TRANSPORT PROCESSES IN PROTOSTELLAR DISKS

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The current understanding of angular momentum transport processes internal to protostellar accretion disks is reviewed. Recent numerical simulations of nonlinear accretion disk hydrodynamics have revealed that vertical convection, and hydrodynamic turbulence in general, are ineffective sources of angular momentum transport. However, the existence of a local, linear instability in weakly magnetized disks has shifted the focus to the role of magnetic fields and the presence of magnetohydrodynamical turbulence. Numerical simulations demonstrate that the instability saturates as MHD turbulence with vigorous angular momentum transport. The existence and strength of the turbulence depends on the ionization fraction within the disk. The instability does not require perfect coupling between the magnetic field and gas, and may operate on scales up to ∼AU in protostellar disks with ionization fractions $f \equiv n_e/n_H$ as low as $f \sim 10^{-13}$. Beyond this radius, however, the existence of MHD turbulence is increasingly problematic. Gravitational torques and global hydrodynamic waves remain as viable transport mechanisms.

I. INTRODUCTION

One of the primary evolutionary stages of star formation is accretion through a disk. Disks are ubiquitous because it is difficult to remove angular momentum from the gas during the infall stage, whereas it is easy to remove thermal energy via radiative cooling. Thus, the end product of infall will often be a rotationally supported thin disk. Disks are important for a variety of reasons: their presence modifies the spectrum of young stellar objects, they regulate the mass accretion rate onto the protostar, they are thought to be the source of powerful winds and outflows, and ultimately they provide the environment in which planetary systems form. Moreover, accretion through a disk seems to be a long-lived stage of star formation. Hydrodynamical studies of the infall stage show that disks form very early in the evolution of a collapsing core (within one free-fall time, e.g. Bodenheimer et al. this volume), while observationally, disks remain associated with objects near the end of the star forming process (weak-line T Tauri stars, e.g. Strom, Edwards, & Skrutskie 1993).

An understanding of the structure and dynamics of protostellar accretion disks is therefore an essential component of a complete theory of star formation. Most importantly, the mechanism for angular momentum transport in disks must be identified, for without it, accretion cannot occur. Moreover, the transport mechanism is intimately linked with basic physical properties of the disk: its vertical structure, magnetization, temperature, and turbulent velocity dispersion.

Understanding angular momentum transport in accretion disks from first principles has proven challenging. Nevertheless, there has been considerable progress in the past few years, driven primarily by two advances. The first is the use of high resolution numerical simulations to study the three-dimensional hydrodynamics and magnetohydrodynamics (MHD) of accretion disks for many dynamical times. As we discuss in this chapter, these simulations have taught us much about the nonlinear dynamics of disks. The second advance is the recognition of the fundamental role of magnetic fields in differentially rotating systems, beginning with the (re)discovery of a powerful, local, linear instability in weakly magnetized accretion disks (Balbus & Hawley 1991; 1992). Although this magnetorotational instability (MRI) had been studied much earlier in its global form within the context of magnetized Couette flows (Velikhov 1959; Chandrasekhar 1960), its importance for accretion disks went unappreciated for decades. Three-dimensional numerical simulations have shown that the instability leads to fully developed MHD turbulence and significant outward angular momentum transport. The implications of this instability to the structure and evolution of protostellar accretion disks is very much a subject of active inquiry.

Most of this review will focus on discoveries made concerning the local fluid dynamical processes that govern *internal* transport. We will discuss the progress made in understanding the roles that convection and nonlinear hydrodynamical instabilities play (or more appropriately, do *not* play) in contributing to transport. We will describe our current understanding of the MRI in ionized, and partially ionized, disks.

We will confine our discussion to the dynamics of Keplerian protostellar disks, which are much less massive than the central, isolated protostar. Of course if the disk is massive, or if tidal interactions are important, gravitational torques can dominate the transport. Although much of the stellar material may be processed this way, the majority of a disk's lifetime is probably spent in the low mass phase. This phase will be the focus of our review. Gravitational mechanisms are briefly discussed in §IIC, and are reviewed more extensively by Bodenheimer et al. (this volume) and Adams & Lin (1993).

Internal torques are not the only possible transport mechanism. Significant angular momentum loss can also occur through winds. Königl & Pudritz (this volume) review the substantial and important progress made in studies of protostellar disk winds and jets. Our current understanding, however, of the relationship between internal disk dynamics and outflow processes remains incomplete.

There are several recent reviews that provide further details on the dynamics of protostellar disks. Adams & Lin (1993) reviewed disks at PPIII. Papaloizou & Lin (1995a) discuss transport processes in disks. Lin & Papaloizou (1996) focuses on observational constraints. Balbus & Hawley (1998; hereafter BH) review accretion disk turbulence and self-consistent disk dynamo theory.

