MULTIPLE FRAGMENTATION OF PROTOSTARS

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Most stars are members of binary or multiple star systems, yet our understanding of multiple star formation is rudimentary at best. Multiple star formation must be understood before we can claim to understand fully the formation of single stars, or of planets around either single or multiple star systems. Theoretical mechanisms for the formation of binary and multiple star systems, as well as recent numerical models, are reviewed here. Comparisons of results obtained with different numerical codes are presented. Observations of main-sequence and pre–main-sequence stars, as well as binary protostars, imply that most binary stars form by fragmentation during the collapse of dense molecular cloud cores.

I. INTRODUCTION

Observations of the properties of binary and multiple star systems provide the ultimate test of any theory of how these systems came into being. Accordingly, we begin with a brief summary of those properties which seem to be the most relevant for choosing between the various mechanisms proposed for binary and multiple star formation.

A. Main-Sequence Binaries

Duquennoy and Mayor (1991; hereafter DM) surveyed all known F7-G9 dwarf stars within about 22 pc of the sun, providing a relatively unbiased sample of main-sequence (MS) stars in the sun's neighborhood. Fischer and Marcy (1992; hereafter FM) performed a similar

survey of M dwarf stars within 20 pc.
1. Frequency. DM found that their sample had a ratio of number of single:binary:triple:quadruple (= s:b:t:q) systems of 57:38:4:1, for companions with a mass ratio $q = M_2/M_1 > 0.1$. DM also found hints for very low mass companions ($q < 0.1$) in about 10% of the primaries, and concluded that each primary star had about 0.5 companions, on average. FM found that the M dwarfs had a similar average number of companions (0.55) and a similar system ratio of s:b:t:q = 58:33:7:1. However, Leinert et al. (1997) found the average number of companions to be ~ 0.32 for their sample of M dwarfs within 5 pc of the sun.
2. Distribution with period. The orbital period (P) distribution of the DM sample was Gaussian-like as a function of log P with a median $P \approx 180$ yr. The periods ranged from less than a day to over 10^6 yr. A similar distribution was found by FM for the M dwarfs. Multiple systems were invariably hierarchical in structure, with both long and short period components.
3. Mass ratios. The number of systems as a function of q in the DM sample is Gaussian with a maximum near $q = 0.23$, similar to a distribution drawn from the field star initial mass function (Kroupa et al. 1990). FM found that their sample of low mass dwarfs was deficient in companions of even lower mass (below $q \sim 0.5$), suggesting that brown-dwarf companions are relatively rare. Mazeh et al. (1992) reanalyzed DM's data for short period (< 3000 days) binaries and found that the distribution was relatively uniform or even rising with q for $0.1 < q < 0.9$, implying a different distribution for long period and short period systems.
4. Orbital eccentricity. Binaries with $P < 11$ days in the DM sample had circular orbits, consistent with circularization produced by tidal dissipation during the lifetimes ($10^9 - 10^{10}$ yr) of these stars. Longer period systems had a range of eccentricities scattered between $e \approx 0.1$ and $e \approx 0.9$.
5. Age. Patience et al. (1998) searched 167 bright stars in the Hyades cluster (age $\approx 6 \times 10^8$ yr) for companions with a mass ratio as low as $q \approx 0.23$. They found that each primary had an average of at least 0.46 companions, close to the results for G and M dwarfs, which have a mean age of several Gyr. A sample of 144 primaries in the even younger ($\sim 10^8$ yr) Pleiades cluster also showed a binary frequency similar to that of the solar neighborhood (Bouvier et al. 1997).

B. Pre–Main-Sequence Binaries

1. Frequency. The binary frequency for young stars in the Taurus, Ophiuchus, Lupus, Chameleon, and Corona Australis star-forming regions is about two times higher than that of the DM sample for separations of 15 AU to 1800 AU (Ghez et al. 1997). Köhler and Leinert (1998) found the same enhancement factor for PMS stars in Taurus

in about the same range of separations. However, the Orion Trapezium cluster has a binary frequency in good agreement with that of MS stars (Petr et al. 1998; Simon et al. 1999). Brandner and Köhler (1998) found statistically different distributions of binary separations in different regions of the Upper Scorpius OB association, and suggested that the MS distribution could result from the superposition of distinct distributions produced in varied star-forming regions.
2. Distribution with period. Mathieu (1994) showed that pre–main-sequence (PMS) stars have a distribution of periods similar to that of MS stars, given the limitations of the PMS sample at the time.
3. Mass ratios. These have not yet been determined reliably for PMS stars, for a variety of reasons (see Mathieu 1994).
4. Orbital eccentricities. Mathieu (1994) showed that PMS stars have a distribution of eccentricities as a function of P that is very similar to that of MS stars, except that circular orbits occur only for P less than ~ 5 days, consistent with the reduced amount of time for tidal circularization in these young stars.
5. Ages. T Tauri stars have ages on the order of a few times 10^6 yrs, and their high binary frequency implies that binary formation has already finished by this age. Brandner and Zinnecker (1997) found that of eight spatially-resolved PMS binaries that could be compared to theoretical evolutionary tracks, all eight pairs were coeval to within theoretical and observational uncertainty (see also Hartigan et al. 1994 and Chapter by Mathieu et al.).

C. Protostellar Binaries

Observations of binary and multiple protostars are as yet too few in number to provide a statistical summary of their properties. Fuller et al. (1996) found an embedded binary protostar in Taurus with a separation of 2800 AU and an age of $\sim 5 \times 10^3$ yr, much less than its $P \sim 10^5$ yr. Lay et al. (1995) detected a hierarchical quadruple (or higher order) protostar, while Wootten (1989; see also Mundy et al. 1992 and Walker et al. 1993), Looney et al. (1997), and Terebey et al. (1998) found protostellar binaries with separations of about 840 AU, 50 AU, and 42 AU, respectively.

II. INITIAL CONDITIONS

The observational evidence strongly suggests that binary formation is closely related to the star formation process itself. Although it is well established that star formation takes place in molecular cloud cores (Myers 1985, 1987), the details of the initial conditions are still not well understood. The typical core, which is close to virial equilibrium, has a scale of about 0.1 pc, a mass in the range one to a few solar masses, and a composition near solar. The hydrogen is in the form of H_2, the total number density is $10^4 - 10^5$ cm^{-3}, the temperature is about 10 K, and the core is slowly rotating. These conditions give a thermal Jeans mass also of order 1 $M_\odot$ (Larson 1985). The parameters for describing the initial conditions include α and β, which are, respectively, the ratios of thermal and rotational energy to the absolute value of the gravitational potential energy, as well as the cloud shape, the density distribution $\rho(r)$, and the angular momentum distribution. High-resolution observations of star-less, pre-collapse cores in regions of isolated star formation show that all of them are centrally condensed with Gaussian or power-law density profiles (Walker et al. 1990; Ladd et al. 1991; Fuller and Myers 1992; Ward-Thompson et al. 1994; Henriksen et al. 1997). For example, the isolated pre-stellar core L1689B (André et al. 1996) has a core ($r \leq 4000$ AU) with $\rho \propto r^{-0.4}$ or $\rho \propto r^{-1.2}$, depending upon the assumed geometry, and a mass of order 0.3 $M_\odot$. Outside of the core region the density decreases as r^{-2}. In contrast, Motte et al. (1998) find that the clustered star formation region ρ Ophiuchi has 10 compact pre-stellar clumps with steep r^{-2} density profiles everywhere, although 19 other cores in the region have the less-steep profiles in the central regions. These results indicate that the density profiles for pre-stellar cores could depend on the environment.

In general, pre-stellar cores appear to be elongated and irregular. Myers et al. (1991) deduced that many cloud cores are actually prolate with apparent aspect ratios of 2:1; some evidence for oblate cores also exists (Boss 1996). On scales of molecular cores, rotation has been detected in about 50% of all cases with the values for the angular velocity ranging between $\Omega \approx 10^{-13}$ to 3×10^{-15} s^{-1} (Goodman et al. 1993; see review by Bodenheimer 1995). The dependence of angular velocity on the radius of a core is $\Omega \sim R^{-0.4}$, with a large scatter. The typical ratio $\beta \sim 0.02$ is too small to explain the observed axis ratios; also the observed velocity gradients within individual cores are consistent with the assumption of uniform rotation.

The initial conditions for protostellar collapse could be strongly affected by magnetic fields, which can support molecular cloud regions against collapse and which control the rate at which they evolve into dense cores. As described by Shu et al. (1993), the material gradually drifts relative to the magnetic field due to ambipolar diffusion, the turbulence decays, the cloud undergoes a slow contraction on the diffusion time scale, and a quiet, dense protostellar core forms. This conclusion is in agreement with observations that dense pre-stellar cores ($n \approx 10^5$ cm^{-3}) are characterised by low linewidths of order 0.2 to 0.5 km/s (Benson and Myers 1989; Myers and Goodman 1988; Lemme et al. 1995) in which the thermal contribution dominates. During the phase of magnetically supported contraction the core acquires an r^{-2} density distribution (Lizano and Shu 1989; Tomisaka et al. 1990). Starting from the center, magnetic effects gradually become unimportant and the cloud experiences an inside-out collapse. If the initial conditions are produced in this manner, the portion of the cloud that begins to collapse will contain close to 1 thermal Jeans mass, and depending on the details of the initial structure, may not fragment.

