THE DYNAMICAL STRUCTURE AND EVOLUTION OF GIANT MOLECULAR CLOUDS

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1. Introduction: The Observed Characteristics of GMCs

The interstellar medium (ISM) of galaxies contains gas that spans a wide range of physical conditions, from hot X-ray emitting plasma to cold molecular gas. The molecular gas is of particular importance because it is believed to be the site of all the star formation that occurs in galaxies. In the Milky Way, molecular gas constitutes about half the total mass of gas within the solar circle. Much of this gas is concentrated in large aggregations called giant molecular clouds (GMCs), which have masses $M \gtrsim 10^4\ M_\odot$. Smaller molecular clouds are also observed, such as the high latitude clouds discovered by Blitz et al. [9] and the small molecular clouds in the Galactic plane cataloged by Clemens & Barvainis [19]. GMCs have internal structure, and I shall follow the terminology of Williams et al. [114] in describing this: *Clumps* are coherent regions in longitude–latitude–velocity space that are generally identified from spectral line maps of molecular emission. *Star–forming clumps* are the massive clumps out of which stellar clusters form. Finally, *cores* are the regions out of which single stars (or multiple stellar systems like binaries) are formed. These characteristics, together with the important observational properties of GMCs, are reviewed elsewhere in this volume by Blitz. In this review I shall first briefly summarize some of the key properties of GMCs and then attempt to account for the dynamical properties theoretically.

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1.1. CHEMICAL AND THERMAL PROPERTIES

Molecular clouds are composed primarily of H_2, but this is relatively difficult to observe. The next most abundant molecule is generally CO, which can be readily observed in both emission and absorption. It is surprising that such molecules can exist in the harsh environment of interstellar space, particularly because of the destructive effects of ultraviolet radiation. Atomic hydrogen (H^0) shields most interstellar gas from EUV photons (those with energies 100 eV $\gtrsim h\nu \geq 13.6$ eV), but FUV photons (those with 13.6 eV$> h\nu \gtrsim 5$ eV) are far more penetrating. These photons ionize atoms such as C, Mg, S, and Fe, and photodissociate molecules. The photodissociation of H_2 and CO occurs in a two step process: first, the molecule undergoes an electronic excitation by absorption of a resonance line photon; then some fraction of the molecules radiate into a state in the vibrational continuum and fly apart [23].

A significant column density of molecules can build up only when the absorption lines become optically thick, or, if this is inadequate, when the FUV radiation is sufficiently attenuated by dust. Under typical conditions in the local ISM, observations show that the extinction to the cloud surface must exceed about 0.1 mag in order for a significant column density of H_2 to be observed [11] (the extinction through the entire cloud is then twice this, or about 0.2 mag). Since CO is less abundant than H_2, a significantly larger column density is required in order for the carbon to become incorporated into molecules. The calculations of van Dishoeck & Black [109] show that a GMC in the local ISM has a layer of C^+ and C^0 corresponding to a column density $N_{\rm H} = 1.4 \times 10^{21}$ cm^{-2}. (Note that we shall measure all densities in terms of the total hydrogen density, $n_{\rm H} = 2n_{\rm H_2}$ for molecular gas, and similarly for column densities.) For the dust to gas ratio observed in the local ISM, the relation between extinction and hydrogen column is [102]

$$A_V = \frac{N_{\rm H}}{2.0 \times 10^{21}\ {\rm cm}^{-2}}, \qquad (1)$$

so this column is equivalent to an extinction of 0.7 mag.

An interstellar cloud with a mean extinction significantly greater than 2×0.7 mag is thus expected to have a thin outer layer of H^0 ($\Delta A_V \simeq$ 0.1 mag), a thicker layer of H_2, C^+, and C^0 ($\Delta A_V \simeq 0.6$ mag), and an interior that is nearly fully molecular. The temperature of the outer atomic layer is of order 50-100 K. In the deep interior, where the gas is fully molecular, the temperature of about 10 K is set by the balance between heating due to cosmic ray ionization and cooling due to CO emission. Because the chemical and thermal structure of the edge of the cloud is dominated by photodissociation, it is termed a photodissociation region [42].

The ionization in most of the volume of a molecular cloud is due to FUV photons; under typical conditions cosmic rays dominate the ionization only for regions in a cloud such that the extinction to the surface exceeds about 4 mag [60]. The ionization in the cosmic ray ionized region of a molecular cloud can be expressed as

$$x_e \equiv \frac{n_e}{n_{\rm H}} = \frac{C_i}{n_{\rm H}^{1/2}}. \tag{2}$$

Williams et al. [112] found $C_i = (0.5 - 1.0) \times 10^{-5}$ cm$^{-3/2}$ from chemical modeling of observations of a number of low mass cores, where the factor of two uncertainty arises from uncertainties in the chemical reaction rates. In arriving at this result, they estimated that the typical density in the cores they observed is $n_{\rm H} \simeq 2 - 6 \times 10^4$ cm^{-3} and that the cosmic ray ionization rate is $\zeta_{\rm H} = 2.5 \times 10^{-17}$ s^{-1}. This value for the ionization is in good agreement with theoretical expectations [65].

1.2. DYNAMICAL PROPERTIES

Some of the most salient characteristics of GMCs were summarized in 1981 by Larson [48], and these results are sometimes referred to as "Larson's laws". The first relation is the *line width–size relation*: molecular clouds are supersonically turbulent with line widths Δv that increase as a power of the size, $\Delta v \propto R^p$. Larson himself estimated that $p \simeq 0.38$, close to the value 1/3 appropriate for turbulence in incompressible fluids. Subsequent work has distinguished between the relation valid for a collection of GMCs and that valid within individual GMCs or parts of GMCs. For different GMCs inside the solar circle, Solomon et al [100] found

$$\sigma = (0.72 \pm 0.07) R_{\rm pc}^{0.5\pm0.05} \quad {\rm km\ s^{-1}}, \tag{3}$$

where σ is the one–dimensional velocity dispersion; it is related to the full-width half-maximum of the line by $\sigma = \Delta v/2.355$. By comparison, the thermal velocity dispersion of molecular gas is $\sigma_{\rm th} = 0.188(T/10\ {\rm K})^{1/2}$ km s^{-1}. Within low–mass cores, Caselli & Myers [15] found that the non–thermal velocity dispersion (i.e, what remains after eliminating the thermal contribution) is

$$\sigma_{\rm nt} \simeq 0.55 R_{\rm pc}^{0.51} \quad {\rm km\ s^{-1}}, \tag{4}$$

which is quite similar to that found for GMCs by Solomon et al. For "high–mass cores", however, Caselli & Myers found

$$\sigma_{\rm nt} \simeq 0.77 R_{\rm pc}^{0.21} \quad {\rm km\ s^{-1}}, \tag{5}$$

a substantially flatter relation. Although these cores are forming more massive stars than the low mass cores, they are not forming OB stars. Plume et al [87] have surveyed clumps in which OB star formation is believed to be occurring. They found that such clumps do not obey the line width–size relation, and furthermore, that $\sigma_{\rm nt}$ is greater than indicated by equation (5).

Larson's second result was that GMCs are gravitationally bound. We shall discuss this in §2.2 below. He concluded that clumps within GMCs are also gravitationally bound, but for ^{13}CO clumps this appears to be true only for the most massive ones [5].

His third conclusion was that all GMCs have about the same column density. As he pointed out, only two of these three conclusions are independent; any one of them can be derived from the other two. For example, if the clouds are gravitationally bound, then $\sigma^2 \propto GM/R \propto \bar N_{\rm H} R$, where $\bar N_{\rm H} = M/(\pi\mu_{\rm H} R^2)$ is the mean column density of the cloud, and $\mu_{\rm H} = 2.34\times 10^{-24}$ g is the mean mass per hydrogen. The line width–size relation for GMCs, $\sigma^2 \propto R$ [100] then implies $\bar N_{\rm H} =$ const. For GMCs inside the solar circle, Solomon et al [100] found

$$\bar N_{\rm H22} = (1.5\pm 0.3) R_{\rm pc}^{0.0\pm 0.1} \qquad {\rm cm}^{-2}, \tag{6}$$

where $\bar N_{\rm H22} \equiv \bar N_{\rm H}/(10^{22}$ Hydrogen nuclei cm$^{-2})$; this corresponds to an extinction $\bar A_V = 7.5$ mag with the local gas to dust ratio. This result does not apply to unbound clumps in GMCs, which can have lower column densities [5], nor to the OB star–forming clumps studied by Plume et al [87], which have $\bar N_{\rm H22} \sim 60$.