II. BASIC TRANSPORT MECHANISMS

Because of the ubiquity of disk accretion in systems ranging from protostars to AGN, it has long been assumed that outward angular momentum transport must be an unavoidable consequence of the internal dynamics of disks. Recognizing that molecular viscosity was many orders of magnitude too small to account for observed accretion rates, Shakura & Sunyaev (1973) introduced the ansatz, based on dimensional arguments, that there must exist an "anomalous" stress tensor component $T_{R\phi}$ whose magnitude scales with the gas pressure P according to

$$T_{R\phi} = \alpha P \tag{1}$$

where α is a dimensionless constant, and R and ϕ are the radial and angular coordinates. A similar approach was taken by Lynden-Bell & Pringle (1974) who assumed an anomalous Stokes viscosity

$$\nu_t = \alpha c_s H \tag{2}$$

where c_s is the sound speed in the disk, and H is the vertical scale height. The subscript t emphasizes that from the beginning, the anomalous viscosity was assumed to arise from *turbulence* in the disk. This parameterization made possible the construction of vertically averaged "α-disk" models (see review of Pringle 1981) which have proved extremely useful. Nonetheless, a fundamental question went unanswered in these early models: what is the source of this anomalous (turbulent) viscosity? Three categories seem natural: (A) hydrodynamic turbulence driven either by vertical convection or nonlinear instabilities, (B) MHD turbulence, and (C) nonlocal mechanisms (e.g., gravitational torques in self-gravitating disks, and nonaxisymmetric waves). We consider each in turn.

A. Hydrodynamical Turbulence

There are two primary difficulties with the suggestion that purely hydrodynamic turbulence drives angular momentum transport in protostellar accretion disks. The first is to decide what drives the turbulence. This is a non-trivial issue since hydrodynamic disks are linearly stable by the Rayleigh criterion $\partial L/\partial R > 0$, where L is the specific angular momentum. The second is to ensure turbulence actually transports angular momentum outwards. Again, this is not immediately obvious, since outward transport requires more than the presence of radial and azimuthal velocity fluctuations. It requires that they be well-correlated (Prinn 1990; BH).

In protostellar disks, there is a simple mechanism for hydrodynamic turbulence, namely vertical convection driven by efficient radiative cooling of the surface layers of the disk. Adams & Lin (1993) suggested that nonlinear dissipation of modes identified through a linear analysis (Ruden, Papaloizou, & Lin 1988) might give rise to $\alpha \sim 10^{-2}$ to 10^{-3}. But a linear analysis of nonaxisymmetric disturbances in the shearing-sheet approximation (Ryu & Goodman 1992) showed that these modes actually transport angular momentum *inward*. Two-dimensional axisymmetric simulations of the nonlinear evolution of convective modes by Kley et al. (1993) produced inward transport, but the authors were careful to note that nonaxisymmetric structure might lead to something very different.

With the increase in available computational power, direct hydrodynamical simulations of three dimensional convective disk flows became possible, and several studies were carried out (Cabot & Pollack 1992; Cabot 1996; Stone & Balbus 1996). These simulations begin with a three-dimensional patch of a shearing disk which uses Cartesian geometry while retaining gravitational and Coriolis forces. The disk is initially convectively unstable. Stone & Balbus (1996) found that at late time the convective cells in the $R-Z$ plane look much the same as those in a non-rotating fluid. However in the $\phi - Z$ plane, the cells are stretched by shear into long sheets. Figure 1 shows the angular momentum transport rate (characterized by the α-parameter) over 40 orbits in a disk in which vertical convective motions are sustained by heat input at the equator. Two results are immediately obvious. The first is that the magnitude of the transport rate is very small, generally $|\alpha| < 10^{-4}$. The second is that the time-averaged value of α is negative, implying net inward angular momentum transport. These results show that even when vigorous convective motions are driven in a protostellar disk, the turbulence does not produce significant angular momentum transport.

Convective instabilities are one way to drive hydrodynamic turbulence in an accretion disk; nonlinear shear instabilities have been proposed as another (Dubrulle 1993). The idea is that the nonlinear instability observed in certain types of high Reynolds number Couette flows is a generic breakdown; Keplerian disks should be similarly unstable. Recently, it has become possible to test this conjecture directly with numerical hydrodynamical experiments. Balbus, Hawley, & Stone (1996) computed a series of local simulations of both differentially rotating and simple shear flows. In these Keplerian flow simulations, despite substantial initial perturbations, turbulence inevitably decayed.

Nonlinear shear instabilities are known to have a critical Reynolds number above which instability sets in. In fact, the *linear* Rayleigh instability itself has a large critical Reynolds number in Couette experiments (Drazin & Reid 1981). Thus one might be concerned that numerical simulations have insufficient resolution (or equivalently, too low an effective Reynolds number) to capture the development of the nonlinear instability. However, such concerns can be addressed. First, relatively low resolution simulations have no difficulty reproducing high Reynolds number nonlinear instability where it is known to exist from laboratory experiments, namely in a simple Cartesian shear flow. Since the Keplerian system differs only by the addition of orbital dynamical forces, the absence of nonlinear instabilities in differentially rotating flows cannot be due entirely to the presence of numerical viscosity. Second, experiments with a wide range of numerical resolutions (as many as 256^3 grid zones), and with both the ZEUS (Stone & Norman 1992) and the PPM (Colella & Woodward 1984) hydrodynamics algorithms (which have very different numerical dissipation properties) all give the same result: finite amplitude initial perturbations quickly die out in Keplerian shear flows, and they die out in remarkably detailed numerical agreement, regardless of which code is used (Hawley, Balbus, & Winters 1999).