Shu et al. (1993) note that the theoretical r^{-2} density profile represents an asymptotic result and that in reality cores would become gravitationally unstable in their central regions before the singular density cusp develops. The results of detailed 2-dimensional cloud contraction models (Ciolek and Mouschovias 1995; Hujeirat 1998; Ciolek and Königl 1998) indeed show that the density profile becomes quite close to $\rho \propto r^{-2}$ at least in the outer regions, but that the central regions retain a relatively uniform density. As a result, magnetically regulated pre-stellar cores will enter the dynamical collapse phase with $\rho(r)$ flatter than r^{-2} in the central region, at least qualitatively in agreement with observations. Note also that magnetic braking induces solid body rotation during the equilibrium contraction phase, as long as the magnetic braking time is shorter than the ambipolar diffusion time (Basu and Mouschovias 1994).

III. PHYSICAL PROCESSES

We discuss now the important physical processes that can influence the occurrence of fragmentation during the collapse of a cloud starting from initial conditions similar to those just described.

A. Radiation Effects

Two different phases of protostellar collapse are generally considered in fragmentation simulations. The first, the isothermal phase, corresponds to low-density ($\rho = 10^{-19}$–10^{-13} g cm^{-3}) gas which is optically thin to infrared radiation. A balance between the rates of heating by gravitational compression and cooling through emission by grains is obtained at a temperature of about 10 K over this entire density range (Hayashi 1966; de Jong et al. 1980; Whitworth and Clarke 1997). This phase is favorable for fragmentation. The general criterion for a uniform-density cloud with uniform rotation to fragment in the isothermal phase is $\alpha\beta < 0.12$ (Hayashi et al. 1982; Miyama et al. 1984). However if the cloud is centrally condensed, the region of (α, β) space where fragmentation occurs is reduced (Boss 1993).

The second phase starts in the central regions of the protostar or fragment when it becomes dense enough to be optically thick. The density is high enough by this point so that the free-fall time rapidly becomes short compared with the diffusion time for the radiation, so the radiation becomes trapped in the infalling matter and the collapse is essentially adiabatic. The cloud becomes less susceptible to fragmentation because increased pressure effects reduce the perturbation amplitude during this phase. The general criterion for fragmentation in a cloud of initially uniform density becomes $\alpha < 0.09\beta^{0.2}$ for a gas with $\gamma = 1.4$ (Hachisu et al. 1987; Tohline 1981; Boss 1981; Miyama 1992). The dust grains are important both for the cooling during the isothermal phase and for the radiative opacity during the optically thick phases; opacity calculations for the relevant temperature regime and with approximately solar composition are provided by Pollack et al. (1985), Alexander and Ferguson (1994), and Preibisch et al. (1993).

The transition between isothermal and adiabatic phases is important with regard to the minimum mass of a fragment produced during the collapse. During the isothermal phase the Jeans mass decreases as the density increases; however once the fragment becomes optically thick and heats, the Jeans mass begins to rise. Thus opacity effects determine the minimum mass of a fragment, which is estimated to be ≈ 0.01 $M_\odot$ (Low and Lynden-Bell 1976, Rees 1976, Silk 1977, Boss 1988a). Although grain opacities in the range 10 K $<$ T $<$ 100 K are not well known, this result is not sensitive to the precise value.

B. Equation of State and Thermodynamics

The generally cool and low-density protostellar material obeys an ideal-gas equation of state during the phases when fragmentation occurs. At temperatures below about 2000 K, the hydrogen is in molecular form. With a chemical composition close to solar, the equation of state will be an ideal gas with $\gamma \approx 1.67$ in the range $10 < T < 100$ K, and $\gamma \approx 1.4$ in the range $100 < T < 2000$ K. During the process of H_2 dissociation the value of γ falls to about 1.1, and the gas becomes dynamically unstable and undergoes a second collapse. When dissociation is nearly complete (T $\approx 10^4$ K), $\gamma \approx 1.67$ again and the hydrostatic stellar core forms. At higher temperatures the hydrogen ionizes, but no further dynamical collapse results (Larson 1969).

C. Gravitational Effects

At the onset of collapse, the molecular cloud core is expected to have a non-axisymmetric structure, with either ordered or random density and velocity fluctuations. As collapse proceeds in near free-fall, the collapse time scale and the time scale for growth of the perturbations is the same, and the amplitudes do not grow relative to the background, unless the perturbations are very large. The central regions of the cloud collapse most rapidly, leading to a density distribution that is highly centrally peaked at later times. Angular momentum is approximately conserved during this phase for a given mass element, since the gravitational torques associated with the (generally small) non-axisymmetic effects do not transfer much angular momentum.

However once filamentary or flattened structures develop and the collapse is slowed or halted, fragmentation is likely (Larson 1985). The collapse can be stopped, for example, by pressure effects associated with entering the adiabatic phase, or by rotational effects associated with the spinup of the gas combined with pressure effects in the direction parallel to the rotation axis. In the case that rotation becomes important before the adiabatic phase is reached, the central density peak of the cloud flattens into a disk, the overall collapse is halted, and non-axisymmetric structures, such as spiral arms, grow and develop into fragments (Larson 1978). Once pressure effects have become important, if filaments or rings have formed they are likely to fragment; otherwise rotational effects are not likely to be large enough to induce fragmentation during the adiabatic phase, and the end result will be a central star in equilibrium surrounded by a disk.

However this disk can also become gravitationally unstable. Although the indicator (Safronov 1960, Toomre 1964) known as $Q = \kappa c_s/(\pi G\sigma)$ is a local criterion for axisymmetric stability, the minimum Q–value in a nearly Keplerian disk serves as a useful indicator for the outcome of nonaxisymmetric instability (here κ is the epicyclic frequency, c_s is the sound speed, and σ is the local surface density). Extensive analytical and numerical modelling of gravitationally unstable disks (Cassen et al. 1981; Larson 1984; Adams et al. 1989; Papaloizou and Savonije 1991; Tomley et al. 1991, 1994; Heemskerk et al. 1992; Laughlin and Bodenheimer 1994; Miyama et al. 1994; Laughlin and Różyczka 1996; Nelson et al. 1998) all have shown that spiral waves develop and that effective angular momentum transfer takes place for values of Q_{min} between 1 and 3. However the amplitude of the waves saturates (Laughlin et al. 1997, 1998); only if Q_{min} falls below ≈ 1 do the spiral waves grow sufficiently to result in fragmentation. For $1 < Q_{min} < 2$, the disk evolution time is on the order of a few local dynamical times (t_d), and Q_{min} tends to increase. As the disk accretes mass from the infalling envelope, Q_{min} tends to decrease. Throughout most of the evolution of a protostar the mass accretion timescale is considerably longer than t_d, so it is unlikely that conditions for fragmentation of the disk can be reached. Exceptions could occur if the accretion is strongly episodic, or if the disk is in an early phase when there still is rapid infall. Then fragmentation can be driven by the interaction of a disk with an embedded bar or binary (Bonnell 1994; Bonnell and Bate 1994a).

D. Magnetic Effects

As the discussion in Section II indicates, magnetic forces are not particularly important in supporting a cloud core against gravity; the fields are however important in shaping the density profile at the onset of collapse. During collapse, ambipolar diffusion and the decrease in conductivity result in a decrease in the importance of magnetic effects. Because of Ohmic resistivity the time scale for magnetic flux loss becomes shorter than the free-fall time at densities above $\sim 10^{11}$ cm^{-3}

(Umebayashi and Nakano 1990), while fragmentation typically begins at densities somewhat higher than that value. Also during collapse the magnetic braking time becomes longer than the free-fall time, and the interior of the cloud spins up and develops differential rotation (Basu and Mouschovias 1994). Three-dimensional collapse calculations that take into account magnetic pressure effects but not the magnetic stress, and which include an approximate model of ambipolar diffusion (Boss 1997a), show that fragmentation is not prevented by magnetic effects in a rapidly rotating cloud with an initial Gaussian density distribution.

IV. FORMATION MECHANISMS

Although three-dimensional hydrodynamic calculations to examine the collapse and fragmentation of molecular clouds have been in evidence for over a decade, we are still at an early stage in pinpointing the dominant mechanism for binary and multiple star formation. In this section we provide a summary of some of the suggested processes and then consider some of the results of detailed numerical simulations on fragmentation during the collapse phase.

A. Brief Survey

Binary formation may occur through the processes of (i) capture, (ii) fission, (iii) fragmentation during the protostar collapse, (iv) disk fragmentation, and (v) prompt initial fragmentation. Item (iii) will be discussed in more detail in Section IVB.

Binary formation by direct capture, either in the presence of a third body to absorb the excess energy or in the presence of tidal dissipation induced in a close encounter, has been shown to be an inefficient process (Tassoul 1978; Boss 1988b) in the Galactic disk. Larson (1990) suggested that capture could occur in a very compact young cluster such as the Orion Trapezium, with the dissipation provided through direct interaction of the captured star with a circumstellar disk. Clarke and Pringle (1991) pointed out that the rate would be too small to account for all of the Trapezium binaries and that the period distribution would not be as observed. However in small clusters of 4–10 fragments, star–disk interactions could produce a reasonable binary fraction (McDonald and Clarke 1995). Note that in this case the binary components would be essentially coeval, as observed, since the age difference would not be greater than the cluster dynamical time.

