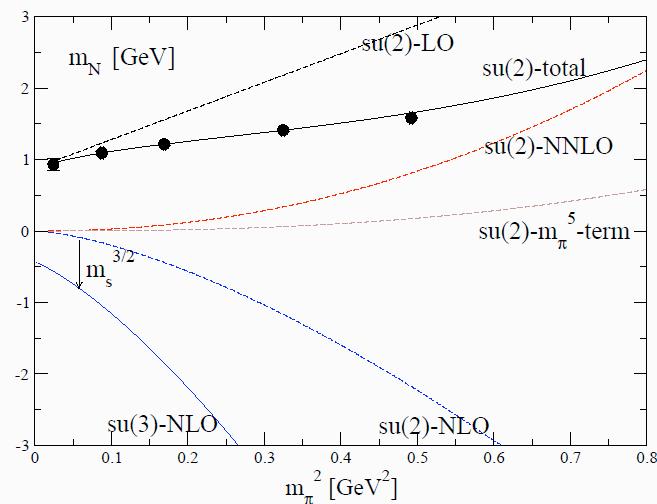
- LECs

QCDSF-UKQCD, 2004

	fit-A(range-I)	fit-B(range-I)	fit-A(range-II)	fit-B(range-II)
$m_0$ [GeV]	0.880 (50)	0.855 (47)	0.850 (27)	0.795 (27)
$c_1$ [ $\text{GeV}^{-1}$ ]	-1.00 (10)	-1.19 (10)	-1.08 (4)	-1.34 (4)
$e_1(1\text{GeV})$ [ $\text{GeV}^{-3}$ ]	3.7 (1.4)	4.2 (1.4)	2.9 (4)	2.4 (3)
$\chi^2/\text{dof}$	0.1 (0.9)	0.0 (0.5)	0.3 (0.9)	1.2 (1.6)
$\sigma_{N\pi}$ [MeV] (**)	51.4 (7.6)	60.1 (7.3)	56.8 (2.7)	70.8 (2.6)

(\*\*) preliminary

- Convergence of SU(2) BChPT for Nucleon



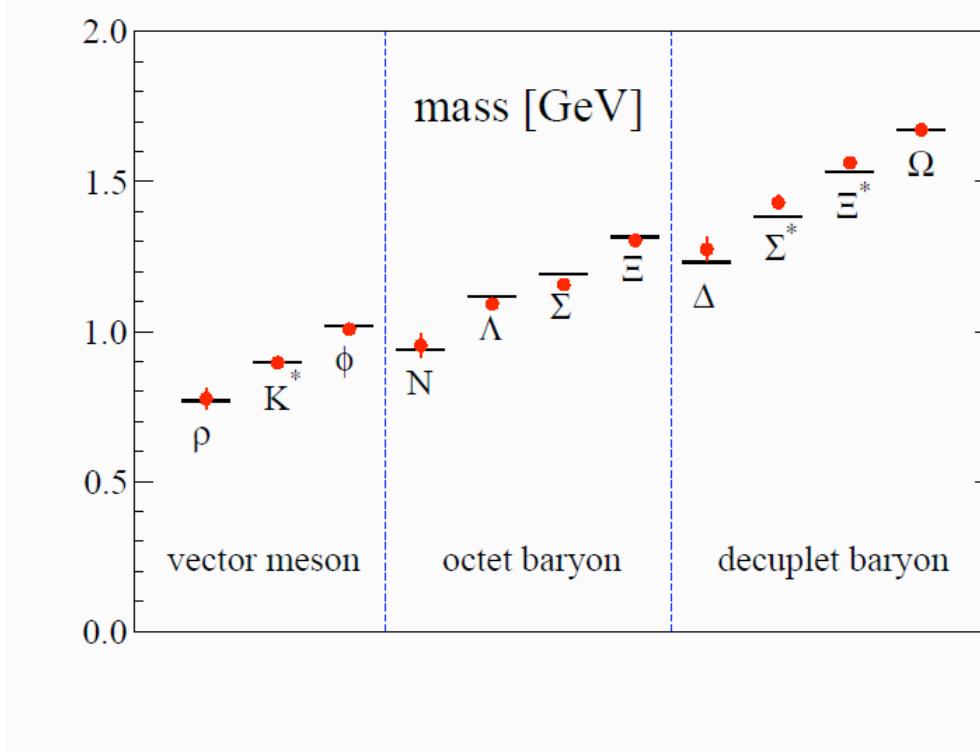
LO and NLO terms cancel each other.  
The convergence is not obvious in large pion mass region.

More drastic cancellation are needed for  
SU(3) BChPT and NNLO would be so large  
due to strange quark mass.

Probably, for 2+1 flavor QCD, it is difficult to obtain the hadron spectrum by performing chiral extrapolation based on vector meson ChPT and baryon ChPT with good convergences. → Simulation at physical point

## Hadron masses at the physical point

- We extrapolate masses of vector mesons, octet baryons and decuplet baryons to the physical point by linear fit formula,  $M = \alpha + \beta m_{ud}^{AWI} + \gamma m_s^{AWI}$



## 6. Summary

- Pion masses are 156–702MeV in our simulation, 156MeV–411MeV points are used for chiral analysis.
- Convergence of chiral expansion is not obvious for SU(3) ChPT and one of SU(2) ChPT is better than SU(3) ChPT.
- Convergences of vector meson ChPT and baryon ChPT would be not obvious because strange quark mass is large even at physical point. So the physical point simulation is more reasonable to obtain spectrum for vector mesons and baryons.