Balbus et al. (1996) show that the simulation results can be understood from a simple analysis of the dynamical equations themselves. The reader may refer to this paper for technical details, but the gist of the argument is as follows. In both simple shear and differentially rotating flows the shear itself provides a source of free energy to drive turbulence. But energy is not the whole story. The critical question is whether or not the turbulent stress tensor can dynamically sustain the velocity fluctuations required to transport angular momentum. The crucial difference between simple shear and disk systems is the streamwise momentum conservation equation. In differential rotation, angular momentum fluctuations feed off the background angular momentum gradient, which has the opposite sign from the angular *velocity* gradient energy source. Outward transport of angular momentum is directed "uphill" against the angular momentum gradient, and this reduces the amplitude of the angular velocity fluctuations needed for the turbulent transport itself. In contrast, a simple shear flow has only a velocity gradient; there is no significant distinction between linear momentum and linear velocity. In simple shear the dynamical fluctuations that produce transport are sustained, since tapping into the cross-stream velocity gradient is always a source for streamwise fluctuations.

The stabilizing influence of differential rotation can be demonstrated by evolving local disk models that vary only in the choice of background angular velocity gradient, characterized by $\Omega \propto R^{-q}$ (Balbus et al. 1996). Keplerian flows ($q = 1.5$) are both linearly and nonlinearly stable. In models where the background angular velocity gradient is small ($q \to 2$) the linear restoring forces are weaker and the turbulence decays more slowly. Finally, when $q = 2$ angular momentum fluctuations can be sustained: the system becomes nonlinearly unstable. Indeed, the moment equations of constant angular momentum flow are formally equivalent to those of simple Cartesian shear. For $q > 2$, both the angular velocity and angular momentum decrease with radius, and the system is Rayleigh unstable. Thus the nonlinear instability manifests itself only very near the boundary between linear instability and linear stability. It is only in this domain, where linear restoring forces are vanishingly small, that nonlinear effects hold sway.

The implications of these results go beyond the assertion that Keplerian flows are not susceptible to the same sort of instability that afflicts shear flows. They indicate that inward convective transport and nonlinear stability of Keplerian flows are different sides of the same coin. The same orbital dynamics that makes the disk stable also prevents significant transport of angular momentum even when fluctuations are maintained from sources other than differential rotation. An explicit demonstration is provided by a simulation in which turbulence is driven by direct random forcing (Hawley & Balbus 1997). In this simulation, there is *no* net angular momentum transport despite the presence of substantial velocity perturbations. To conclude: (1) hydrodynamical turbulence does not develop spontaneously within a Keplerian disk, and (2) if such turbulence is provided by some means (e.g., convection or random "stirring"), significant outward transport of angular momentum is neither guaranteed, nor, it would appear, likely.

B. MHD Turbulence

Unlike purely hydrodynamic disks, weakly magnetized accretion disks are subject to a powerful, local linear instability (Balbus & Hawley 1991; 1992b). The fastest growing modes are amplified at a rate of order the orbital frequency, so that the perturbation energy is amplified by a factor of 10^4 *per orbit*. This enormous growth rate (it is likely that no linear instability driven by shear in the disk can grow faster [Balbus & Hawley 1992a]) implies that the nonlinear stage of the MRI must have important consequences for disks.

Again, numerical simulations are required to study the nonlinear evolution. The first results from two-dimensional simulations showed that in axisymmetry, disks with a mean vertical field evolve into the channel solution, consisting of two oppositely directed streams (Hawley & Balbus 1991; 1992). Goodman & Xu (1994) showed that this channel solution is an exact nonlinear solution to the shearing sheet MHD equations. More importantly, they showed that in three-dimensions, the channel solution is subject to disruptive "parasitic" instabilities.

Because of the importance of nonaxisymmetric modes to the nonlinear outcome of the MRI, fully three-dimensional simulations are essential. The first such results (Hawley, Gammie & Balbus 1995) borrowed a technique from Couette flow studies (Lees & Edwards 1972) to treat a local patch of the disk, incorporating the shear of the radial boundaries. We refer to this system as the shearing-box. Simulations of homogeneous boxes with a variety of field strengths and configurations confirm the results of the linear analyses, and find that the nonlinear outcome of the MRI is MHD turbulence, which amplifies and sustains an initially weak field. It also provides for vigorous angular momentum transport. For example, Figure 2 plots $\alpha = T_{R\phi}/P$, in a typical example studied by Hawley et al. (1995). In contrast to the convective turbulence shown in Figure 1, strong outward transport of angular momentum is sustained by the MRI.

Brandenburg et al. (1995) and Hawley, Gammie & Balbus (1996) addressed the question of dynamo activity. They carried out simulations of initially self-contained magnetic fields in shearing boxes. (The two studies used different boundary conditions.) Both studies found that the MRI leads to dynamo activity and sustains magnetic fields against dissipation for many resistive decay times. Significantly, the dynamo action cannot be described by a kinematic theory. Lorentz forces can never be ignored in determining the velocity field, even when the field is weak. The point is there is nothing for the field to be weak with respect to; the strength of the magnetic field simply sets the length scale at which the Lorentz force is significant.

Vertically-stratified shearing boxes were the subject of further three-dimensional simulations (Brandenburg et al. 1995; Stone et al. 1996). Stratification does not change most of the properties of the instability reported for homogeneous boxes (e.g., MHD turbulence still results). Most importantly, buoyancy is *not* a significant saturation mechanism; instead local dissipation dominates.