Binary formation by fission occurs when a star or newly formed stellar core in its quasi-static contraction phase undergoes a rotational instability and divides into two distinct masses, in which process spin angular momentum is converted into orbital angular momentum. Since such young equilibrium objects in the mass range of 1 $M_\odot$ are known, from protostellar theory and from observations, to have radii less than 10 $R_\odot$, the outcome would be a close binary. Numerous objections to this theory have arisen, which are summarized by Tassoul (1978) and Bodenheimer et al. (1993), and numerical simulations of the process do not show binary formation (Williams and Tohline 1988).

Disk fragmentation can occur in an equilibrium circumstellar disk if the minimum Q value approaches 1 (see Section IIIC, which discusses the important question of whether the required initial condition can be reached.) Numerical examples have been provided by Adams and Benz (1992), Boss (1997b), Laughlin and Różyczka (1996) and Miyama et al. (1994). Typical fragment masses are in the range 0.01 – 0.1 $M_\odot$, that is, in the brown-dwarf range, and orbital separations are 5–100 AU. The calculations have not been followed far enough for orbital properties to be determined. However, external perturbations can also induce fragmentation in a disk; in this case the Q-value could be > 1. Numerical simulations (Bonnell et al. 1992; Boffin et al. 1998, Watkins et al. 1998a,b) show that star-disk encounters, as well as disk-disk encounters, can result in the triggering of new condensations in a disk.

'Prompt initial fragmentation' is a triggered star formation process introduced by Pringle (1989) as a necessary alternative for binary formation in view of the fact that the $\rho \propto r^{-2}$ profiles generated by magnetic effects in contracting cloud cores are unlikely to fragment during the subsequent collapse. He envisions cloud-cloud collisions resulting in binary formation on wide elliptical orbits. Subsequent interactions of the components with each other's disks would reduce the orbital separation on a longer time scale. Numerical simulations (Chapman et al. 1992; Turner et al. 1995) show that cooling by dust, which occurs in the density range 10^{-22}–10^{-20} g cm^{-3}, is an important element of this process. Off-center collisions of gravitationally stable clouds of $\approx 200 M_\odot$ result in a shocked layer which cools and becomes unstable to the formation of several fragments; the system later evolves into a binary protostar, each component with a surrounding disk. Klein and Woods (1998) have considered a modification of this scenario, involving fragmentation by bending mode instabilities in cloud collisions, and leading to porous structure in clouds.

B. Fragmentation

Fragmentation during protostar collapse can produce binaries with a wide range of periods. The nature of the problem can be appreciated when one considers that the phase space of initial parameters for molecular clouds is vast, consisting of possible variations in α, β, the initial distributions of velocity, density, and angular momentum, the form of initial perturbations, and the size and shape of clouds. A wide variety of (often idealized) initial conditions has been employed, for example, a sphere in uniform rotation with a density perturbation in the form of an $m = 2$ mode with 10% amplitude, or a prolate spheroid with a random perturbation (Boss 1993), or a filamentary cloud that is rotating end–over–end (Zinnecker 1990). The phase space would be somewhat reduced if the initial conditions were chosen to approximately match the observed properties of molecular cloud cores (Section II).

Calculations have been performed under several different physical approximations. Many calculations have treated the isothermal phase of collapse (Bonnell et al. 1991; Bonnell and Bastien 1992; Burkert and Bodenheimer 1993, 1996; Nelson and Papaloizou 1993; Monaghan 1994; Sigalotti and Klapp 1994, 1997; Boss 1996; Truelove et al. 1997). Radiation has been treated in simplified approximations, such as the diffusion and the Eddington approximations, applicable to optically thick regions (Boss 1993; Myhill and Kaula 1992). The transition to the optically thick regime has also been approximated (Bonnell 1994; Bate et al. 1995; Klein et al. 1998) by a change in the equation of state from isothermal to adiabatic at the appropriate density ($\rho_{crit} \approx 10^{-13}$ g cm^{-3}). Cooling during collapse has been considered in SPH simulations by Monaghan and Lattanzio (1991), Chapman et al. (1992), and Turner et al. (1995). The cooling that is employed is appropriate to the density regime 10^{-22} – 10^{-20} g cm^{-3}, so such calculations apply to clouds that fall in the mass range 100–200 M$_\odot$ rather than around 1 M$_\odot$. The adiabatic phase has been considered by Miyama (1992).

The resulting fragmentation occurs in various modes. A common occurrence is the simple formation of a wide binary with separation $D \approx 100 - 1000$ AU, on an eccentric orbit. But other results have been found: Monaghan and Lattanzio (1991) find six fragments in a ringlike formation; Bonnell et al. (1991), Boss (1991) and Chapman et al. (1992) find hierarchical multiple systems; Bonnell et al. (1992) find fragmentation of a disk induced by the tidal perturbation from a close companion; Monaghan (1994) finds multiple fragmentation in long filaments; Burkert and Bodenheimer (1993) find that a bar connecting two binary components fragments into several pieces along its length; Boss (1996) and Klapp and Sigalotti (1998) find that a small cluster develops; Bonnell and Bate (1994a) and Burkert and Bodenheimer (1996) find that an interior binary can induce further fragmentation in a circumbinary disk. Problems concerning numerical resolution, which may affect many of these studies, are discussed in Section V.

Two major theoretical problems have been discussed in connection with these simulations. First, strongly peaked, centrally condensed density distributions are relatively stable to fragmentation. In particular, cores with $\rho \propto r^{-2}$ and uniform rotation (Tsai and Bertschinger 1989), which have been observed in some star-forming regions (Section II), appear to be stable against fragmentation. This conclusion is particularly perplexing in view of the fact that all star-forming regions exhibit a substantial fraction of binary systems (Mathieu 1994; Chapter by Mathieu et al.). The discrepancy has been alleviated to some extent by (1) observations that at least some pre-stellar cores have density distributions that become relatively flat near the center (André et al. 1996) and (2) numerical simulations (Burkert et al. 1997) in which an initial density profile $\rho \propto r^{-1}$ with uniform rotation does fragment. The other significant problem is that most simulations produce wide binaries, while close ($D \leq 1$ AU) binaries appear to be difficult to form. Several mechanisms have been suggested. A promising one is the orbital decay of a wide binary by gravitational torques exerted on it by surrounding spiral arms in the circumbinary disk (Artymowicz et al. 1991). Numerical calculations have demonstrated that fission is extremely difficult (Section IVA). Hierarchical fragmentation has not been studied with sufficient numerical resolution to determine whether it is a likely process. One mode of such fragmentation would involve subfragmentation in the adiabatic phase of fragments formed in the isothermal phase; however the range of initial conditions for the subfragmentation is quite limited (Section IIIA). Fragmentation during the second collapse phase (Section IIIB) has been suggested (Bonnell and Bate 1994b; Larson 1972), but more recent 3-D simulations by Bate (1998) indicate that fragmentation may not occur during this phase because of prior efficient transfer of angular momentum by gravitational torques. Further investigation is also needed on other proposed mechanisms, such as fragmentation in a cloud with low initial angular momentum, disk fragmentation induced by a companion, or gravitational interactions in a compact small cluster of fragments (see Bodenheimer 1995).

V. NUMERICAL CALCULATIONS

A. Introduction

The physics of gravitational collapse and fragmentation is essential for obtaining an understanding of the formation of stars and galaxies, yet gaseous flows undergoing fragmentation naturally develop a substantial variation in length scale, up to a factor of 10^4 or more. This enormous dynamic range presents a formidable obstacle to obtaining an accurate numerical solution, as the flow must remain well-resolved throughout the evolution. Fixed-resolution methods are not efficient for such simulations; variable-resolution methods are required. This section discusses a minimum requirement on resolution, known as the Jeans condition, and then explores a series of highly resolved test cases, carried out to determine whether two or more independent codes reach the same result on the same problem.

Four basic approaches have been developed for employing variable resolution in the solution of 3-D self-gravitational problems. The smoothed particle hydrodynamics (SPH) technique is a grid-less, Lagrangian technique in which gravitating particles of fixed mass carry fluid properties (see, e.g., Monaghan 1992). SPH provides a means of achieving a large dynamic range in scale that is much easier to implement than many of the high-order finite-difference approaches. It concentrates its resolution and computational effort in high-density regions. Also, it is easy to track the transfer of angular momentum for a particular mass element. However, for similar computational times, SPH provides poorer resolution of shocks and poorer resolution of low-density regions than do grid-based codes (Kang et al. 1994). Shocks are an integral feature of calculations of astrophysical hydrodynamics, and this restricts the utility of SPH for such problems (Klein and McKee 1994). In addition, while the spatial redistribution of particles at each step yields a variable spatial resolution, the mass resolution of SPH codes remains fixed. Recent SPH work in the study of star formation has been carried out by Bate and Burkert (1997). A second approach to achieve variable resolution is simply to utilize a grid in spherical coordinates that naturally affords increased resolution in the form of decreased cell volume about the origin. When coupled with a nonuniform radial cell dimension and rezoning, this method is useful for studying collapse that produces structure near the origin (Boss 1996; Sigalotti and Klapp 1997). A third approach, a multiple-grid method, is utilized by Burkert and Bodenheimer (1993; 1996). In this scheme, a series of smaller grids of increasingly finer resolution is concentrically positioned about the center of the computational volume. These grids remain fixed throughout the calculation. In typical problems, this method over-resolves the cloud center during the early stages of collapse, and may under-resolve structure outside the preset locations of the finer grids. The fourth approach, Adaptive Mesh Refinement (AMR) was introduced into astrophysical problems by Klein et al. (1990) and Klein and McKee (1994) and generalized to include self-gravity by Truelove et al. (1997, 1998). This scheme utilizes grids at multiple levels of resolution. Linear resolution varies by integral refinement factors—usually 4—between levels, and a given subgrid is always fully contained within one at the next coarser level. The method stems from the seminal work of Berger and Oliger (1984). The AMR method can employ multiple spatially unconnected grids at a given level of refinement. Most importantly, the AMR method dynamically resizes and repositions these grids and inserts new, finer ones within them according to adjustable refinement criteria. Fine grids are automatically removed as flow conditions require less resolution. Thus AMR naturally follows developing structure and fronts, placing high resolution where it is needed most. This powerful approach, when combined with an underlying higher-order hydrodynamics method such as the Godunov scheme, provides a natural means of resolving structure accurately over an enormous dynamic range in scale, and also has the ability to follow strong fronts with great economy.