There are several other characteristics of GMCs that we must take note of. First, GMCs appear to have magnetic fields that are dynamically significant [41]; this will be discussed in §2.4 below. Second, GMCs are highly clumped, in that the typical density $n_{\rm H}$—i.e., the local density around a typical molecule—is significantly greater than the volume–averaged density $\bar n_{\rm H}$ in the cloud. Liszt [51] has summarized studies of the typical density of clouds in the Galactic plane as inferred from their excitation, and cites values of $n_{\rm H}$ ranging from 10^3 to 1.2×10^4 cm^{-3}; in other words, $n_{\rm H} \simeq 3000$ cm^{-3} ± 0.5 dex. However, the mean density of gas in the GMCs, $\bar n_{\rm H}$, is considerably less, at least for the massive ones: since $M \propto \bar n_{\rm H} R^3$ and $\bar N_{\rm H} \propto \bar n_{\rm H} R$, we have

$$\bar n_{\rm H} = \frac{84}{M_6^{1/2}} \left(\frac{\bar N_{\rm H22}}{1.5}\right)^{3/2} \qquad {\rm cm}^{-3}, \tag{7}$$

where $M_6 \equiv M/(10^6\ M_\odot)$. The filling factor of the gas in GMCs is then typically

$$f \equiv \frac{\bar{n}_{\rm H}}{n_{\rm H}} = \frac{0.084}{n_{\rm H3} M_6^{1/2}} \left(\frac{\bar{N}_{\rm H22}}{1.5}\right)^{3/2}, \tag{8}$$

where $n_{\rm H3} = n_{\rm H}/(10^3\ {\rm cm}^{-3})$. Clouds of mass $M \lesssim 10^4\ M_\odot$ must have $n_{\rm H} > 10^3\ {\rm cm}^{-3}$ if they are to have the typical column density found by Solomon et al [100]. The nature of the low density interclump medium is uncertain; for example, it is not even known whether it is atomic or molecular [114]. A possible explanation for the small filling factor of the gas in GMCs will be given below, where we discuss models of turbulence in these clouds.

Finally, GMCs have a power-law mass distribution with a relatively sharp cutoff. Let $d\mathcal{N}_c(M)$ be the number of GMCs with mass between M and $M + dM$. Then the observations of GMCs inside the solar circle (but excluding the Galactic Center) are consistent with the mass distribution [115]

$$\frac{d\mathcal{N}_c}{d\ln M} = 63 \left(\frac{6\times 10^6\ M_\odot}{M}\right)^{0.6} \qquad (M \le 6\times 10^6\ M_\odot), \tag{9}$$

$$= 0 \qquad (M > 6\times 10^6\ M_\odot). \tag{10}$$

The mass distribution is often represented in terms of $d\mathcal{N}_c/dM$ instead of $d\mathcal{N}_c/d\ln M$, which leads to an exponent of 1.6 instead of 0.6. The form adopted here has the advantage that $d\mathcal{N}_c/d\ln M$ represents an actual number of clouds. The fact that the coefficient 63 is so much larger than unity means that it must have a physical significance [62]: If there were no cutoff to the distribution, one would expect $63/0.6 \simeq 100$ GMCs more massive than $6\times 10^6\ M_\odot$ in the Galaxy; in fact, there are none.

1.3. STAR FORMATION IN GMCS

GMCs are the sites of most of the star formation in the Galaxy. A crucial fact about star formation is that it is usually very inefficient: In the absence of support, GMCs would collapse to very high densities and presumably form stars in a free fall time

$$t_{\rm ff} = \left(\frac{3\pi}{32 G\bar{\rho}}\right)^{1/2} = \frac{1.37\times 10^6}{\bar{n}_{\rm H3}^{1/2}}\ \ {\rm yr}. \tag{11}$$

(As we shall see in §2.7, simulations indicate that it is very difficult to maintain the turbulence that supports GMCs.) Now, the total mass of

GMCs inside the solar circle is $10^9\ M_\odot$ [115], and the mean density in GMCs is about 10^2 cm^{-3} (eq. 7). If GMCs collapsed in a free fall time and formed stars, the Galactic star formation rate would be

$$\dot{M}_* \simeq \frac{10^9\ M_\odot}{4 \times 10^6\ \mathrm{yr}} = 250\ M_\odot\ \mathrm{yr}^{-1}, \tag{12}$$

far greater than the observed rate in this part of the Galaxy of 3 $M_\odot$ yr^{-1} [62]. This gross disparity between the potential star formation rate and the actual one was pointed out many years ago by Zuckerman and Evans [116], who concluded that the supersonic motions observed in GMCs do not reflect gravitational infall. Any successful theory of star formation must account for this inefficiency.

2. Dynamical Structure of GMCs

2.1. THE VIRIAL THEOREM

Some insight into the structure of GMCs can be gained from the virial theorem, as shown in the classic studies of self–gravitating gas clouds by McCrea [59] and by Mestel & Spitzer [68]. Define the quantity $I = \int r^2 dm$, which is proportional to the trace of the inertia tensor. Next, evaluate $\ddot{I} \equiv d^2I/dt^2$ from the equation of motion,

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P_{\mathrm{th}} + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} + \rho\mathbf{g}, \tag{13}$$

where $\mathbf{g}$ is the acceleration due to gravity. The result is the virial theorem,

$$\frac{1}{2}\ddot{I} = 2(\mathcal{T} - \mathcal{T}_s) + \mathcal{M} + \mathcal{W}. \tag{14}$$

Alternatively, this relation can be derived by taking the dot product of $\mathbf{r}$ with the equation of motion and integrating over a volume corresponding to a fixed mass. We shall now consider each of the terms in this equation.

The term on the LHS reflects variations in the rate of change of the size and shape of the cloud. This term is usually neglected, but it may be significant for a turbulent cloud. In contrast to the terms on the RHS of the equation, it can be of either sign, and as a result its effects can be averaged out either by applying the virial theorem to an ensemble of clouds or by averaging over a time long compared with the dynamical time.

The first two terms on the RHS contain the effects of kinetic energy, including thermal energy; they do *not* include the energy associated with internal degrees of freedom. The thermal energy density inside the cloud is $(3/2)\rho c^2 = (3/2)P_{\mathrm{th}}$, where c is the isothermal sound speed, and the bulk kinetic energy density is $(1/2)\rho v^2$. The total kinetic energy inside the cloud is then

$$\mathcal{T} = \int_{V_{\rm cl}} \left(\frac{3}{2}P_{\rm th} + \frac{1}{2}\rho v^2\right)\, dV \equiv \frac{3}{2}\bar{P}V_{\rm cl}, \tag{15}$$

where $V_{\rm cl}$ is the volume of the cloud and $\bar{P}$ is the mean pressure of the gas (i.e., it does not include the pressure associated with the magnetic field). Note that the kinetic energy associated with rotation is included in $\mathcal{T}$ and therefore $\bar{P}$; rotation is generally not a dominant effect in molecular clouds, however [39] [65]. The mean pressure can be related to the 1D velocity dispersion in the cloud σ, which is observable:

$$\sigma^2 \equiv \frac{1}{M}\int (c^2 + \frac{1}{3}v^2)\, dM = \frac{\bar{P}}{\bar{\rho}}. \tag{16}$$

The second kinetic energy term is a surface term,

$$\mathcal{T}_s \equiv \frac{1}{2}\int_S P_{\rm th}\mathbf{r}\cdot \mathbf{dS}, \tag{17}$$

where the integral is over the surface of the cloud. If the thermal pressure in the ambient medium is constant at P_s, then $\mathcal{T}_s = (3/2)P_s V_{\rm cl}$. As a result, the two kinetic energy terms combine to give

$$2(\mathcal{T} - \mathcal{T}_s) = 3(\bar{P} - P_s)V_{\rm cl}. \tag{18}$$

Thus, it is the difference between the energy in the cloud and that in the background medium that enters the virial theorem. If the ambient medium is turbulent and has a density much smaller than that of the cloud, one can show that P_s is somewhat less than the total pressure far from the cloud [63]: The stress exerted by the ambient medium normal to the cloud is entirely thermal (and is significantly larger than the thermal pressure far from the cloud if the turbulence in the ambient medium is supersonic), but it is reduced below the value it otherwise would have by flow of intercloud gas along the cloud surface.