Going beyond the local shearing box representation is difficult because of the large dynamic range between the scale height H and the outer radius in a thin disk. Moreover, the very low densities encountered in the envelope of the disk result in very high Alfvén speeds. Miller & Stone (1999), using special numerical techniques that limit the Alfvén speed, have studied a local patch of a disk extending five scale heights above and below the midplane. Although most of the dissipation still occurs within the disk, buoyancy leads to a strongly magnetized, low density corona. This may have important dynamically consequences for magnetically driven winds from the disk.

Fully global simulations are in their infancy, but are also now being carried out. Matsumoto & Shibata (1997) modeled a thick disk embedded in a vertical field and found that rapid infall of the disk led to strong outflows near the axis. More recently, Armitage (1998) and Hawley & Balbus (1999) have followed the development of turbulence and angular momentum transport in global simulations. No calculation to date has been carried through to the point where a classical global α disk has emerged.

To conclude this brief summary, nonlinear MHD simulations all indicate the MRI produces outward angular momentum transport in disks. Values of α found in the current simulations range from 5×10^{-3} to ~ 0.5 (the range in these values is related to whether or not there is a net flux of magnetic field).

Since nearly all of the studies described above use the assumption of ideal MHD, i.e., that the magnetic field is frozen to the fluid, one may ask whether the results are applicable to protostellar disks, which are only weakly ionized. There are two processes capable of suppressing the MRI in circumstellar disks: ambipolar diffusion (Blaes & Balbus 1994) and resistivity (Jin 1996; BH). Ambipolar diffusion tends to be more important at higher field strength and lower density. For parameters typical of circumstellar disk models, both can be important (Hayashi 1981).

The resistivity of partially ionized plasma is (e.g., Blaes & Balbus 1994)

$$\eta = 230 \left(\frac{n_n}{n_e} \right) T^{1/2} \ \ \mathrm{cm\ s}^{-2} \tag{3}$$

where n_n is the neutral number density, n_e is the electron number density, and T is the temperature. A measure of its relative importance is given by the magnetic Reynolds number

$$Re_M = \frac{H c_s}{\eta} = \frac{c_s^2}{\Omega \eta} \tag{4}$$

Now on dimensional grounds, we expect that the dissipation rate associated with a harmonic disturbance of wave number k to be ηk^2. The resistive stabilization of linear disturbances occurs when

$$\eta k^2 \simeq \Omega \tag{5}$$

a result supported by more detailed calculations (Terquem & Papaloizou 1997; BH). The most rapidly growing wavenumber of the MRI is approximately Ω / v_A, therefore the linear stability will be significantly affected by resistivity when

$$\eta \simeq \frac{v_A^2}{\Omega} \tag{6}$$

corresponding to

$$Re_M \simeq \frac{c_s^2}{v_A^2} \sim \beta \tag{7}$$

Thus the linear growth of the instability should be strongly affected when the magnetic Reynolds number is less than the plasma β parameter.

The instability can be completely quenched if

$$\eta \left(\frac{2\pi}{H}\right)^2 \simeq \Omega, \tag{8}$$

since then dissipation rates would exceed growth rates for all vertical wavenumbers. This leads to

$$Re_M \simeq 4\pi^2 \tag{9}$$

as the threshold for damping at all wavenumbers smaller than the disk scaleheight. On scales $\sim$ 1AU in the solar nebula, this requires an ionization fraction $f \lesssim 10^{-13}$. Only a very small ionization fraction is enough to trigger linear instability. In protostellar disks, there are many regions where f is small but larger than the value required for completely suppressing the MRI. The question then becomes whether the resulting turbulence can be sustained. Thus, studies of the nonlinear evolution of the MRI in resistive disks are important for protostellar systems.

Several groups have begun to investigate the nonlinear structure of the MRI in highly resistive disks. Two-dimensional simulations have recently been presented by Sano, Inutsuka, & Miyama (1998) who studied the resistive range $0.3 \leq Re_M' \leq 3$ (where $Re_M' \equiv v_A^2/\eta\Omega$, rather than the definition given in equation [4]) using purely vertical fields and a variety of field strengths. As expected from a linear analysis (Jin 1996), the instability does not grow for sufficiently small Re_M. At Reynolds numbers above this linear limit, Sano et al. (1998) discovered the two-dimensional channel solution can saturate via reconnection across the radial streams. Fleming, Stone & Hawley (1999) have studied the three-dimensional evolution of the instability in resistive disks for a variety of initial field strengths and topologies. When the disk contains a mean vertical field supported by currents external to the computational domain, ohmic dissipation can never completely destroy the background field. In this case, MHD turbulence is sustained above a Reynolds number of around 10^3. Between this value and $Re_M \sim 4\pi^2$, the instability is present and produces periodic behavior characterized by growth of the channel solution, saturation, and then decay of the fluctuations until the field again becomes weak. At this point, the channel solution emerges again, and the cycle repeats. At low Re_M, reconnection across the streams in the channel solution is an effective saturation mechanism, as was seen in the two-dimensional simulations of Sano et al. (1998).

For simulations with no mean field, however, the evolution can be affected by much larger values of the Reynolds number. Turbulence initially generated by the instability dies away due to field dissipation below a magnetic Reynolds number between 10^4 and 10^3. This suggests that in the absence of a mean field, MHD turbulence in protostellar disks can be inhibited by ohmic dissipation at much larger magnetic Reynolds numbers than expected from the linear analysis.