B. Jeans Condition

Recently Truelove et al. (1997, 1998) introduced the Jeans condition as an important grid refinement criterion for self-gravitational hydrodynamics to avoid artificial fragmentation in numerical simulations. This condition arises because perturbations on scales above the Jeans length, $\lambda_J \equiv (\pi c_s^2/G\rho)^{1/2}$, are physically unstable (Jeans 1902, 1928). Discretization of the equations for self-gravitational hydrodynamics introduces perturbations on scales of order the grid resolution Δx. If $\Delta x > \lambda_J$ these perturbations might result in artificial fragmentation. It is therefore essential to keep λ_J as resolved as possible in order to diminish the initial amplitude of perturbations that exceed this scale. Resolution of λ_J also suppresses numerical effects which simulate a viscosity and its accompanying effect of artificially slowing the collapse.

Defining the Jeans number $J \equiv (\Delta x/\lambda_J)$, Truelove et al. (1997) found keeping $J \leq 0.25$ avoided artificial fragmentation in the particular case of an isothermal collapse spanning 7 decades of density, the approximate range separating typical molecular cloud cores from non-isothermal protostellar fragments. It is important to note, however, that the Jeans condition is a necessary but not, in general, sufficient condition to ensure convergence. It is necessary to perform an appropriate convergence study to determine the necessary and sufficient value for J required to avoid artifical fragmentation in a particular simulation.

For a uniform Cartesian grid, the condition $J \leq 0.25$ is equivalent to requiring that the mass of a cell should not exceed 1/64 of the Jeans mass (note that the precise value required will depend on the problem being considered). Boss (1998) extended the Jeans condition to the case of a non-uniformly-spaced, spherical coordinate grid, where there are three grid spacings, $\Delta x_r = \Delta r$, $\Delta x_\theta = r\Delta\theta$, and $\Delta x_\phi = r \sin\theta \Delta\phi$, and a fourth grid length that is related to the cell mass, $\Delta x = (\Delta x_r \Delta x_\theta \Delta x_\phi)^{1/3}$. Boss (1998) found that as long as $\Delta x < \lambda_J/4$, artificial fragmentation could be avoided, even if Δx_r, Δx_θ, or Δx_ϕ exceeded $\lambda_J/4$, implying the primacy of a Jeans-mass condition over a Jeans-length condition for a spherical grid; for a Cartesian grid they are equivalent. A closely related Jeans condition has been developed for SPH calculations (Bate and Burkert 1997). Here the minimum resolvable mass must be less than the Jeans mass. In practice, if N_{neigh} is the number of particles within 2 smoothing lengths (typically 50), then 2 N_{neigh} times the particle mass must be less than the Jeans mass.

C. Code Intercomparisons for Isothermal Collapse

The uniform-density cloud studied by Burkert and Bodenheimer (1993), Bate and Burkert (1997), and Truelove et al. (1998), and the Gaussian profile studied by Boss (1991; 1993; 1998), Klapp et al.

(1993), Burkert and Bodenheimer (1996), and Truelove et al. (1997) have emerged as important benchmark test cases for code intercomparisons because of the diversity of the results obtained. The test involving the uniform cloud has the following parameters: $M = 1M_{\odot}$, $R = 5 \times 10^{16}$ cm, $\rho_0 = 10^{-17.4}$ g cm^{-3}, $\alpha = 0.26$, $\beta = 0.16$, $c_s = 0.167$ km s^{-1}, and a rotation rate $\Omega = 7.2 \times 10^{-13}$ rad s^{-1}. The cloud is perturbed by $\rho \rightarrow \rho \times [1 + 0.1\cos(2\phi)]$, an $m = 2$ mode with 10% amplitude. This cloud begins with $\lambda_J = 1.17R$.

Burkert and Bodenheimer (1993) calculated the isothermal collapse over a density increase of more than 6 decades. The result was two local density maxima (a binary) separated by about 10^{16} cm. A bar subsequently formed between the components and fragmented into 9 small condensations. Truelove et al. (1998) attacked this problem using an initial resolution of R_{32} (32 cells per initial cloud radius), and dynamically refined so as to ensure $J_{\max} = 0.25$ throughout the simulation. They found a binary with characteristics similar to that of Burkert and Bodenheimer (1993). Each fragment (Figure 1A) has a mass $M_f = 0.032M_{\odot}$, with a binary separation of 8.6×10^{15} cm.

Truelove et al. (1998) continued the calculation until ρ had increased by 8.1 decades. At this point, the code used resolution at the level of R_{131072}, representing six levels of refinement beyond the original R_{32}; the finest cells are only 5.4 solar radii in size. Each member of the binary continues to collapse into a thinner and thinner filamentary structure and does not subfragment. Thus the solution to the purely isothermal, inviscid collapse is a pair of singular filaments.

As in Burkert and Bodenheimer (1993) the binary is connected by a thin bar (Figure 1A). The bar is also expected to collapse to a linear singularity in the absence of "arresting" agents of heating and viscosity (both numerical and artificial). The high-resolution calculations show no evidence of subfragmentation in the bar, in contrast to Burkert and Bodenheimer (1993) who used fixed finest resolution and thus eventually violated the Jeans condition in the highest-density zones. The resulting poor resolution most likely slowed the collapse of the bar onto itself and allowed numerical perturbations to grow. Thus in the pure isothermal case the bar fragmentation is most likely a numerical artifact, although in calculations in which heating is included above ρ_{crit}, and in which the Jeans condition is satisfied, the collapse of the bar is slowed and fragmentation does result (Bate and Burkert 1997).

Recent work on the same problem by Boss (Figure 1B), with sufficiently high resolution in his spherical code to satisfy the Jeans condition for most of the simulation, finds excellent agreement with Truelove et al. (1998) with a basic morphology of a thin bar with binary fragments. In a lower-resolution calculation in which the Jeans condition is not satisfied in the vicinity of the forming binary pair, the bar begins to develop a third (low mass) clump at its center. Thus it appears that in this particular test case all groups find a binary pair connected by a bar with similar morphology. Multiple fragmentation along the bar is not found in purely isothermal calculations which satisfy the Jeans condition.

The Gaussian cloud has initial conditions identical to those of the uniform cloud except that the radial density dependence is $\rho(r) = \rho_c e^{-(r/R_1)^2}$, where $\rho_c = 10^{-16.8}$g cm^{-3} and $R_1 = 0.58R$. The sound speed $c_s = 0.19$ km s^{-1}. Boss (1991, 1993) found the Gaussian cloud to fragment into a binary, then into a hierarchical quadruple system. Klapp et al. (1993) and Burkert and Bodenheimer (1996) also obtained the quadruple system, but in the latter paper the quadruple then evolved into an inner triple and an outer binary. Truelove et al. (1997) found that with their highest spatial resolution, ensuring that $\Delta x < \lambda_J/4$, the Gaussian cloud collapsed to form a very thin filament without fragmentation (Figure 2A). Boss (1998) obtained a binary system while maintaining $\Delta x < \lambda_J/4$.

Clearly the earlier calculations did not have sufficient resolution to satisfy the Jeans condition. The main reason the Burkert and Bodenheimer (1996) results differ from those of Truelove et al. (1997) is that in the former simulation an artificial viscosity was specifically introduced to halt the collapse of the very densest regions, to allow the calculations to continue to later times. The effect is roughly similar to that of including heating. However in the pure isothermal case the collapse of the filament cannot be stopped, and it continues, on ever shorter time scales, until a singularity is reached.

The isothermal collapse of the Gaussian cloud has been recalculated by Boss with considerably higher spatial resolution than he previously used. The grid is moved inward during the collapse, keeping all four spherical Jeans lengths less than $\lambda_J/4$. The result is shown in Figure 2B. At a maximum density comparable to that of the AMR calculation, a collapsing isothermal bar is seen with similar morphology.

Thus it appears that the evolutionary outcome of the isothermal collapse of a Gaussian cloud is the singular isothermal filament. This result is consistent with Inutsuka and Miyama (1992, 1997), who show that perturbed, unstable, inviscid, isothermal clouds form filaments that increase in density faster than linear perturbations along them can grow to fragments. Neither set of calculations shows the complex multiple fragmentation seen, for example, by Burkert and Bodenheimer (1996). However the latter case is not comparable, because the isothermal assumption was in fact broken by the introduction of artificial viscosity, which allowed the calculation to proceed to a time beyond the bar-formation stage.