The magnetic term in the virial theorem is fairly complicated,

$$\mathcal{M} = \frac{1}{8\pi}\int_V B^2 dV + \frac{1}{4\pi}\int_S \mathbf{r}\cdot \left(\mathbf{BB} - \frac{1}{2}B^2\mathbf{I}\right)\cdot \mathbf{dS}, \tag{19}$$

where $\mathbf{I}$ is the unit tensor. If the cloud is immersed in a low density ambient medium and if the stresses due to MHD waves in the ambient medium are negligible, then the field outside the cloud will be approximately force free. In this case, one can show that the magnetic term becomes [63]

$$\mathcal{M} = \frac{1}{8\pi}\int (B^2 - B_0^2) dV, \tag{20}$$

where B_0 is the field strength in the ambient medium far from the cloud. Thus, $\mathcal{M}$ is the difference between the total magnetic energy with the cloud and that in the absence of the cloud.

Finally, we consider the gravitational term $\mathcal{W}$. In the absence of an external gravitational field, this is the gravitational energy of the cloud [102],

$$\mathcal{W} = \int \rho \mathbf{r} \cdot \mathbf{g} dV = -\frac{3}{5} a \left(\frac{GM^2}{R} \right), \tag{21}$$

where a is a numerical factor of order unity that has been evaluated by Bertoldi and McKee [5]. In order to relate this term to the other terms in the virial theorem, we define the "gravitational pressure" P_G by

$$\mathcal{W} \equiv -3 P_G V_{\rm cl}; \tag{22}$$

intuitively, P_G is just the mean weight of the material in the cloud. The gravitational pressure can be evaluated as

$$P_G = \left(\frac{3\pi a}{20} \right) G \Sigma^2 \rightarrow 1.39 \times 10^5 \bar{N}_{\rm H22}^2 \quad {\rm K\ cm}^{-3}, \tag{23}$$

where $\Sigma \equiv M/\pi R^2 \equiv \mu_{\rm H} \bar{N}_{\rm H}$ is the mean surface density of the cloud. The numerical evaluation is for a spherical cloud with a $1/r$ density profile.

These results enable us to express the steady–state, or time–averaged, virial theorem (eq. 14 with $\ddot{I} = 0$) as

$$\bar{P} = P_s + P_G \left(1 - \frac{\mathcal{M}}{|\mathcal{W}|} \right). \tag{24}$$

In this form, the virial theorem has an immediate intuitive meaning: the mean pressure inside the cloud is the surface pressure plus the weight of the material inside the cloud, reduced by the magnetic stresses.

2.2. ARE GMCS GRAVITATIONALLY BOUND?

With these results in hand, we can now address the issue of whether molecular clouds and their constituents are gravitationally bound. We assume that the cloud is large enough that the motions are highly supersonic (§1.2), and as a result the energy in internal degrees of freedom is negligible. The total energy is then $E = \mathcal{T} + \mathcal{M} + \mathcal{W}$, which can be expressed as

$$E = \frac{3}{2} \left[P_s - P_G \left(1 - \frac{\mathcal{M}}{|\mathcal{W}|} \right) \right] V_{\rm cl}. \tag{25}$$

with the virial theorem (eq. 24). In the absence of a magnetic field, the condition that the cloud be bound (i.e., $E < 0$) is simply $P_G > P_s$. We shall use this criterion even for magnetized clouds, bearing in mind that using the total ambient gas pressure (thermal plus turbulent) for P_s is an overestimate and that our analysis is approximate because we have used the time-averaged virial theorem. In the opposite case, in which $P_s \gg P_G$, the cloud is said to be *pressure–confined.* In the typical case in which the pressure is largely turbulent, the "pressure–confined cloud" is likely to be transient.

For GMCs, the surface pressure is that of the ambient ISM. In the solar vicinity, the total interstellar pressure is about 2.8×10^4 K cm^{-3}, which balances the weight of the ISM [14]. Of this, about 0.7×10^4 K cm^{-3} is due to cosmic rays; since they pervade both the ISM and a molecular cloud, they do not contribute to the support of a cloud and may be neglected. The magnetic pressure is about 0.3×10^4 K cm^{-3} [40], leaving $P_s \simeq 1.8 \times 10^4$ K cm^{-3} as the total ambient gas pressure.

What is the minimum value of P_G for a molecular cloud? According to van Dishoeck and Black [109], molecular clouds exposed to the local interstellar radiation field have a layer of C^+ and C^0 corresponding to a visual extinction of 0.7 mag (§1.1). If we require at least 1/3 of the carbon along a line of sight through a cloud to be in the form of CO in order to term the cloud "molecular", then the total visual extinction must be $\bar{A}_V > 2$ mag (allowing for a shielding layer on both sides). According to equation (23), this gives $P_G \gtrsim 2 \times 10^4$ K cm^{-3} $\sim P_s$, verifying that molecular clouds as observed in CO are at least marginally bound. Larson's "second law" is thus seen to be a consequence of the relationship between the column density required for CO to be significant and the pressure in the ISM. Note that if we defined molecular clouds as having a significant fraction of H_2 rather than CO, the minimum column density required would be substantially less and the clouds might not be bound. Furthermore, the conclusion that CO clouds are bound depends on the metallicity, the interstellar pressure and the strength of the FUV radiation field, so that CO clouds may not be bound everywhere in the Galaxy or in other galaxies [28].

GMCs in the solar neighborhood typically have mean extinctions significantly greater than 2 mag, and as a result P_G is generally significantly greater than P_s. Indeed, observations show $P_G \sim 2 \times 10^5$ K cm^{-3}, an order of magnitude greater than P_s [5] [8] [113]. Thus, if GMCs are dynamically stable entities (the crossing time for a GMC is about 10^7 yr, smaller than the expected lifetime [10] [115]), then GMCs must be self-gravitating. In the inner galaxy, where P_s is expected to be greater, the typical GMC linewidths also appear to be somewhat greater than those found locally [90], and thus P_G is still comfortably greater than P_s.

In order to determine whether clumps within GMCs are gravitationally bound, it is convenient to work in terms of the *virial parameter* [5] [76]

$$\alpha \equiv \frac{5\sigma^2 R}{GM}, \tag{26}$$

which is readily determined from observation. With the aid of equations (21) and (22), this parameter can be expressed as

$$\alpha = a\left(\frac{2\mathcal{T}}{|\mathcal{W}|}\right) = a\left(\frac{\bar{P}}{P_G}\right). \tag{27}$$

Bertoldi & McKee [5] show how this result can be applied to ellipsoidal clouds; if α is interpreted as an average over the orientation of the cloud, then the factor a is generally within a factor of about 1.3 of unity. In terms of the virial parameter, the net energy of the cloud is

$$E = |\mathcal{W}|\left[\frac{\alpha}{2a} - \left(1 - \frac{\mathcal{M}}{|\mathcal{W}|}\right)\right]. \tag{28}$$

A clump is therefore bound for $\alpha \lesssim 2$, although the exact value depends on the strength of the field. For $\alpha \gg 1$, clumps are pressure–confined.

Whether a clump is pressure–confined or gravitationally bound is directly related to its surface density [7]. The surface pressure on a clump is just the mean pressure inside the GMC,

$$P_s(\mathrm{clump}) \simeq P_G(\mathrm{GMC}) \propto \Sigma^2(\mathrm{GMC}), \tag{29}$$

where we have assumed that the GMC is strongly bound, so that the pressure acting on it can be ignored. As a result, the virial theorem for a clump becomes

$$\bar{P}(\mathrm{clump}) \propto \Sigma^2(\mathrm{GMC}) + \Sigma^2(\mathrm{clump}), \tag{30}$$

where we have assumed that the mass of an individual clump is small, so that it does not significantly affect the surface density of the cloud. Pressure–confined clumps have Σ(clump)$\ll \Sigma$(GMC) since $P_G \ll P_s$. On the other hand, gravitationally bound clumps have $\Sigma(\mathrm{clump}) \gtrsim \Sigma(\mathrm{GMC})$. The studies of the stability of gas clouds discussed below show that P_G cannot be too much greater than P_s if the cloud is to be gravitationally stable; correspondingly, Σ(clump) cannot be much greater than Σ(GMC) if the clump is stable. We conclude that gravitationally bound clumps have column densities similar to that of the GMC in which they are embedded, $\Sigma(\mathrm{clump}) \sim \Sigma(\mathrm{GMC})$, as is often observed [7].