In low density regions of the disk, where the neutral-ion collision time is longer than a gyroperiod, the dynamics of a weakly ionized disk are described by ambipolar diffusion. MacLow et al. (1995) assumes that the ion inertia can be ignored, that the ion density is a power law of the neutral density, and treats the effect of ambipolar diffusion as nonlinear diffusion term to the induction equation (e.g., Shu 1992). The simulations show the suppression of the linear MRI when the ion-neutral diffusion rate is sufficiently large. Similar results with similar assumptions were obtained by Brandenburg et al. (1995) in three-dimensional simulations. More recently, Hawley & Stone (1998) have reported comprehensive three-dimensional simulations of two-fluid disks in which the ions and neutrals are evolved as separate fluids coupled through a drag term. They find that significant turbulence in the neutral fluid is produced only when the ion-neutral collision frequency is $\gtrsim 100$ times the orbital frequency. The criterion for the linear modes to grow is only that these frequencies be comparable (Blaes & Balbus 1994); again, the nonlinear criterion for significant transport is more stringent.

C. Other Mechanisms

Gravitational instability can also create nonaxisymmetric structures that transfer angular momentum by gravitational and Reynolds stresses. The key parameter that determines whether or not gravitationally driven transport is possible is Toomre's Q parameter, where $Q \leq 1$ implies local gravitational instability:

$$\begin{aligned} Q \equiv \frac{c_s \Omega}{\pi G \Sigma} = 56 \left(\frac{M_*}{M_\odot}\right)^{\frac{1}{2}} \left(\frac{R}{AU}\right)^{-\frac{3}{2}} \left(\frac{T}{100}\right)^{\frac{1}{2}} \left(\frac{\Sigma}{10^3 \mathrm{gm\ cm}^{-2}}\right) \\ \simeq \left(\frac{H}{R}\right) \left(\frac{M_*}{M_{disk}}\right), \end{aligned} \tag{10}$$

where for the last equality we have taken $M_{disk} \simeq \pi R^2 \Sigma$. Local or global gravitational instability usually requires Q close to 1 somewhere in the disk, i.e., a cold, massive disk.

The nonlinear outcome of gravitational instability in circumstellar disks is not yet understood. Several ideas have been suggested, including fragmentation (e.g. Nelson et al. 1998; Bodenheimer et al., this volume), rapid rearrangement of the disk material to a more stable configuration (e.g. Laughlin & Bodenheimer 1994), and persistent instability due to cooling (e.g., Gammie 1999). There is little question that fully developed nonaxisymmetric gravitational instabilities can be a potent source of angular momentum transport. Unlike shear driven turbulence it is not clear how gravitational disturbances approach a dynamically steady state.

Nonaxisymmetric hydrodynamical waves provide another possible mechanism to transport both energy and angular momentum throughout the body of the disk. This is an extensive subject; here we offer only a brief summary of key points. A comprehensive review of tidally excited waves in disks is given in Lin & Papaloizou (1993).

The hydrodynamic turbulent stress tensor is $T_{R\phi} \equiv \langle \rho u_R u_\phi \rangle$ where ρ is the mass density, and $\mathbf{u}$ is the noncircular component of the velocity $\mathbf{v}$, i.e., $\mathbf{v} = R\Omega\hat{\phi} + \mathbf{u}$. The angle brackets represent an average over angle ϕ and suitable radial and vertical scales. In shear turbulence (although not in unmagnetized disks!) the turbulent stress tensor is a significant source of angular momentum transport because the radial and azimuthal velocity fluctuations are well correlated. Wave transport depends on precisely the same correlation tensor. For axisymmetric waves, the velocity components are uncorrelated (more accurately, they are destructively out of phase); for nonaxisymmetric waves, the stress is typically linear in the azimuthal to radial wavenumber:

$$\langle u_R u_\phi \rangle \sim (m/kR)u^2 \tag{11}$$

The equivalent α parameter (pertaining to transport but not to energy dissipation) would be

$$\alpha \sim (m/kR)\mathcal{M}^2, \tag{12}$$

where $\mathcal{M}$ is the Mach number of the velocity fluctuation. Trailing spiral waves transport angular momentum outwards. Even in the extreme case of velocity amplitudes which approach c_s, the α parameter of WKB waves will only be of order $1/kR$. When the wave velocity amplitudes are highly subsonic, the resulting α is even smaller yet.

Other potential difficulties faced by any wave transport mechanism are that waves must be present throughout the bulk of the disk (either because they can propagate or because they are directly excited in situ), and that such disturbances need to be dissipated to exchange angular momentum with the disk. Because waves tend to refract, it is not a simple matter for them to propagate throughout the interior of the disk, if they are excited near the outer edge by a companion. Longer radial wavelength disturbances fare better in this regard, as they are less prone to refraction.