D. Uniform Clouds with Heating

Recent work using SPH (Bate and Burkert 1997) and high resolution AMR (Klein 1998; Klein et al. 1998; Fisher et al. 1998) have followed the collapse of an initially rigidly rotating, uniform isothermal cloud. They use an equation of state which makes the transition from an isothermal equation in the optically thin regime to a polytrope in the optically thick regime in a smooth fashion. The initial conditions are identical to the isothermal uniform cloud previously discussed, including the $m = 2$ perturbation of 10% amplitude. The AMR calculations of Klein (1998) and Fisher et al. (1998) followed the subsequent evolution of the fragments over dynamical (orbital) timescales, while still adhering to the Jeans condition. The cloud initially collapses to an isothermal disk, a strong isothermal shock above the disk plane is established, and an elongated filamentary bar forms, with the first signs of fragmentation in it occurring by 1.41×10^{12} s. At about $t = 1.46 \times 10^{12}$ s the isothermal bar becomes optically thick and the accretion flow onto the bar is arrested, because of pressure effects, resulting in the growth of non-axisymmetric perturbations in the bar. Fragmentation in the bar results in the formation of binary, spherical cores. The core-bar system is embedded in an outer two-armed spiral, derivative of the initial $m = 2$ perturbation. The low angular momentum of shocked gas accreted in the bar allows the bar to be directly accreted onto the cores. The binary separation decreases as this mass is accreted, and the bar starts to dissipate.

Later at $t = 1.51 \times 10^{12}$ s, protostellar disks have formed around the cores; the cores continue to grow by direct accretion from these surrounding disks. The disks are attached to the long outer spiral. The scale of the disks, of order 100 AU, is consistent with observations of gaseous disks surrounding single T-Tauri stars and debris disks surrounding systems such as β Pictoris. The situation is shown at $t = 1.6 \times 10^{12}$ s (Figure 3). The cores, spiral arms, and disks have 20%, 27%, and 2%, respectively, of the total mass of the cloud at this time. The fact that the initial bar did not fragment further after binary formation is consistent with the results of Bate and Burkert (1997), who, using a slightly different equation of state, found that such fragmentation occurred only if $\rho_{crit} > 3 \times 10^{-14}$ g cm^{-3}.

E. Power-Law Clouds with Heating

A further significant test case, involving the comparison of an SPH code with a grid code, was provided by Burkert et al. (1997). This work also considers the problem of fragmentation in clouds with significant density concentration toward the center. The initial core had mass 1.0 $M_\odot$, $R = 5 \times 10^{16}$ cm, and a density distribution $\rho \propto r^{-1}$. They showed that with an initial density perturbation of the form $m = 2$ and a 10% amplitude, and with $\alpha = .25, \beta = .23$, fragmentation into four objects occurs in the inner region of the cloud, on a scale of 100 AU. The calculation was done with an Eulerian grid code, with fixed nested grids such that the resolution on the innermost grid was $10^{-3}R$, where R is the total radius of the cloud. The same problem was calculated by use of an SPH code with 200 000 particles. The resolution element in such a calculation is the smoothing length, which varies through the volume; in this case it was adjusted to be $10^{-3}R$ at a density of 10^{-12} g cm^{-3}, which is about the point where fragmentation starts. However the spatial resolution of the two codes can be quite different at other densities. Figure 4A shows the onset of fragmentation in the grid code, in a region of the core that contains about 12% of the total mass. Interactions among the spiral arms result in the formation of an inner unstable triple, which ejects one component which later merges with one of the binary components that has formed in the outer spiral arm. At the end of the simulation the inner binary has masses 0.055 and 0.025 $M_\odot$, and the outer binary has masses 0.017 and 0.011 $M_\odot$. Clearly there will be further orbital evolution and accretion with possible mergers, further fragmentation, and ejections. Figure 4B shows the parallel evolution in the SPH code, which shows fragmentation on the same scale and by the same process but with some differences in detail. Both calculations satisfy the Jeans condition, although the form assumed for the heating above ρ_{crit} was somewhat different in the two calculations.

VI. SUMMARY

The goal of detailed fragmentation calculations is to explain the properties of the multiple systems that are described in Section I. Although various plausible ideas have been investigated, an overall comparison between theory and observations has not yet been established. It is true that the relatively broad distribution of rotational velocities of molecular cloud cores could in part explain the broad period distribution of binary stars, but there is probably not a one-to-one correspondence of cloud spin angular momentum and resulting binary orbital angular momentum. Orbital interactions in a system of multiple fragments in a dense protostellar cluster could also play a role in the determination of the period distribution. Such interactions could also be responsible for the broad eccentricity distribution, although theoretical models have not progressed far enough to provide a statistical comparison. It is particularly difficult for theory to explain the very closest binary systems; possibilities include orbital decay of long-period systems, hierarchical fragmentation, or three-body captures in a dense system of protostellar fragments. The distribution of mass ratios likewise has not been well explained. Although numerical simulations do show that fragments with a mass difference of roughly a factor 10 can form in a collapsing cloud, it is not clear how the mass ratio depends on initial conditions, or how the mass ratio will evolve as a result of subsequent accretion (Chapter by Lubow and Artymowicz). Very generally, the observational evidence is consistent with the idea that fragmentation during collapse of a rotating cloud is the basic process.

A further important issue is the binary frequency and the conditions under which single stars are formed. It has been suggested (Chapter by Mathieu et al.) that the density of the star-forming region has an effect on the binary fraction. However in a given star formation region it is still not clear under what conditions single stars are formed. The possibilities include (1) ejection from a system of multiple fragments as a result of a close encounter, (2) a low-angular momentum initial cloud that fails to fragment in the isothermal collapse phase and evolves to a single stellar core plus disk, (3) variations in the initial conditions of standard cloud cores, which on the average, observationally, have $\alpha = 0.4$ and $\beta = 0.01$. Boss (1993) calculated the collapse of initially prolate clouds with α and β in this range. Initial axis ratios of 1.5 and 2 led to an outcome of a single star and a binary, respectively. All of these possibilities require further investigation.

What is the future for fragmentation calculations? Computing power is now becoming sufficient so that, coupled with the numerical techniques developed over the past few years, the demanding three-dimensional collapse calculations can be performed with adequate spatial resolution and improved physics. Actual cloud cores are not in general represented by the highly idealized and symmetric initial conditions assumed in many of the test calculations done so far. Initial clouds with irregular shapes, with a residual spectrum of turbulent velocities, and with observed values of α and β should be considered. The earlier parameter surveys with radiation transport should be recalculated with improved spatial resolution, because the transition from the isothermal regime to the adiabatic regime is of particular significance with regard to the outcome of fragmentation. Of less importance are magnetic fields, which have possible influence on the earlier stages of collapse, but probably not on the actual fragmentation phase. Finally, a much closer connection between the results of theoretical calculations and observational quantities must be established. Detailed line profiles for the high-density tracers such as CS and H_2CO, which are being used to establish the existence of collapse as well as rotational motions in protostars, should be calculated from the 3-D models. Further diagnostics include synthetic infrared spectral energy distributions as well as continuum maps which correspond to the observed bands in the submillimeter and millimeter regions. Such detailed observational comparisons should provide further clues as to the origin of multiple systems.

REFERENCES

Adams, F. C., and Benz, W. 1992. Gravitational instabilities in circumstellar disks and the formation of binary companions. In *Complementary Approaches to Double and Multiple Star Research*, eds. H. McAllister and W. Hartkopf (San Francisco: Astronomical Society of the Pacific), pp. 185–194.

Adams, F. C., Ruden, S. P., and Shu, F. H. 1989. Eccentric gravitational instabilities in nearly Keplerian disks. *Astrophys. J.* 347:959–975.

Alexander, D. R., and Ferguson, J. W. 1994. Low-temperature Rosseland opacities. *Astrophys. J.* 437:879–891.

André, P., Ward-Thompson D., and Motte, F. 1996. Probing the initial conditions of star formation: the structure of the prestellar core L1689B. *Astron. Astrophys.* 314:625–635.

Artymowicz, P., Clarke, C. J., Lubow, S. H., and Pringle, J. E. 1991. The effect of an external disk on the orbital elements of a central binary. *Astrophys. J.* 370:L35–L38.

Basu, S., and Mouschovias, T. Ch. 1994. Magnetic braking, ambipolar diffusion, and the formation of cloud cores and protostars. I. Axisymmetric solutions. *Astrophys. J.* 432:720–741.

Bate, M. R. 1998. Collapse of a molecular cloud core to stellar densities: the first three-dimensional calculations. *Astrophys. J.* 508:L95–L98.

Bate, M. R., Bonnell, I., and Price, N. 1995. Modelling accretion in protobinary systems. *Mon. Not. R. Astron. Soc.* 277:362–376.

Bate, M. R., and Burkert, A. 1997. Resolution requirements for smoothed particle hydrodynamics calculations with self-gravity. *Mon. Not. R. Astron. Soc.* 288:1060–1072.

Benson, P. J., and Myers, P. C. 1989. A survey for dense cores in dark clouds. *Astrophys. J. Suppl.* 71:89–108.

Berger, M. J., and Oliger, J. 1984. Adaptive mesh refinement for hyperbolic partial differential equations. *J. Comput. Phys.* 53:484–512.