From an analysis of ^{13}CO clumps in Ophiuchus, Orion B, the Rosette, and Cepheus OB3, Bertoldi & McKee [5] concluded that most clumps are pressure–confined; however, most of the mass is in clumps that appear to be gravitationally bound, or nearly so. Pressure–confined clumps do not satisfy the line width–size relation; instead, the velocity dispersion and mean density in the clumps are about constant. As a result, the virial parameter scales with mass as $M^{-2/3}$, with the most massive clumps having $3 \gtrsim \alpha \gtrsim 1$ (with the exception of Cepheus, for which the data are of low resolution). Furthermore, the observable star formation appears to be confined to these massive clumps. Thus, not only are GMCs as a whole gravitationally bound, but there is a mass scale $M(\mathrm{SFC})$, the mass of the star–forming clumps, such that structures with $M \sim M(\mathrm{SFC})$ are bound as well.

On still a smaller scale, cloud cores (out of which individual stars or stellar systems like binaries form) are also observed to be gravitationally bound. These cores exist in both star–forming clumps and, in star–forming clouds like Taurus, in relative isolation. Thus, there appears to be a hierarchy of bound structures: GMCs, the star–forming clumps within them, and cloud cores (cf [33]). As we shall see in §2.4.1, this hierarchy may be mirrored in the magnetic properties of GMCs.

2.3. ISOTHERMAL CLOUDS

Molecular gas is often observed to be at a temperature of about 10 K. The simplest model for a molecular cloud is thus an isothermal cloud. In fact, the cores of molecular clouds often have thermal pressures that are greater than the nonthermal pressures, so the model of an isothermal cloud may apply approximately to such cores. (For non–isothermal models, see [12] and [32].) For now, we shall neglect the effect of a static magnetic field.

Using the virial theorem, we can infer the basic properties of an isothermal, self–gravitating cloud [59] [101]. Setting $\mathcal{M} = 0$ in equation (24) and evaluating P_G with the aid of equation (23), we find

$$P_s = \frac{3M\sigma^2}{4\pi R^3} - \frac{3aGM^2}{20\pi R^4}, \tag{31}$$

where we have written the mean surface density as $\Sigma = M/\pi R^2$. If the cloud radius R is large, the second term on the RHS is negligible, and the mean pressure in the cloud (represented by the first term) is approximately equal to the ambient pressure P_s. Now consider a sequence of equilibria of smaller and smaller R. Initially, reducing R requires a higher ambient pressure P_s. Eventually, the second term on the RHS, which is P_G, becomes comparable to the first and the increase in P_s is halted. This occurs when the escape velocity is comparable to the velocity dispersion, $GM/R \sim \sigma^2$, as can be seen by comparing the two terms on the RHS of equation (31). Further reductions in R lead to a decrease in surface pressure, which is unstable. The point at which P_s is a maximum therefore represents a *critical point* that separates stable from unstable solutions. Using this result for the radius at the critical point, we find that the maximum pressure is of order

$$P_{\rm cr} \propto \frac{M\sigma^2}{(GM/\sigma^2)^3} \propto \frac{\sigma^8}{G^3M^2}. \tag{32}$$

Equivalently, this relation can be interpreted as giving the maximum mass that can be supported against gravity in a medium of pressure $P_{\rm cr}$. This critical mass is termed the Bonnor-Ebert mass after the two individuals who first worked out the structure of isothermal spheres [13] [25]:

$$M_{\rm BE} = 1.18\,\frac{\sigma^4}{(G^3P_s)^{1/2}}, \tag{33}$$

$$= 1.15\,\frac{(T/10\ {\rm K})^2}{(P_s/10^5\ {\rm K\ cm^{-3}})^{1/2}} \quad M_\odot. \tag{34}$$

The numerical value is for the conditions typical of a low mass core, with $n \sim 10^4$ cm^{-3} and $T \sim 10$ K, and it is significant that the result is of order the typical stellar mass.

The maximum central density of a stable isothermal sphere is only $\rho_c = 14.0\rho_s$, where ρ_s is the density at the surface. The mean density is $2.5\rho_s$. Equilibria at lower mass have significantly lower central concentrations: for example, the central density is only $2.2\rho_s$ for $M = 0.5M_{\rm BE}$.

2.4. MAGNETIC FIELDS VS GRAVITY

Magnetic fields are believed to play a crucial role in the structure and evolution of molecular clouds. Theorists concluded that magnetic fields are important even before molecular clouds were discovered because simple estimates showed that interstellar gas is far more highly magnetized than stars are (e.g., [68]), and they therefore turned their attention to how this excess flux could be lost. The ongoing efforts to observe magnetic fields in molecular clouds will be discussed in §2.4.5 below.

2.4.1. *Magnetic Critical Mass*

The simplest case of a magnetically supported cloud is one in which the field is purely poloidal and the kinetic energy vanishes ($\mathcal{T} = 0$); since the gas is cold and there are no bulk motions, it settles into a thin disk. We assume the field is fully connected with that in the ambient medium. (The effects of closed field lines are discussed in reference [65].) For a thin disk, the volume of the cloud vanishes, and P_G goes to infinity; the virial theorem (24) yields

$$\mathcal{M} = |\mathcal{W}|. \tag{35}$$

Let the magnetic flux threading the cloud be

$$\Phi = \int 2\pi r B dr \equiv \pi R^2 \bar{B}. \tag{36}$$

Now consider a sequence of equilibria in which the flux is constant and the functional form of the mass–to–flux ratio is constant, but the value of the mass–to–flux ratio increases. At some point the mass–to–flux ratio will become large enough that gravity will overwhelm the magnetic stresses, and the cloud will collapse. The point at which the mass–to–flux ratio reaches the maximum is the critical point. We write the net magnetic energy there as

$$\mathcal{M}_{\rm cr} = \frac{1}{8\pi} \int (B^2 - B_0^2) dV \Big|_{\rm cr} \equiv \left(\frac{b}{3}\right) \bar{B}^2 R^3 = \left(\frac{b}{3\pi^2}\right) \frac{\Phi^2}{R}, \tag{37}$$

where b is a numerical factor of order unity. We can evaluate the *magnetic critical mass* M_Φ by inserting this relation into equation (35) and using equation (21):

$$M_\Phi = \left(\frac{5b}{9\pi^2 a}\right)^{1/2} \frac{\Phi}{G^{1/2}} \equiv c_\Phi \left(\frac{\Phi}{G^{1/2}}\right). \tag{38}$$

So long as the magnetic flux is frozen to the matter, M_Φ is a constant. For $M < M_\Phi$, the cloud is said to be *magnetically subcritical*: the mass–to–flux ratio is small enough that magnetic stresses always exceed gravity, so such a cloud can never undergo gravitational collapse. Conversely, if $M > M_\Phi$, the cloud is *magnetically supercritical*, and magnetic fields cannot prevent gravitational collapse.

If the cloud has a constant mass–to–flux ratio (which Shu and Li [96] term "isopedic"), then the numerical factor $c_\Phi = 1/2\pi$ [81]. Isopedic disks are highly idealized: they can be finite in extent only if the field vanishes outside the disk, and they can be in equilibrium only if they are critical ($M = M_\Phi$). Precisely because they are so idealized, it is possible to obtain useful results even if they are non-axisymmetric and time-dependent [96]. The ratio M_Φ/M is constant in such a disk. The distinction between magnetically subcritical and supercritical disks is particularly clear in this case since the ratio of the magnetic force to the gravitational force on a mass M in the disk is (in our notation) $(M_\Phi/M)^2$.

The field strength in a cloud can be expressed in terms of its column density by noting that $M/M_\Phi \propto \Sigma/\bar{B} \propto \bar{N}_{\rm H}/\bar{B}$, so that

$$\bar{B} = 50.5 \left(\frac{\bar{N}_{\rm H22}}{M/M_\Phi}\right) \ \mu{\rm G} = 10.1 \left(\frac{\bar{A}_V}{M/M_\Phi}\right) \ \mu{\rm G}. \tag{39}$$

It is sometimes convenient to have an alternative expression for the magnetic critical mass that is independent of the mass of the cloud. Mouschovias

& Spitzer [71] showed that such an alternative critical mass, denoted M_B, is related to M_Φ by

$$\frac{M_B}{M} \equiv \left(\frac{M_\Phi}{M}\right)^3 . \tag{40}$$

For an ellipsoidal cloud of size $2Z$ along the axis of symmetry and radius R normal to the axis, we have [5]

$$M_B = 512 \left(\frac{R}{Z}\right)^2 \frac{\bar{B}_{1.5}^3}{\bar{n}_{\rm H3}^2} \quad M_\odot, \tag{41}$$

where $\bar{B}_{1.5} \equiv \bar{B}/(10^{1.5}\ \mu{\rm G})$. This form for the magnetic critical mass is similar in form to the Bonnor–Ebert mass, since

$$M_B \propto \frac{\bar{B}^3}{\bar{n}^2} \propto \frac{1}{\bar{B}}\left(\frac{\bar{B}^4}{\bar{\rho}^2}\right) \propto \frac{v_A^4}{P_B^{1/2}}, \tag{42}$$

where $v_A \equiv \bar{B}/(4\pi\bar{\rho})^{1/2}$ is the Alfvén velocity; here v_A plays the role of the isothermal sound speed and the magnetic pressure $P_B \equiv \bar{B}^2/8\pi$ that of the external pressure.