Different types of waves display different propagation behavior. Near the midplane of a disk, the local dispersion relation for hydrodynamical WKB waves is

$$(\omega - m\Omega)^4 - (\omega - m\Omega)^2(k_Z^2 a^2 + k_R^2 a^2 + \kappa^2) + k_Z^2 a^2 \kappa^2 = 0 \qquad (13)$$

where k_R and k_Z are the radial and vertical wavenumbers, ω the wave frequency, Ω the local rotation frequency, a the adiabatic sound speed, and κ is the epicyclic frequency. At large wavenumbers, two distinct branches of solutions emerge from the dispersion relation: density waves (rotationally modified sound waves),

$$(\omega - m\Omega)^2 - (k_Z^2 a^2 + k_R^2 a^2 + \kappa^2) = 0 \qquad (14)$$

and inertial waves (Vishniac & Diamond 1989),

$$(\omega - m\Omega)^2 = \left(\frac{k_Z^2}{k_Z^2 + k_R^2} \right) \kappa^2. \qquad (15)$$

Density waves can steepen and form shocks (Sawada, Matsuda, & Huchisa 1986; Różyczka & Spruit 1993); the idea that shocks might be a possible source of angular momentum transport in disk galaxies is an old one (Roberts 1969; Shu, Milione, & Roberts 1973). Two-dimensional simulations suppress the effects of refraction and show relatively strong spiral structure (Różyczka & Spruit 1993). More recent results of tidally-driven three-dimensional simulations (Yukawa, Boffin, & Matsuda 1997) are less dramatic. The status of spiral shock accretion remains open pending more refined numerical studies.

Inertial waves were first considered in the context of accretion disks by Vishniac & Diamond (1989), and can be resonantly excited (Goodman 1993; Lubow & Ogilvie 1998). But these waves also propagate slowly and tend to be refracted upwards. A strong case for substantial angular momentum transport by inertial waves has yet to be made.

Long wavelength bending waves of the disk, which will be excited when the orbital plane of the companion differs from the disk plane (Papaloizou & Lin 1995b; Papaloizou & Terquem 1995; Terquem 1998), represent yet another possibility. These waves are not internally propagating WKB modes, hence they are not represented by equation (13). It is a complicated matter simply to write down a general dispersion relation for bending waves ("warps"), and we shall not present such a formula here. (See, e.g., Papaloizou & Lin 1995b).

Terquem (1998) has recently analyzed the properties of bending waves in a viscous Keplerian disk. Because such waves need to be damped to interact with the disk, some form of dissipation needs to be present. For linear waves some viscosity is presumably associated with the turbulent transport in the disk, but this means that the waves themselves cannot be the entire basis of disk transport. But they can alter the effective α, as a matter of principle, by a factor of order unity. If the waves reach nonlinear amplitude they may be self-dissipating (Papaloizou & Terquem 1998), leading to a scenario in some respects similar to spiral shocks.

To summarize, wave propagation, by itself, is less efficient than fully developed MHD turbulence as angular momentum transport mechanism. But the excitation process is strongest in the outer regions of protostellar disks, where MHD turbulence may by ineffective. The interplay between waves and MHD turbulence is an underexplored—indeed largely unexplored—process which may be quite important for the evolution of binary disks.

III. TRANSPORT PROCESSES AND THE RADIAL STRUCTURE OF DISKS

¿From the results discussed in §II, it is clear that the nature and efficacy of the angular momentum transport mechanism strongly depends on physical conditions within the protostellar disk, which will vary rapidly with both radius and height. Thus, transport across the entire disk, from protostar to outer edge, will not be mediated by a single mechanism. In the following subsections, we divide the disk into three regions where the dynamics of angular momentum transport may be dominated by significantly different processes. The divisions are somewhat arbitrary, and the actual structure is almost certainly more complex than we have assumed. Figure 3 is a sketch of the radial structure described below.

A. The Circumstellar Region: Star-Disk Interaction

Within the innermost few stellar radii, the dynamics of the disk will be strongly influenced by interaction with the central star. There is substantial evidence that protostars have strong stellar magnetic fields (e.g., Montmerle et al. 1993). Thus, the star–disk interaction will be a complex interplay driven by the large inertia of the accretion flow and the strong magnetic forces within the protostellar magnetosphere.

The theory of the interaction of a stellar magnetosphere and an accretion disk is still being developed (Pringle & Rees 1972; Ghosh & Lamb 1979; Lovelace, Romanova, & Bisnovatyi-Kogan 1995; Shu et al. 1994). It seems likely, both on theoretical and observational grounds, that within the Alfvén radius R_A (at which point the Alfvén speed in the disk is comparable to the orbital velocity) the disk will be disrupted by the stellar field. The accretion flow will no longer be in the equatorial plane but instead will flow along stellar field lines to the magnetic poles. Precisely how plasma is loaded onto the stellar field lines is poorly understood; it may require a large effective (perhaps turbulent) resistivity. Near R_A, the disk will be threaded by stellar field lines, and a strong magnetically driven outflow may be produced (Shu et al., this volume). Magnetic coupling between the star and disk can control the rotation rate of the central star (Königl 1991; Lovelace et al. 1995).

Angular momentum transport in this star–disk interface can be regulated by several processes. The most likely are (1) MHD turbulence; (2) torques exerted by the rigid magnetospheric field; and (3) magnetically driven winds and outflows. The large anomalous resistivity required to load the disk plasma onto stellar field lines suggests the internal dynamics and magnetic field in this region is turbulent and highly tangled, implying that MHD turbulence must be present to some degree.