Bodenheimer, P. 1995. Angular momentum evolution of young stars and disks. *Annu. Rev. Astron. Astrophys.* 33:199–238.

Bodenheimer, P., Ruzmaikina, T., and Mathieu, R. 1993. Stellar multiple systems: constraints on the mechanism of origin. In *Protostars* *and Planets III*, eds. E. H. Levy and J. I. Lunine (Tucson: Univ. of Arizona), pp. 367–404.

Boffin, H. M. J., Watkins, S. J., Bhattal, A. S., Francis, N., and Whitworth, A. P. 1998. Numerical simulations of protostellar encounters. I. Star–disc encounters. *Mon. Not. R. Astron. Soc.* 300:1189–1204.

Bonnell, I. 1994. A new binary formation mechanism. *Mon. Not. R. Astron. Soc.* 269:837–848.

Bonnell, I., Arcoragi, J.-P., Martel, H., and Bastien, P. 1992. Fragmentation of elongated cylindrical clouds. IV. Clouds with solid-body rotation about an arbitrary axis. *Astrophys. J.* 400:579–594.

Bonnell, I., and Bastien, P. 1992. Fragmentation of elongated cylindrical clouds. V. Dependence of mass ratios on initial conditions. *Astrophys. J.* 401:654–666.

Bonnell, I., and Bate, M. R. 1994a. Massive circumbinary disks and the formation of multiple systems. *Mon. Not. R. Astron. Soc.* 269:L45–L48.

Bonnell, I., and Bate, M. R. 1994b. The formation of close binary systems. *Mon. Not. R. Astron. Soc.* 271:999–1004.

Bonnell, I., Martel, H., Bastien, P., Arcoragi, J.-P., and Benz, W. 1991. Fragmentation of elongated cylindrical clouds. III. Formation of binary and multiple systems. *Astrophys. J.* 377:553–558.

Boss, A. P. 1981. Collapse and fragmentation of rotating adiabatic clouds. *Astrophys. J.* 250:636–644.

Boss, A. P. 1988a. Protostellar formation in rotating interstellar clouds. VII. Opacity and fragmentation. *Astrophys. J.* 331:370–376.

Boss, A. P. 1988b. Binary stars: formation by fragmentation. *Comment. Astrophys.* 12:169–190.

Boss, A. P. 1991. Formation of hierarchical multiple protostellar cores. *Nature* 351:298–300.

Boss, A. P. 1993. Collapse and fragmentation of molecular cloud cores. I. Moderately centrally condensed cores. *Astrophys. J.* 410:157–167.

Boss, A. P. 1996. Collapse and fragmentation of molecular cloud cores. IV. Oblate clouds and small cluster formation. *Astrophys. J.* 468:231–240.

Boss, A. P. 1997a. Collapse and fragmentation of molecular cloud cores. V. Loss of magnetic field support. *Astrophys. J.* 483:309–319.

Boss, A. P. 1997b. Giant planet formation by gravitational instability. *Science* 276:1836–1839.

Boss, A. P. 1998. The Jeans mass constraint and the fragmentation of molecular cloud cores. *Astrophys. J.* 501:L77–L81.

Bouvier, J., Rigaut, F., and Nadeau, D. 1997. Pleiades low-mass binaries: do companions affect the evolution of protoplanetary disks? *Astron. Astrophys.* 323:139–150.

Brandner, W., and Köhler, R. 1998. Star formation environments and the distribution of binary separations. *Astrophys. J.* 499:L79–L82.

Brandner, W., and Zinnecker, H. 1997. Physical properties of 90 AU to 250 AU pre–main-sequence binaries. *Astron. Astrophys.* 321:220–228.

Burkert, A., and Bodenheimer, P. 1993. Multiple fragmentation in collapsing protostars. *Mon. Not. R. Astron. Soc.* 264:798–806.

Burkert, A., and Bodenheimer, P. 1996. Fragmentation in a centrally condensed protostar. *Mon. Not. R. Astron. Soc.* 280:1190–1200.

Burkert, A., Bate, M. R., and Bodenheimer, P. 1997. Protostellar fragmentation in a power-law density distribution. *Mon. Not. R. Astron. Soc.* 289:497–504.

Cassen, P. M., Smith, B. F., Miller, R. H., and Reynolds, R. T. 1981. Numerical experiments on the stability of preplanetary disks. *Icarus* 48:377–392.

Chapman, S., Pongracic, H., Disney, M., Nelson, A., Turner, J., and Whitworth, A. 1992. The formation of binary and multiple star systems. *Nature* 359:207-210.

Ciolek, G., and Königl, A. 1998. Dynamical collapse of nonrotating magnetic molecular cloud cores: evolution through point-mass formation. *Astrophys. J.* 504: 257–279.

Ciolek, G., and Mouschovias, T. Ch. 1995. Ambipolar diffusion, interstellar dust, and the formation of cloud cores and protostars. IV. Effect of ultraviolet ionization and magnetically controlled infall rate. *Astrophys. J.* 454:194–216.

Clarke, C. J., and Pringle, J. E. 1991. Star-disk interactions and binary star formation. *Mon. Not. R. Astron. Soc.* 249:584–587.

de Jong, T., Dalgarno, A., and Boland, W. 1980. Hydrostatic models of molecular clouds. *Astron. Astrophys.* 91:68–84.

Duquennoy, A., and Mayor, M. 1991. Multiplicity among solar-type stars in the solar neighbourhood. II. Distribution of the orbital elements in an unbiased sample. *Astron. Astrophys.* 248:485–524.

Fischer, D. A., and Marcy, G. W. 1992. Multiplicity among M dwarfs. *Astrophys. J.* 396:178–194.

Fisher, R., Klein, R. I., and McKee, C. F. 1998. in preparation.

Fuller, G. A., Ladd, E. F., and Hodapp, K.-W. 1996. Lynds 1527: An embedded protobinary system in Taurus. *Astrophys. J.* 463:L97–L100.

Fuller, G. A., and Myers, P. C. 1992. Dense cores in dark clouds. VII. Line width–size relation. *Astrophys. J.* 384:523–527.

Ghez, A. M., McCarthy, D. W., Patience, J. L., and Beck, T. L. 1997. The multiplicity of pre–main-sequence stars in southern star-forming regions. *Astrophys. J.* 481:378–385.

Goodman, A., Benson, P., Fuller, G., and Myers, P. 1993. Dense cores in dark clouds. VIII. Velocity gradients. *Astrophys. J.* 406:528–547.

Hachisu, I., Tohline, J. E., and Eriguchi, Y. 1987. Fragmentation of rapidly rotating gas clouds. I. A universal criterion for fragmentation. *Astrophys. J.* 323:592–613.

Hartigan, P., Strom, K. M., and Strom, S. E. 1994. Are wide pre-main-sequence binaries coeval? *Astrophys. J.* 427:961–977.

Hayashi, C. 1966. Evolution of protostars. *Annu. Rev. Astron. Astrophys.* 4: 171–192.

Hayashi, C., Narita, S., and Miyama, S. M. 1982. Analytic solutions for equilibrium of rotating isothermal clouds. *Prog. Theor. Phys.* 68: 1949–1966.

Heemskerk, M. H. M., Papaloizou, J. C. B., and Savonije, G. J. 1992. Non-linear development of $m = 1$ instabilities in a self-gravitating gaseous disk. *Astron. Astrophys.* 260:161–174.

Henriksen, R., André, P., and Bontemps, S. 1997. Time-dependent accretion and ejection implied by pre-stellar density profiles. *Astron. Astrophys.* 323:549–565.

Hujeirat, A. 1998. Ambipolar diffusion in star-forming clouds. *Astron. Astrophys.* 334:742-745.

Inutsuka, S., and Miyama, S. M. 1992. Self-similar solutions and the stability of collapsing isothermal filaments. *Astrophys. J.* 388:392–399.

Inutsuka, S., and Miyama, S. M. 1997. A production mechanism for clusters of dense cores. *Astrophys. J.* 480:681–693.

Jeans, J. H. 1902. *Phil. Trans. A* 199:1.

Jeans, J. H. 1928. *Astronomy and Cosmogony* (London: Cambridge Univ. Press.)

Kang, H., Ostriker, J. P., Cen, R., Ryu, D., Hernquist, L., Evrard, A. E., Bryan, G. L., and Norman, M. L. 1994. A comparison of cosmological hydrodynamic codes. *Astrophys. J.* 430:83–100.

Klapp, J., and Sigalotti, L. DiG. 1998. Collapse and fragmentation models of oblate molecular cloud cores. III. Formation of small protostellar clusters. *Astrophys. J.* 504:158–169.

Klapp, J., Sigalotti, L. DiG., and DeFelice, F. 1993. Formation of multiple protostellar systems. *Astron. Astrophys.* 273:175–184.

Klein, R. I. 1998. Self-gravitational hydrodynamics with 3-D adaptive mesh refinement: the collapse and fragmentation of molecular clouds. *Journal of Computational and Applied Mathematics*, ed. H. Riffert, Elsevier Press, in press.

Klein, R. I., Fisher, R., McKee, C. F., and Truelove, K. 1998. Gravitational collapse and fragmentation in molecular clouds with adaptive mesh refinement hydrodynamics. In *Numerical Astrophysics 1998*, eds. S. Miyama, K. Tomisaka, and T. Hanawa (Dordrecht: Kluwer), in press.