Equation (41) for the magnetic critical mass naturally suggests three very different mass scales for molecular clouds [5]: (1) On the largest scales, clouds are formed from compression of the diffuse ISM [26] [67], which has a mean density $\bar{n}_{\rm H} \sim 1$ cm^{-3} and a field $\bar{B} \sim 3\ \mu$G; this gives $M_B \sim 5\times10^5$ $M_\odot$, a typical mass for a GMC (cf. eq. 10). Since M_B is constant so long as the mass is constant and flux freezing holds, this value will be preserved as the gas is compressed and the GMC forms. (2) On intermediate scales, the clumps within GMCs might well have originated as the diffuse clouds in the gas that formed the GMC [26]. In this case, the density is about 30–100 times greater than the average interstellar density, but the field is about the same, since diffuse clouds are not gravitationally bound. As a result, we expect $M_B \sim 50 - 500\ M_\odot$ in clumps, which is $\lesssim$ the mass of star–forming clumps [M(SFC)] in nearby GMCs [5]. On the other hand, M_B is substantially greater than the mass of a typical star: Stellar mass clumps at the density of star–forming clumps are magnetically subcritical. (3) Finally, in regions in which the density of the gas can grow so that the thermal pressure is comparable to the magnetic pressure, either by ambipolar diffusion or by flow along field lines, then $M_B \sim M_{\rm BE} \sim 1 M_\odot$ from equation (34).

2.4.2. *Toroidal Fields*

Toroidal fields can provide a confining force, thereby reducing the magnetic critical mass [36] [107]. To see how this occurs, consider a current $I(r)$ in the z direction, which generates a toroidal field $\mathbf{B}_\phi = 2I/cr$. The resulting force per unit volume is $\mathbf{J}\times\mathbf{B}/c = -(J_z B_\phi/c)\hat{\mathbf{r}}$, which indeed provides a pinching force. To determine the contribution to the magnetic term in the virial theorem, it is best to recall that the virial theorem can be derived by taking the dot product of the equation of motion with $\mathbf{r}$ and integrating over the volume; the contribution of the toroidal field to the magnetic term is then

$$\mathcal{M}_{\rm toroidal} = \frac{1}{c}\int (\mathbf{J}_z\times\mathbf{B}_\phi)\cdot\mathbf{r}dV \simeq -\frac{LI^2}{c}, \tag{43}$$

where L is the length of the cylinder in which $\mathcal{M}$ has been evaluated. Thus, whereas poloidal fields give a positive definite contribution to $\mathcal{M}$ (provided the field decreases outward), toroidal fields give a negative definite contribution.

Several caveats about toroidal fields should be kept in mind: First, the ratio of the toroidal field to the poloidal field cannot become too large without engendering instabilities (e.g., [44]). Second, the current that generates the toroidal field must return to where it started, since currents do not have sources or sinks in MHD [44]. Once the current density reverses direction, so does the force, and the toroidal field ceases to be confining. If the virial theorem is applied to a volume large enough to encompass the entire return current, then $I = 0$ and the toroidal field has no net effect. In astrophysical MHD, it is convenient to think of the current as being generated by the field rather than vice versa; it follows that if the toroidal field is restricted to a finite volume, then it has no net effect on the virial theorem applied to any larger volume. Finally, a purely toroidal field is subject to dissipation by magnetic reconnection, which can occur with reasonable efficiency according to Lazarian & Vishniac [49]. However, such reconnection can be avoided if there is a sufficiently strong poloidal field along the axis, as in the protostellar wind model of Shu et al [98].

2.4.3. *Clouds Supported by both Magnetic and Gas Pressure*

Let us return to the case of a poloidal field, and ask, what is the critical mass for a cloud supported by gas pressure as well as magnetic stresses? We can use the same approach that we adopted to estimate the Bonnor–Ebert mass [71]. If we retain the magnetic term in the virial theorem (24), the same steps that led to equation (31) give

$$P_s = \frac{3M\sigma^2}{4\pi R^3} - \frac{3aGM^2}{20\pi R^4}\left(1 - \frac{\mathcal{M}}{|\mathcal{W}|}\right). \tag{44}$$

For a cloud at the critical point, $\mathcal{M} = \frac{3}{5}aGM_\Phi^2/R$ and $|\mathcal{W}| = \frac{3}{5}aGM^2/R$, so that $\mathcal{M}/|\mathcal{W}| = (M_\Phi/M_{\rm cr})^2$ there. As discussed in reference [65], this remains a good approximation even for subcritical clouds. We then have

$$1-\frac{\mathcal{M}}{|\mathcal{W}|}\simeq 1-\left(\frac{M_\Phi}{M_{\rm cr}}\right)^2\equiv k_m. \qquad (45)$$

Generalizing the argument that led to the Bonnor–Ebert mass, we consider a sequence of equilibria in which the ambient pressure P_s is increased while the mass and flux remain constant. The maximum value of P_s is reached when the two terms become comparable, which occurs at a radius such that $\sigma^2 \sim GMk_m/R$. The maximum pressure is then of order

$$P_{\rm cr}\propto\frac{M\sigma^2}{(GMk_m/\sigma^2)^3}\propto\frac{\sigma^8}{G^3k_m^3M^2}. \qquad (46)$$

For a given ambient pressure, this gives a cubic equation for the critical mass. If the gas is isothermal, one obtains [71]

$$M_{\rm cr}=c_1M_{\rm BE}\left[1-\left(\frac{M_\Phi}{M_{\rm cr}}\right)^2\right]^{-3/2}, \qquad (47)$$

where c_1 is a numerical constant and where we have substituted back for k_m from its definition in equation (45). Tomisaka et al [106] find that their numerical results are best fit by $c_1 = 1.18$, quite close to the value estimated by Mouschovias & Spitzer [71] many years earlier.

An approximate solution to equation (47) for $M_{\rm cr}$ is $M_{\rm cr} \simeq M_{\rm BE} + M_\Phi$, which is accurate to within about 5% for $M \lesssim 8M_\Phi$ [60]; for weaker fields, this approximation predicts that $M_{\rm cr} \to M_{\rm BE}$, which is presumably more accurate than the solution of equation (47). We anticipate that this result will remain approximately valid in the more general case in which the gas is not isothermal. Defining the Jeans mass $M_{\rm J}$ as the critical mass associated with thermal and nonthermal motions of the gas, we then have

$$M_{\rm cr}\simeq M_{\rm J}+M_\Phi. \qquad (48)$$

Axisymmetric numerical models for magnetized clouds were first constructed by Mouschovias [69] [70]. He focused on the case of a mass–to–flux distribution corresponding to a uniform field threading a uniform, spherical cloud. For this case, Mouschovias & Spitzer [71] found $c_\Phi \simeq 0.126$; subsequent calculations by Tomisaka et al. [106] found $c_\Phi \simeq 0.12$, which we adopt for numerical estimates. Tomisaka et al. showed that for a range of mass–to–flux distributions it is the mass–to–flux ratio on the central flux tube that controls the stability, with the critical value being $(dM_\Phi/d\Phi)_c \simeq 0.17/G^{1/2}$. (For Mouschovias' case, they found a coefficient $0.18 = 1.5 \times 0.12$; the factor 1.5 is just the ratio of the central mass–to–flux ratio to the mean.)

Tomisaka et al's result is quite close to the value for an isopedic disk, $1/(2\pi G^{1/2}) \simeq 0.16/G^{1/2}$.