The dynamics of the circumstellar region may be the most complex in the entire system. For this reason, numerical simulations have become an increasingly important investigative tool. Hayashi, Shibata, & Matsumoto (1996) have used simulations to study the evolution of a Keplerian disk embedded in a pure dipole field anchored in the central star. Shearing of the field lines produces expansion of the magnetosphere (Lovelace et al. 1995; Lynden-Bell & Boily 1994), and reconnection ejects isolated plasmoids. As evidence for such reconnection events, these authors point to recently observed X-ray flares in T Tauri stars. Goodson, Winglee, & Boehm (1997) use a nested mesh code to follow the evolution of a similar problem over large spatial scales. As a result, they find not only plasmoid ejection, but also global collimation of these objects into jet-like structures. Finally, Miller & Stone (1997) followed the axisymmetric evolution of a Keplerian disk in the magnetosphere of a central star. Rapid radial collapse of the disk driven by the MRI or magnetic braking is always observed, and polar cap accretion flows occur in some circumstances. Although substantial progress has been made, important questions (such as identifying the *dominant* transport mechanism in this region) remain for future studies.

B. The Inner Disk: MHD Turbulence

Since the transport properties of the disk depend crucially upon the degree of magnetization, protostellar disks must have complex radial structure. The key parameter is the ionization fraction, f. If the resistivity of the gas is small enough, and the neutral–ion collision rate rapid enough, the MRI is triggered. As discussed in §IIB, an ionization fraction as low as 10^{-13} is sufficient to trigger the linear instability on

AU scales in standard solar nebula models. However, linear instability is not necessarily a guarantee that MHD turbulence will be maintained at significant levels. Nonlinear MHD simulations of resistive plasmas show that in the absence of a net vertical field, turbulence triggered by the instability ultimately dies away for Reynolds numbers below $Re_M \lesssim 10^4$.

¿From these results it seems that the extent over which MHD turbulence will be important in a typical protostellar disk is likely to be rather small, somewhere between 0.1 and 1 AU. To see this, write the magnetic Reynolds number in the form

$$Re_M \simeq T^{1/2} R_{AU}^{3/2} \left(\frac{n_e}{n_n}\right)_{-12} (M_\odot/M)^{1/2} \tag{17}$$

where R_{AU} is the disk radius in AU units, $M/M_\odot$ is the central mass in units of solar mass, and n_e/n_n is the ionization fraction in units of 10^{-12}. At solar abundances, thermal ionization of the disk at the onset of magnetic coupling will be controlled almost entirely by potassium, with an ionization potential $\Phi = 4.341$eV (Allen 1973). The solution to the Saha equation may be simply expressed in this regime as

$$\frac{n_e}{n_n} = a^{1/2} T^{3/4} \left(\frac{2.4 \times 10^{15}}{n_n}\right)^{1/2} \exp(-50,370/2T) \tag{18}$$

where a is the abundance of potassium ($\sim 10^{-7}$ if roughly solar), and n_n is the density of the dominant neutral species, generally molecular hydrogen for $T < 2000$ K. The Boltzmann factor makes the ionization exquisitely sensitive to temperature, acting as an "on-off" switch for $T \sim 1500$.

Take a simple disk model described by a surface temperature T_s, surface density Σ, midplane temperature T, and midplane optical depth τ,

$$T_s^4 = \frac{3}{4}\tau T^4 = \frac{3}{4}\tau \frac{3GM\dot{M}}{8\pi R^3 \sigma}, \quad \dot{M} = 3\pi\Sigma\alpha c_S H. \tag{19}$$

This leads to

$$T = 1100 \left(\frac{\alpha}{10^{-2}}\right)^{1/3} \left(\frac{\Sigma\tau}{10^6}\right)^{1/3} R_{AU}^{-1/2}. \tag{20}$$

The principal uncertainty is the combination $\Sigma\tau$; fortunately T depends rather weakly on the disk parameters. A midplane temperature as high as 1700 K would give rise to $Re_M \simeq 4 \times 10^6$ at 1 AU; such a disk would be easily magnetized within this radius. But if the temperature drops below 1400, Re_M approaches 10^4, the regime of suppressed turbulence.

This result is appropriate to disks with no net vertical field. If the disk contains a net field from an external source (e.g. a dipole field from the central protostar, or the remnant interstellar field), then, as discussed in §IIB, significant transport can persist at a much lower Re_M (Fleming et al. 1999).

In summary, the heating of the disk by its own dynamical dissipation permits MHD turbulence on scales up to 1 AU or so. In principle, this region could be as small as 0.1 AU if turbulence cannot sustain thermal ionization levels; at this radius, stellar heating alone will maintain requisite ionization levels.

C. The Outer Disk: Layered Accretion

While the thermally ionized inner portions of the disk can be understood with some confidence, the ionization fraction is much less certain in those regions of the disk where nonthermal ionization dominates. Cosmic rays may be an important source of ionization, if they are not excluded from the surface of the disk surrounding magnetic structures. Once in the disk, cosmic rays are exponentially attenuated below a column of $\simeq 10^2$g cm^{-2}, thus suggesting the possibility of "layered accretion" (Gammie 1996), in which the surface layers are turbulent, but the center of the disk is a quiescent "dead zone." X-rays are another possible source of ionization (e.g. Glassgold, Najita, & Igea 1997), and they are likewise strongly attenuated toward the midplane. Decay of ^{26}Al and ^{40}K is another possible source of ionization (Umebayashi 1983), although estimates suggest that it is not quite sufficient to provide good coupling of disk to magnetic field. Of course the mean abundance of these elements is uncertain, and they may be concentrated by dust settling into particular regions in the disk (although not self-consistently if the disk is turbulent!). Uncertainties also remain on the recombination side. In particular, small dust grains are capable of soaking up most of the free charges in the gas and effecting the ion chemistry. In sum, the ionization state of the bulk of circumstellar disks is uncertain and an interesting area for future research; it may govern the evolution of circumstellar disks.