Klein, R. I., and McKee, C. F. 1994. The hydrodynamics of cloud interactions. In *Numerical Simulations in Astrophysics,* eds. J. Franco, S. Lizano, L. Aguilar, and E. Daltabuit (Cambridge: Cambridge Univ. Press), pp. 251–266.

Klein, R. I., McKee, C. F., and Colella, P. 1990. Turbulent stripping of interstellar clouds by interaction with supernova remnants. In *The Evolution of the Interstellar Medium*, ASP Conf. Ser. 12, ed. L. Blitz (San Francisco: ASP), pp. 117–136.

Klein, R. I., and Woods, D. T. 1998. Bending mode instabilities and fragmentation in interstellar cloud collisions: a mechanism for complex structure. *Astrophys. J.* 497:777–799.

Köhler, R., and Leinert, C. H. 1998. Multiplicity of T Tauri stars in Taurus after ROSAT. *Astron. Astrophys.* 331:977–988.

Kroupa, P., Tout, C. A., and Gilmore, G. 1990. The low-luminosity stellar mass function. *Mon. Not. R. Astron. Soc.* 244:76–85.

Ladd, E. F. et al. 1991. Far-infrared and submillimeter wavelength observations of star-forming dense cores. II. Images. *Astrophys. J.* 382:555–569.

Larson, R. B. 1969. The dynamics of a collapsing proto-star. *Mon. Not. R. Astron. Soc.* 145:271–295.

Larson, R. B. 1972. The collapse of a rotating cloud. *Mon. Not. R. Astron. Soc.* 156:437–458.

Larson, R. B. 1978. Calculations of three-dimensional collapse and fragmentation. *Mon. Not. R. Astron. Soc.* 184:69–85.

Larson, R. B. 1984. Gravitational torques and star formation. *Mon. Not. R. Astron. Soc.* 206:197–207.

Larson, R. B. 1985. Cloud fragmentation and stellar masses. *Mon. Not. R. Astron. Soc.* 214:379–398.

Larson, R. B. 1990. Formation of star clusters. In *Physical Processes in Fragmentation and Star Formation*, eds. R. Capuzzo-Dolcetta, C. Chiosi, and A. DiFazio (Dordrecht: Kluwer), pp. 389–400.

Laughlin, G., and Bodenheimer, P. 1994. Nonaxisymmetric evolution in protostellar disks. *Astrophys. J.* 436:335–354.

Laughlin, G., Korchagin, V., and Adams, F. C. 1997. Spiral mode saturation in self-gravitating disks. *Astrophys. J.* 477:410–423.

Laughlin, G., Korchagin, V., and Adams, F. C. 1998. The dynamics of heavy gaseous disks. *Astrophys. J.* 504:945–966.

Laughlin, G., and Różyczka, M. 1996. The effect of gravitational instabilities on protostellar disks. *Astrophys. J.* 456:279–291.

Lay, O. P., Carlstrom, J. E., and Hills, R. E. 1995. NGC 1333 IRAS 4: Further multiplicity revealed with the CSO-JCMT interferometer. *Astrophys. J.* 452:L73–L76.

Leinert, C. H., Henry, T., Glindemann, A., and McCarthy, D. W., Jr. 1997. A search for companions to nearby southern M dwarfs with near-infrared speckle interferometry. *Astron. Astrophys.* 325:159–166.

Lemme, C., Walmsley, C.M., Wilson, T.L., and Muders, D. 1995. A detailed study of an extremely quiescent core: L1498. *Astron. Astrophys.* 302:509–520.

Lizano, S., and Shu, F. H. 1989. Molecular cloud cores and bimodal star formation. *Astrophys. J.* 342:834–854.

Looney, L. W., Mundy, L. G., and Welch, W. J. 1997. High-resolution $\lambda = 2.7$ millimeter observations of L1551 IRS 5: A protobinary system? *Astrophys. J.* 484:L157–L160.

Low, C., and Lynden-Bell, D. 1976. The minimum Jeans mass, or when fragmentation must stop. *Mon. Not. R. Astron. Soc.* 176:367–390.

Mathieu, R. 1994. Pre-main-sequence binary stars. *Ann. Rev. Astron. Astrophys.* 32:465–530.

Mazeh, T., Goldberg, D., Duquennoy, A., and Mayor, M. 1992. On the mass ratio distribution of spectroscopic binaries with solar-type primaries. *Astrophys. J.* 401:265–268.

McDonald, J. M., and Clarke, C. J. 1995. The effect of star-disc interactions on the binary mass-ratio distribution. *Mon. Not. R. Astron. Soc.* 275:671–684.

Miyama, S. M. 1992. Criteria for the collapse and fragmentation of rotating clouds. *Publ. Astron. Soc. Japan* 44:193–202.

Miyama, S. M., Hayashi, C., and Narita, S. 1984. Criteria for collapse and fragmentation of rotating isothermal clouds. *Astrophys. J.* 279:621–632.

Miyama, S. M., Nakamoto, T., Kikuchi, N., Inutsuka, S., Kobayashi, K., and Takeuchi, T. 1994. The stability of circumstellar disks. In *Numerical Simulations in Astrophysics*, eds. J. Franco, S. Lizano, L. Aguilar, and E. Daltabuit (Cambridge: Cambridge Univ. Press), pp. 305–312.

Monaghan, J. J. 1992. Smoothed particle hydrodynamics. *Ann. Rev. Astron. Astrophys.* 30:543–574.

Monaghan, J. J. 1994. Vorticity, angular momentum, and cloud fragmentation. *Astrophys. J.* 420:692–704.

Monaghan, J. J., and Lattanzio, J. C. 1991. A simulation of the collapse and fragmentation of cooling molecular clouds. *Astrophys. J.* 375:177-189.

Motte, F., Andre, P., and Neri, R. 1998. The initial conditions of star formation in the ρ Ophiuchi main cloud: wide-field millimeter continuum mapping. *Astron. Astrophys.* 336:150–172.

Mundy, L. G., Wootten, A., Wilking, B. A., Blake, G. A., and Sargent, A. I. 1992. IRAS 16293-2422: A very young binary system? *Astrophys. J.* 385:306–313.

Myers, P. C. 1985. Molecular cloud cores. In *Protostars and Planets II*, eds. D. C. Black and M. S. Matthews (Tucson: Univ. of Arizona), pp. 81–103.

Myers, P. C. 1987. Dense cores and young stars in dark clouds. In *Star Forming Regions*, eds. M. Peimbert and J. Jugaku (Dordrecht: Reidel), pp. 33–44.

Myers, P. C., Fuller, G. A., Goodman, A. A., and Benson, P. J. 1991. Dense cores in dark clouds. VI. Shapes. *Astrophys. J.* 376:561–572.

Myers, P. C., and Goodman, A. A. 1988. Evidence for magnetic and virial equilibrium in molecular clouds. *Astrophys. J.* 326:L27–L30.

Myhill, E., and Kaula, W. M. 1992. Numerical models for the collapse and fragmentation of centrally condensed molecular cloud cores. *Astrophys. J.* 386:578–586.

Nelson, A. F., Benz, W., Adams, F. C., and Arnett, D. 1998. Dynamics of circumstellar disks. *Astrophys. J.* 502:342–371.

Nelson, R. P., and Papaloizou, J. C. B. 1993. Three-dimensional hydrodynamic simulations of collapsing prolate clouds. *Mon. Not. R. Astron. Soc.* 265:905–920.

Papaloizou, J. C. B., and Savonije, G. J. 1991. Instabilities in self-gravitating gaseous disks. *Mon. Not. R. Astron. Soc.* 248:353–369.

Patience, J., Ghez, A. M., Reid, I. N., Weinberger, A. J., and Matthews, K. 1998. The multiplicity of the Hyades and its implications for binary star formation and evolution. *Astron. J.* 115:1972–1988.

Petr, M. G., Du Foresto, V. C., Beckwith, S. V. W., Richichi, A., and McCaughrean, M. J. 1998. Binary stars in the Orion Trapezium cluster core. *Astrophys. J.* 500:825–837.

Pollack, J. B., McKay, C. P., and Christofferson, B. M. 1985. A calculation of the Rosseland mean opacity of dust grains in primordial solar system nebulae. *Icarus* 64:471–492.

Preibisch, T., Ossenkopf, V., Yorke, H. W., and Henning, Th. 1993. The influence of ice-coated grains on protostellar spectra. *Astron. Astrophys.* 279:577–588.

Pringle, J. E. 1989. On the formation of binary stars. *Mon. Not. R. Astron. Soc.* 239:361–370.

Rees, M. J. 1976. Opacity-limited hierarchical fragmentation and the masses of protostars. *Mon. Not. R. Astron. Soc.* 176:483–486.

Safronov, V. S. 1960. On the gravitational instability in flattened systems with axial symmetry and non-uniform rotation. *Ann. Astrophys.* 23:901–904.

Shu, F. H., Najita, J., Galli, D., Ostriker, E., and Lizano, S. 1993. The collapse of cores and the formation and evolution of stars and disks. In *Protostars and Planets III*, eds. E. H. Levy and J. I. Lunine (Tucson: Univ. of Arizona), pp. 3–45.

Sigalotti, L. DiG., and Klapp, J. 1994. Gravitational collapse and fragmentation of centrally condensed protostellar cores. *Mon. Not. R. Astron. Soc.* 268:625–640.