2.4.4. *Are Clouds Magnetically Supercritical? Theory*

With the framework we have developed, we can ask the question, are GMCs magnetically supercritical or subcritical? McKee [60] argued that GMCs are magnetically supercritical based on the following line of argument: GMCs must be approximately critical ($M \simeq M_{\rm cr}$) since they are highly pressured relative to their environment, and calculations show that this is possible only for nearly critical clouds [70] [106]. The large nonthermal motions observed in GMCs and the fact that the clouds do not appear to be highly flattened imply that $M_{\rm J}$ is not small compared to M_Φ. Hence, from equation (48), we have $M \simeq M_{\rm cr} \simeq M_{\rm J} + M_\Phi > M_\Phi$. This argument does not set a lower bound on M_Φ, but if initially a cloud had $M_{\rm J} \gg M_\Phi$ and the gas pressure was dominated by nonthermal motions, then the field would be amplified into approximate equipartition, giving $M_\Phi \sim M_{\rm J}$. Altogether, then, one expects GMCs to have $M \sim 2M_\Phi$ on theoretical grounds. This argument was extended to the gravitationally bound clumps within GMCs by Bertoldi & McKee [5]. These conclusions are not universally accepted, however [73].

Nakano [80] has used similar reasoning to conclude that the observed cores in molecular clouds should also be magnetically supercritical. The general argument that applies to all three cases—GMCs, star–forming clumps, and cores—is that if a cloud is clearly gravitationally bound, then its mass must be nearly equal to the critical mass; if furthermore, the cloud has more than one significant source of pressure support, then it is supercritical with respect to each of the sources of support individually. Nakano then used this argument to question the standard paradigm of low–mass star formation, which is based on ambipolar diffusion in magnetically subcritical clumps [72] [95]. However, this criticism may be unwarranted: about half the observed cores already contain embedded stars [4], so in the conventional interpretation such cores would have already experienced substantial ambipolar diffusion. The issue that remains to be resolved by observation is whether the *protocores*—i.e., the precursors of the observed cores—are magnetically subcritical.

2.4.5. *Observation of Magnetically Supercritical Clouds*

What do observations say about the strength of magnetic fields in molecular clouds? The data on magnetic field strengths come from observations of Zeeman splitting of molecular lines, which determine $B_\parallel$, the component of the magnetic field along the line of sight (see Heiles et al. [41] for a discussion of techniques for measuring magnetic fields). Myers & Goodman

[76] summarized the data available on magnetic field strengths in 1988, and concluded that the data were consistent with approximate equipartition among magnetic, kinetic, and gravitational energies. Their results are consistent with $M \simeq 2M_\Phi$ [60]. Since in their sample the line widths vary substantially less than the density in the clouds, the approximate equality between kinetic and magnetic energies ($\rho {v_{\rm rms}}^2/2 \sim B^2/8\pi$) implies the approximate relation $B \propto \rho^{1/2}$, as advocated by Mouschovias [73]. This in turn means that the Alfvén velocity is independent of the density of the cloud; Heiles et al. [41] found that $\langle v_A\rangle \sim 2$ km s^{-1}. Subsequently, Crutcher et al. [21] studied the cloud B1 in some detail. They found that the inner envelope was marginally magnetically subcritical, whereas the densest region was somewhat supercritical. The observational results were shown to be in good agreement with a numerical model that, however, did not include the observed nonthermal motions.

The most comprehensive study of magnetic fields in molecular clouds available to date is that by Crutcher [20]. He concludes that molecular clouds are generally magnetically supercritical: His sample, which tends to focus on the central regions of clouds, has no clear case in which a cloud is magnetically subcritical. In order to reach this conclusion, he had to allow for projection effects: If the magnetic field makes an angle θ with respect to the line of sight, then the observed field $B_\parallel$ is related to the true field B by $B_\parallel = B\cos\theta$, so that on average $\langle B_\parallel\rangle = B/2$. After allowing for this, he finds that $\langle M/M_\Phi\rangle \simeq 2.4$ for clouds with measured fields. Twelve of the 27 clouds in his sample have only upper limits on the field strength; these clouds, which typically have both lower densities and lower column densities than the clouds with measured field strengths, are also magnetically supercritical. If the clouds are flattened along the field lines, then the observed area is smaller than the true area by factor $\cos\theta$ as well, so that $M/M_\Phi \propto \cos^2\theta$; on average, this is a factor 1/3. However, since clouds are observed to have substantial motions, they are unlikely to be highly flattened along field lines, so Crutcher concludes that $\langle M/M_\Phi\rangle \simeq 2$, consistent with the theoretical arguments advanced above. Many of the upper limits come from a study of the Zeeman effect in OH; if these data apply to the "protocores" discussed at the end of §2.4.4, then they indicate that the fields there are supercritical as well, contrary to the assumption underlying many theories of low mass star formation. The clouds with detected fields are gravitationally bound, with a mean value for the virial parameter $\alpha \simeq 1.4$. He also finds that the Alfvén Mach number of the turbulent motions, $m_A = \sigma_{\rm nt}\sqrt{3}/v_A$, is about unity in the clouds with measured fields, as inferred previously by Myers & Goodman [76] on the basis of less complete data. Two points should be kept in mind, however: First, these data do not address the issue of the strength of the field on large scales (i.e., the field threading an entire GMC). Second, since the data for the detected regions deal with dense regions in molecular clouds, it is possible that the observed mass–to–flux ratio has been altered by ambipolar diffusion.

2.5. MHD WAVES IN MOLECULAR CLOUDS

The velocity dispersions in molecular clouds are typically $\sigma \sim 1-2$ km s^{-1}, whereas the thermal velocity dispersion is only about 0.2 km s^{-1} at a temperature of 10 K. The fact that the motions in molecular clouds are highly supersonic led Arons & Max [1] to suggest that the motions in molecular clouds are MHD waves. This was a prescient suggestion, made before the magnetic field measurements discussed above. Subsequent discussions of MHD waves in molecular clouds have been given by Zweibel & Josefatsson [117], Falgarone & Puget [33], Pudritz [88], and McKee & Zweibel [64].

2.5.1. *Wave Pressure*

There are three types of MHD waves: fast, slow, and Alfvén. Alfvén waves are particularly simple because they have an *isotropic* pressure [24]. At first sight, this result is surprising, since the stress exerted by a static magnetic field is anisotropic. The isotropy of the pressure due to the Alfvén waves can be understood as follows [64]: In an Alfvén wave the perturbation in the field $\delta\mathbf{B}$ is orthogonal to the background field; the stress associated with δB gives a pressure $\delta B^2/8\pi$ in the direction of the background field $\mathbf{B}_0$ and in the direction orthogonal to both $\mathbf{B}_0$ and $\delta\mathbf{B}$. In the direction of $\delta\mathbf{B}$, the net stress is $-\delta B^2/8\pi$ due to the tension in the field. However, it is just in this direction that the motion of the gas contributes a dynamic pressure $\rho\delta v^2$. Since the kinetic and magnetic energies are in equipartition for MHD waves [118], we have $\rho\delta v^2/2 = \delta B^2/8\pi$; the net stress along the direction of the perturbed field is then $-\delta B^2/8\pi + \rho\delta v^2 = \delta B^2/8\pi$, which is identical to that in the other two directions. The wave pressure is then

$$P_{\rm w} = \frac{\delta B^2}{8\pi} = \frac{3}{2}\rho\sigma_{\rm nt}^2, \tag{49}$$

where in the last step we have assumed that δB has a random orientation with respect to the observer so that $\delta v^2 = 3\sigma_{\rm nt}^2$.

Understanding the dynamics of Alfvén waves in molecular clouds is a complex problem in radiation magnetohydrodynamics. However, we can gain some insight into the general problem by considering simple limiting cases. First, consider the question of how the wave pressure varies during an adiabatic compression [64]. For an adiabatic process, we have $P_{\rm w} \propto \rho^{\gamma_{\rm w}}$ for some $\gamma_{\rm w}$. The pressure is related to the energy density $u_{\rm w}$ by $P_{\rm w} =$

$(\gamma_{\rm w}-1)u_{\rm w}$. Now, equipartition implies

$$u_{\rm w}=\frac{1}{2}\rho\delta v^2+\frac{\delta B^2}{8\pi}=\frac{\delta B^2}{4\pi}=2P_{\rm w}, \tag{50}$$

which in turn implies

$$\gamma_{\rm w}=\frac{3}{2}. \tag{51}$$

Thus, in a medium of uniform density $\rho(t)$, the wave pressure varies as $P_{\rm w}(t)\propto\rho(t)^{3/2}$. Since $\gamma_{\rm w}$ is greater than unity, the waves heat up during a compression (i.e., the wave frequency increases).