In any event, it seems likely that beyond $R \gtrsim 0.1$–1 AU, substantial portions of the center of circumstellar disks are not well coupled to the magnetic field and are therefore not heated by MHD turbulence. Since these parts of the disk can suddenly *become* well coupled once the temperature rises above 10^3 K, there is the possibility of unstable accretion. An excursion in temperature above 10^3 K will be rewarded by turbulence and further heating, causing surrounding regions also to rise in temperature. This nonlinear switch is a robust feature of any model that incorporates angular momentum transport by MHD turbulence. Its consequences have been explored by Gammie (1996), who finds that, combined with gravitational instability (which provides the "match" to ignite the disk), it leads to unsteady accretion similar to that observed in FU Orionis objects.

Many aspects of the outer disk region remain to be studied. The vertical structure produced by layered accretion in the disk has yet to be explored. As shown by two-fluid studies of the MRI in weakly ionized disks (Hawley & Stone 1998), or in the presence of large ohmic dissipation (Sano et al. 1998; Fleming et al. 1999), the nonlinear saturation may not always result in fully developed MHD turbulence. Moreover, transport by hydrodynamic waves is always a possibility, especially if a companion object is present. Finally, in the outer regions of the disk, infall may be occurring. The distinction between the disk and its environment becomes blurred, and it is no longer tenable to view the disk as an isolated structure.

IV. PROBLEMS FOR THE FUTURE

A detailed understanding of the internal dynamics of protostellar accretion disks is fundamental and essential to a theory of star formation. There are several areas where progress is needed before this can be achieved. Several of the most important areas include:

Better Treatment of the Thermodynamics. Because the ionization fraction of the disk is so sensitive to temperature, it is important to improve the treatment of the thermodynamics of the disk in MHD studies. The problem is difficult, because it requires modeling the microphysical dissipation properly (which supplies the heat input), as well as heat transport via both turbulence in the optically thick core of the disk and radiation in the optically thin corona (making the problem in many ways similar to stellar convection).

Modeling the Ionization Fraction. Whenever the MRI can operate, it should dominate over all other internal transport processes. Thus, an important question is, in what regions of the disk is the ionization fraction is too low to support the MRI? This question requires coupling the ionization balance equations with the dynamics (which will necessarily have to include a precise treatment of the thermodynamics given the sensitivity of the ionization rate on temperature). Moreover, the outcome will depend on uncertain quantities such as cosmic ray fluxes at the disk surface, irradiance by high energy photons, and ionization and recombination processes in a dusty gas. Nonetheless, modeling the ionization fraction will be an important extension of current MHD studies.

Global Dynamical Models. Almost all the dynamical models to date focus on local patches of the disk. Fully global models of thin disks have yet to be constructed, although global cylindrical flows have just begun to be studied (Armitage 1998). This is a promising area for rapid progress in the next few years.

Relative Importance of Internal and External Torques. Once fully global models incorporating both the internal dynamics and external envelope are possible, investigating the relative transport contributions of magnetically driven outflows (i.e., external torques) and MHD turbulence (i.e., internal torques) becomes feasible. Can the MHD dynamo in the disk generate an external field which drives an outflow? Or is an externally supplied field, perhaps the remnant from the molecular cloud, required? How does the vertical structure of the disk affect the launching of the wind? Given the importance of protostellar outflows to star forming regions, this is one of the major outstanding questions of the field.

Disk Formation and Evolution. Time-explicit MHD simulations can only follow the evolution of disks for relatively short times, perhaps hundreds of orbits, simply because of the limits imposed by current computer performance. But there are several key issues which can only be addressed through much longer time evolution. It may soon be possible to improve on the standard α formalism, and provide a better parameterization of angular momentum transport in terms of the physical quantities in the disk. Time-implicit methods could then follow disk evolution on much longer time scales (much in the same way that stellar evolution is studied using a parametrization for convective heat transport).

While many challenges remain, our understanding of local transport processes in circumstellar disks has improved considerably since PPIII. We know the mechanism by which accretion disks become turbulent. It is possible, in principle, to construct global dynamical models in which angular momentum transport arises spontaneously. We foresee a time when predictive protostellar disk studies can be pursued *ab initio.* This would truly be a milestone.

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FIGURE CAPTIONS

Figure 1. Evolution of the Reynolds stress normalized by the pressure at the disk midplane (equivalent to the α parameter) in a disk in which vertical convection is driven. Note α is both very small and, on average, negative.

Figure 2. Evolution of the total (Reynolds plus Maxwell) stress normalized by the pressure at the disk midplane (equivalent to α) in a three-dimensional local simulation of a weakly magnetized disk. The MRI is able to sustain strong angular momentum transport apparently indefinitely.

Figure 3. Possible radial structure of a protostellar accretion disk, as discussed in the text. Region A is dominated by the interaction of the stellar magnetosphere and disk. In region B the ionization fraction is large enough that the disk is dominated by MHD turbulence throughout. In region C the central regions of the disk are decoupled from the magnetic field and therefore quiescent, resulting in layered accretion. Beyond region C, the disk merges with the ambient cloud core, and can no longer be considered as an isolated structure. The location of the transitions between regions is highly uncertain.