Sigalotti, L. DiG., and Klapp, J. 1997. Collapse and fragmentation models of prolate molecular cloud cores. I. Initial uniform rotation. *Astrophys. J.* 474:710–718.

Silk, J. 1977. On the fragmentation of cosmic gas clouds. II. Opacity-limited star formation. *Astrophys. J.* 214:152–160.

Simon, M., Close, L. M., and Beck, T. L. 1999. Adaptive optics imaging of the Orion Trapezium cluster. *Astron. J.* , in press.

Tassoul, J.-L. 1978. *Theory of Rotating Stars* (Princeton: Princeton Univ. Press).

Terebey, S., Van Buren, D., Padgett, D. L., Hancock, T., and Brundage, M. 1998. A candidate protoplanet in the Taurus star-forming region. *Astrophys. J.* 507:L71–L74.

Tohline, J. E. 1981. The collapse to equilibrium of rotating adiabatic spheroids. I. Protostars. *Astrophys. J.* 248:717–726.

Tomisaka, K., Ikeuchi, S., and Nakamura, T. 1990. The equilibria and evolutions of magnetized, rotating isothermal clouds. IV. Quasistatic evolution. *Astrophys. J.* 362:202–214.

Tomley, L., Cassen, P., and Steiman-Cameron, T. 1991. On the evolution of gravitationally unstable protostellar disks. *Astrophys. J.* 382:530–543.

Tomley, L., Steiman-Cameron, T., and Cassen, P. 1994. Further studies of gravitationally unstable protostellar disks. *Astrophys. J.* 422:850–861.

Toomre, A. 1964. On the gravitational stability of a disk of stars. *Astrophys. J.* 139:1217–1238.

Truelove, J. K., Klein, R. I., McKee, C. F., Holliman, J. H., Howell, L. H., and Greenough, J. A. 1997. The Jeans condition: a new constraint on spatial resolution in simulations of isothermal self-gravitational hydrodynamics. *Astrophys. J.* 489:L179–L183.

Truelove, J. K., Klein, R. I., McKee, C. F., Holliman, J. H., Howell, L. H., Greenough, J. A., and Woods, D. T. 1998. Self-gravitational hydrodynamics with three-dimensional adaptive mesh refinement: methodology and applications to molecular cloud collapse and fragmentation. *Astrophys. J.* 495:821–852.

Tsai, J., and Bertschinger, E. 1989. Stability of the expansion wave solution of collapsing isothermal spheres. *Bull. Amer. Astron. Soc.* 21:1089 (abstract).

Turner, J. A., Chapman, S. J., Bhattal, A. S., Disney, M. J., Pongracic, H., and Whitworth, A. P. 1995. Binary star formation: gravitational fragmentation followed by capture. *Mon. Not. R. Astron. Soc.* 277:705–726.

Umebayashi, T., and Nakano, T. 1990. Magnetic flux loss from interstellar clouds. *Mon. Not. R. Astron. Soc.* 243:103–113.

Walker, C.K., Adams, F. C., and Lada, C. J. 1990. 1.3 millimeter continuum observations of cold molecular cloud cores. *Astrophys. J.* 349:515–528.

Walker, C.K., Carlstrom, J.E. and Bieging, J.H. 1993. The IRAS 16293–2422 cloud core: a study of a young binary system. *Astrophys. J.* 402:655–666.

Ward-Thompson, D., Scott, P. F., Hills, R. E., and André, P. 1994. A submillimetre continuum survey of pre-protostellar cores. *Mon. Not. R. Astron. Soc.* 268:276–290.

Watkins, S. J., Bhattal, A. S., Boffin, H. M. J., Francis, N., and Whitworth, A. P. 1998a. Numerical simulations of protostellar encounters. II. Coplanar disc-disc encounters. *Mon. Not. R. Astron. Soc.* 300:1205–1213.

Watkins, S. J., Bhattal, A. S., Boffin, H. M. J., Francis, N., and Whitworth, A. P. 1998b. Numerical simulations of protostellar encounters. III. Non-coplanar disc-disc encounters. *Mon. Not. R. Astron. Soc.* 300:1214–1224.

Whitworth, A. P., and Clarke, C. J. 1997. Cooling behind mildly supersonic shocks in molecular clouds. *Mon. Not. R. Astron. Soc.* 291:578–584.

Williams, H. A., and Tohline, J. E. 1988. Circumstellar ring formation in rapidly rotating protostars. *Astrophys. J.* 334:449–464.

Wootten, A. 1989. The duplicity of IRAS 16293–2422: a protobinary star? *Astrophys. J.* 337:858–864.

Zinnecker, H. 1990. Pre-main-sequence binaries. In *ESO Workshop on Low-Mass Star Formation and Pre-main-sequence Objects*, ed. B. Reipurth (Garching: ESO), pp. 447–469.

FIGURE CAPTIONS

Figure 1A. Equatorial slice of log density (in g cm^{-3}) at a time $t = 1.415 \times 10^{12}$ s after the onset of collapse of a uniform isothermal cloud subject to a 10% m =2 perturbation (AMR code). The fragments have begun to evolve into filamentary structures. Density range: log $\rho_{max} = -12.41$ (red); log $\rho_{min} = -15.57$ (violet); resolution 2.44 $\times 10^{13}$ cm. Velocity vectors are shown with length proportional to speed; maximum velocity 0.824 km s^{-1}.

Figure 1B. Density contours in the equatorial plane at time $t = 1.376 \times 10^{12}$ s for the same cloud that is shown in Fig. 1A, calculated with the Boss code. A binary protostar has formed. The maximum density $= 6.3 \times 10^{-14}$ g cm^{-3}, the box radius $= 5.8 \times 10^{15}$ cm, and the contours are separated by factors of two.

Figure 2A. Equatorial slice of log density (in g cm^{-3}) at a time $t = 7.23 \times 10^{11}$ s after the onset of collapse of an isothermal cloud, initially with a Gaussian density profile subject to a 10% $m = 2$ perturbation (AMR code). Density range: log $\rho_{max} = -10.33$ (red); log $\rho_{min} = -13.11$ (violet); resolution 1.526 $\times 10^{12}$ cm; maximum velocity 0.742 km s^{-1}. The single filament morphology was reproduced in a run at eight times the spatial resolution (twice in linear dimension) verifying the convergence of the filamentary structure.

Figure 2B. Density contours in the equatorial plane at $t = 7.050 \times 10^{11}$ s, for the same cloud that is shown in Fig. 2A, calculated with several times the spatial resolution of model I of Boss (1998). An isothermal filament has formed with maximum density $= 4.0 \times 10^{-11}$ g cm^{-3}. The box radius is 3.0×10^{14} cm and the contours are separated by factors of two.

Figure 3. Equatorial slice in log density (in g cm^{-3}) at $t = 1.60 \times 10^{12}$ s. for a non-isothermal cloud initially having uniform density and an $m = 2$ perturbation of 10% amplitude. Density range: log $\rho_{max} = -10.28$ (red); log $\rho_{min} = -16.48$ (violet); resolution 9.76 $\times 10^{13}$ cm; maximum velocity 2.75 km s^{-1}. The protostellar disks have interacted with the surrounding spiral arms. 50% of the initial mass of the cloud has accreted into the binary cores, protostellar disks and spiral arms at this time.

Figure 4A. Contours of equal density in the equatorial (x, y) plane at four times during the fragmentation of a cloud with an initial $\rho \propto r^{-1}$, calculated with a nested grid code. Velocity vectors are shown with length proportional to speed. The times, maximum densities, contour intervals, and maximum velocities are as follows: (a; upper left) t = 1.176 $\times 10^{12}$ s; log $\rho_{max} = -10.3$; Δ log $\rho = 0.166$; $V_{max} = 1.95 \times 10^5$ cm s^{-1}. (b; upper right) t = 1.190 $\times 10^{12}$ s; log $\rho_{max} = -10.2$; Δ log $\rho = 0.17$; $V_{max} = 2.57 \times 10^5$ cm s^{-1}. (c; lower left) t = 1.209 $\times 10^{12}$ s; log $\rho_{max} = -10.1$; Δ log $\rho = 0.173$; $V_{max} = 3.17 \times 10^5$ cm s^{-1}. (d; lower right) t = 1.217 $\times 10^{12}$ s; log $\rho_{max} = -10.1$; Δ log $\rho = 0.173$; $V_{max} = 3.67 \times 10^5$ cm s^{-1}. ¿From Burkert et al. (1997).

Figure 4B. The evolution of the same cloud that is shown in Fig. 4A, calculated with an SPH code. Symbols and curves have the same meaning as in Fig. 4A. (a; upper left) t=1.135 $\times 10^{12}$ s; log $\rho_{max} = -10.0$; Δ log

$\rho = 0.25$; $V_{max} = 2.44 \times 10^5$ cm s^{-1}. (b; upper right) t=1.160×10^{12} s; log $\rho_{max} = -9.7$; Δ log $\rho = 0.25$; $V_{max} = 2.88 \times 10^5$ cm s^{-1}. (c; lower left) t=1.175×10^{12} s; log $\rho_{max} = -9.6$ Δ log $\rho = 0.25$; $V_{max} = 3.66 \times 10^5$ cm s^{-1}. (d; lower right) t=1.190×10^{12} s; log $\rho_{max} = -9.5$; Δ log $\rho = 0.25$; $V_{max} = 3.38 \times 10^5$ cm s^{-1}. ¿From Burkert et al. (1997).