Next consider how the Alfvén wave pressure varies with position in a static cloud with a density $\rho(\mathbf{r})$. We assume a steady state, with no sources, sinks, or losses by transmission through the surface of the cloud. (The latter approximation is reasonably good if there is a large density drop at the cloud surface, since then the transmission coefficient is small.) We anticipate that $P_{\rm w}\propto\rho^{\gamma_{\rm pw}}$, where the polytropic index $\gamma_{\rm pw}$ may differ from the adiabatic index $\gamma_{\rm w}$. The answer to this problem for electromagnetic radiation is simple: the radiation pressure would be constant ($\gamma_{\rm pw}=0$). To determine the answer for Alfvén waves, we use the energy equation for Alfvén waves [24] [64] (the $\pm$ determines the direction of propagation),

$$\frac{\partial u_w}{\partial t}+\nabla\cdot u_{\rm w}(\mathbf{v}\pm\mathbf{v}_A)+\frac{1}{2}u_{\rm w}\nabla\cdot\mathbf{v}=0. \tag{52}$$

In a steady state in a static cloud, this reduces to

$$\nabla\cdot u_{\rm w}\mathbf{v}_A=\nabla\cdot\frac{u_{\rm w}\mathbf{B}}{(4\pi\rho)^{1/2}}=0, \tag{53}$$

which implies

$$\mathbf{B}\cdot\nabla\frac{P_{\rm w}}{\rho^{1/2}}=0 \tag{54}$$

since $\nabla\cdot\mathbf{B}=0$ and $P_{\rm w}\propto u_{\rm w}$. Based on this argument, McKee & Zweibel [64] concluded that $P_{\rm w}(\mathbf{r})\propto\rho(\mathbf{r})^{1/2}$ along any field line. If the constant of proportionality is the same for all the field lines, then the Alfvén wave pressure satisfies a polytropic relation $P_{\rm w}(\mathbf{r})\propto\rho(\mathbf{r})^{\gamma_{\rm pw}}$ with

$$\gamma_{\rm pw}=1/2. \tag{55}$$

This result is consistent with that of Fatuzzo & Adams [34], who studied the particular case of Alfvén waves in a self-gravitating slab threaded by a uniform vertical magnetic field. The fact that $\gamma_{\rm pw}$ is less than unity means that the velocity dispersion increases as the density decreases: the surface of the cloud is "hotter" than the center, which is consistent with the observed line width–size relation. The fact that the polytropic index $\gamma_{\rm p}$ differs from the adiabatic index γ introduces a complication into modeling molecular clouds, as we shall see below.

2.5.2. *Wave Damping*

MHD waves are subject to both linear and nonlinear damping (see §2.7 for the latter). The linear damping is due to ion–neutral friction, the same process that governs ambipolar diffusion. At sufficiently low frequencies, the ions and neutrals are well coupled so that the Alfvén velocity is determined by the density of the entire medium, $\rho = \rho_n + \rho_i$. For transverse waves the equation of motion for the neutrals is

$$\rho_n \frac{d\mathbf{v}_n}{dt} = \rho_n \nu_{ni} \mathbf{v}_D, \tag{56}$$

where ν_{ni} is the neutral–ion collision frequency and $\mathbf{v}_D$ is the velocity of the ions with respect to the neutrals. For a linear wave of frequency ω and velocity amplitude δv, this yields $v_D = \omega \delta v / \nu_{ni}$. The specific energy of the waves (including both kinetic and magnetic energy) is $\epsilon = \delta v^2$; the rate at which this energy is damped out is $\dot{\epsilon} = \nu_{ni} v_D^2$. In terms of the damping rate for the wave amplitude Γ, we have $\dot{\epsilon} = -2\Gamma\epsilon$, so that

$$\Gamma = \frac{\omega^2}{2\nu_{ni}}. \tag{57}$$

This heuristic argument, which implicitly assumes $\Gamma \ll \omega$, suggests that the waves are critically damped (i.e., $\Gamma = \omega$) at a frequency $\omega_{\rm cut} = 2\nu_{ni}$, corresponding to a wavenumber $k_{\rm cut} = \omega_{\rm cut}/v_A = 2\nu_{ni}/v_A$. A more precise calculation [54] shows that the real part of the frequency vanishes at $k = k_{\rm cut}$. MHD waves in which the motion of the ions and neutrals is coupled cannot propagate if $k > k_{\rm cut}$, and they therefore cannot provide pressure support on scales smaller than about $R_{\rm cut} \equiv \pi / k_{\rm cut}$.

The numerical value of $R_{\rm cut}$ depends on the neutral–ion collision frequency, $\nu_{ni} = n_i \langle \sigma v \rangle = 1.5 \times 10^{-9} n_i$ s^{-1} [79]. Using equation (2) for the ionization, we find that for a cloud of radius R and mass–to–flux ratio governed by M/M_Φ,

$$\frac{R}{R_{\rm cut}} = 7.7 \left(\frac{C_i}{10^{-5}\ {\rm cm}^{-3/2}} \right) \frac{M}{M_\Phi}. \tag{58}$$

Thus, for typical levels of ionization produced by cosmic rays (§1.1), clouds large enough that they cannot be supported by static magnetic fields alone ($M > M_\Phi$) are large enough to support a modest spectrum of MHD waves ($R \gg R_{\rm cut}$) [65] [112].

2.6. POLYTROPIC MODELS FOR MOLECULAR CLOUDS

Polytropic models, in which the pressure varies as a power of the density,

$$P = K_{\rm p}\rho^{\gamma_{\rm p}}, \tag{59}$$

have long been used to model stars. The power $\gamma_{\rm p}$ is often expressed in terms of an index n,

$$\gamma_{\rm p} \equiv 1 + \frac{1}{n}. \tag{60}$$

Prior to the advent of computers, polytropic models were the best available for studying stellar structure, and even now they are useful for gaining insight. Models of molecular clouds are decades behind those for stars, with computational models only now beginning to be developed (§2.7). Thus, polytropic models can be expected to be of use here too, in order to shed light on the density structure of molecular clouds, the line width—size relation, the precollapse conditions for star formation, and the relation between the properties of GMCs and the medium in which they are embedded.

The isothermal Bonnor–Ebert models discussed above are the simplest examples of polytropes. Non-isothermal polytropes ($\gamma_{\rm p} \neq 1$) have been discussed by Shu et al. [97], Viala & Horedt [111], and Chiéze [17]. Maloney [57] pointed out that the line width—size relation demanded a "negative–index" polytrope, in which $\gamma_{\rm p} < 1$ so that n is negative. In order to treat the nonthermal motions in molecular clouds, Lizano & Shu [52] assumed that the pressure associated with these motions is proportional to the logarithm of the density—the limiting case of a negative index polytrope in which $\gamma_{\rm p} \rightarrow 0$. This "logatropic" form for the turbulent pressure gives a sound speed $(dP/d\rho)^{1/2} \propto 1/\rho^{1/2}$, which is consistent with Larson's laws (§1.2); on the other hand, it as yet has no physical basis. The logatropic equation of state has been studied further by Gehman et al. [38] and, in a different form, by McLaughlin & Pudritz [66].

2.6.1. *Structure of Polytropes*

The structure of a polytrope is controlled by the value of $\gamma_{\rm p}$. Some insight into the behavior of spherical polytropes can be gained by considering the limiting case of singular polytropic spheres, which have power law solutions. From the equation of hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}, \tag{61}$$

one readily finds [16]

$$\rho \propto r^{-2/(2-\gamma_{\rm p})}, \quad P \propto r^{-2\gamma_{\rm p}/(2-\gamma_{\rm p})}, \quad c \propto r^{(1-\gamma_{\rm p})/(2-\gamma_{\rm p})}, \tag{62}$$

where $c^2 = P/\rho$ is the generalized isothermal sound speed. A singular isothermal sphere [94], for example, has $\gamma_{\rm p} = 1$ so that $\rho \propto P \propto 1/r^2$ and c =const. For a cloud supported by Alfvén waves, we have $\gamma_{\rm p} = 1/2$ so that $\rho \propto r^{-4/3}$ and $c \propto r^{1/3}$. In the limit as $\gamma_{\rm p} \to 0$, which is an approximation for a logatrope, we have $\rho \propto 1/r$ and $c \propto r^{1/2}$. Note that in the last two cases the velocity dispersion increases outward, as observed. However, these simple power law models cannot be used to determine the nature of the pressure support in molecular clouds, both because actual polytropes are not power laws and because actual clouds have more than one source of support.

Since the Bonnor–Ebert mass scales as $c^3/(G^3\rho^{1/2})$, it is convenient to introduce a dimensionless mass [61] [103]

$$\mu \equiv \frac{M}{c^3/(G^3\rho)^{1/2}} = \frac{M}{c^4/(G^3P)^{1/2}}. \tag{63}$$

For stars, which have $\gamma_{\rm p} > 4/3$, this quantity can go to infinity at the surface: stars are supported by the hot gas in their interiors. However, the sources of support for molecular clouds have $\gamma_{\rm p} \leq 4/3$, and for such clouds there is an upper limit on μ of 4.555, so that stable clouds satisfy [61]

$$M < 4.555 \left(\frac{c_s^4}{G^{3/2}P_s^{1/2}}\right), \tag{64}$$

where the subscript "s" emphasizes that the sound speed and pressure are evaluated at the surface of the cloud. Thus, the mass of a molecular cloud is limited by conditions at its *surface*. By contrast, the maximum mass of a star is set by conditions at or near its center. This upper limit on μ is a monotonically increasing function of $\gamma_{\rm p}$; for negative index polytropes ($\gamma_{\rm p} < 1$), it must be less than the Bonnor–Ebert value, $\mu = 1.18$.

2.6.2. *Stability of Polytropes: Locally Adiabatic Components*

In order to determine the stability of a cloud, we need to know how it will respond to a perturbation. We shall assume that the perturbation can be modeled as being adiabatic, in the sense that there is no heat exchange between pressure components (e.g., we ignore wave damping) and there is no heat exchange with the environment (e.g., we ignore the loss of wave energy by transmission into the ambient medium). McKee & Holliman [61] distinguish two cases: if the heat flow associated with the given pressure component is very inefficient, then the component is *locally adiabatic*. In the opposite limit of very efficient heat flow, the component is *globally adiabatic*. An adiabatic gas in conventional parlance is locally adiabatic. For a locally adiabatic component, the entropy parameter

$$K_i \equiv \frac{P_i}{\rho^{\gamma_i}} \tag{65}$$

remains constant during the perturbation, where the adiabatic index γ_i is distinct from the polytropic index $\gamma_{\mathrm{p}i}$. (The actual entropy is proportional to the logarithm of the entropy parameter.) The magnetic field can be modeled approximately as a locally adiabatic component with $\gamma_B = 4/3$; the corresponding entropy parameter is $K_B \propto B^2/\rho^{4/3} \propto M_B^{2/3}$, which indeed is constant so long as flux freezing holds.

If the adiabatic and polytropic indexes are the same ($\gamma_i = \gamma_{\mathrm{p}i}$), then the gas is *isentropic* since the entropy parameter $K_i \propto \rho^{\gamma_{\mathrm{p}i}-\gamma_i}$ is spatially and temporally constant. Even if the gas is subject to heating and cooling, it may be possible to model it as an isentropic gas: If the heating rate scales as nT^a and the cooling rate as n^2T^b, where a and b are constant, then the gas can be modeled as isentropic with $\gamma_i = \gamma_{\mathrm{p}i} = 1 + 1/(a-b)$. A discussion of the value of γ_i based on molecular cooling curves and allowing for variable b is given by Scalo et al. [92]. Note that if the gas is not isentropic, then perturbation of a polytrope leads to a configuration that is not polytropic.

A polytrope supported by a locally adiabatic pressure component is stable for $\gamma_i > 4/3$. For $\gamma_i < \gamma_{\mathrm{p}i}$, the polytrope is convectively unstable. Along the line $\gamma_i = \gamma_{\mathrm{p}i}$ (isentropic polytropes), the critical point that divides unstable clouds from stable ones lies at the maximum value of μ; equation (64) thus gives an upper limit on M_{cr}. As γ_i increases above $\gamma_{\mathrm{p}i}$, the value of μ at the critical point (μ_{cr}) changes, but it remains close to, and somewhat less than, the maximum value of μ. The magnitude of the density contrast between the center of the cloud and the surface, ρ_c/ρ_s, increases dramatically as γ_i increases, and can become infinite even if γ_i is less than 4/3 [61].

For isothermal polytropes, M_{cr} is reduced somewhat by an increase in the external pressure, $M_{\mathrm{cr}} \propto P_s^{-1/2}$ (eq. 64). For negative–index polytropes, however, M_{cr} can decrease much more sharply with P_s due to the decrease in c_s [97]. Since the decrease in the temperature is bounded (it is difficult to cool below 10 K in a typical molecular cloud, for example), it is more convenient to express the critical mass in terms of quantities after the compression,

$$M_{\mathrm{cr}} = \mu_{\mathrm{cr}} \left(\frac{c_{s,f}^4}{G^{3/2} P_{s,f}^{1/2}} \right), \tag{66}$$

where $c_{s,f}$ is the final value of c_s, etc. This result shows that the reduction in the critical mass due to cooling, which reduces c_s, can be large, but the reduction due to the compression is limited by the weak $P_{s,f}^{-1/2}$ dependence. For example, in a radiative shock the final pressure is related to the initial value $P_{s,i}$ by $P_{s,f} = (v_{\rm shock}/c_{s,i})^2 P_{s,i}$. In spherical implosions even higher compressions, and correspondingly greater reductions in $M_{\rm cr}$, are possible [105], although in practice it may be difficult to maintain the high degree of spherical symmetry required to achieve very large compressions.

2.6.3. *Stability of Polytropes: Globally Adiabatic Components*
MHD waves are not locally adiabatic since they can move in response to a perturbation. In the absence of damping and losses, or in the case in which sources balance damping and losses, it is the wave action integrated over the cloud that is conserved [24], and the waves can be said to be "globally adiabatic." The wave action is related to the entropy parameter for the waves [64], and McKee & Holliman [61] have determined how to treat the stability of a globally adiabatic pressure component. This is a generalization of the problem of the stabiity of globular clusters considered by Lynden–Bell & Wood [53], with the complication that the gas is not isothermal. Lynden–Bell & Wood modeled globular clusters as polytropes with $\gamma_{\rm p} = 1$ and $\gamma = 5/3$. Whereas locally adiabatic polytropes are stable for $\gamma > 4/3$, this is not the case for globally adiabatic polytropes. If the density contrast between the center and edge becomes too great, the cluster is subject to core collapse, in which the stars carry heat from the core to the envelope and allow the core to collapse while the envelope expands. This is a generic property of globally adiabatic polytropes: for $\gamma_{\rm p} < 6/5$, such polytropes are unstable for arbitrary values of γ, with global collapse occurring for $\gamma < 4/3$ and core collapse for $\gamma > 4/3$. Since $\gamma_{\rm w} = 3/2$, a polytrope supported by Alfvén waves is subject to core collapse. Note that for $\gamma > 4/3$, the cloud heats up and becomes more stable if it is compressed; it is *decompression* that leads to instability.

Because $\gamma_{\rm pw}$ is only 0.5, the critical mass for a cloud supported by Alfvén waves is smaller than that for an isothermal cloud. Calculations show that the critical mass for such a cloud is [61]

$$M_{\rm w} = 0.39 \left(\frac{\sigma_{{\rm nt},s}^3}{G^{3/2}\rho_s^{1/2}} \right) = 0.65 \left(\frac{\langle \sigma_{\rm nt}^2 \rangle^{3/2}}{G^{3/2}\rho_s^{1/2}} \right) . \qquad (67)$$

A cloud supported by the pressure of both an isothermal gas and Alfvén waves has a critical mass [61]

$$M_{\rm J} = 1.18 \left(\frac{\sigma_{\rm eff}^3}{G^{3/2}\rho_s^{1/2}} \right) , \qquad (68)$$

where the effective velocity dispersion is

$$\sigma_{\rm eff}^2 \equiv c_{\rm th}^2 + 0.67 \langle \sigma_{\rm nt}^2 \rangle . \qquad (69)$$

Insofar as it is accurate to represent the nonthermal motions in clouds as Alfvén waves, they are less effective at supporting clouds than thermal motions because they tend to concentrate in the low density envelope, as indicated by their polytropic index $\gamma_{\rm pw} = 1/2$.

2.6.4. *Multi–Pressure and Composite Polytropes*

Real clouds are supported by thermal pressure, magnetic stresses, and wave pressure. Polytropic models with multiple components can be classified into three types [61]: *Composite polytropes*, in which the pressure components are spatially separated (an example is the core–envelope model for red giant stars of Schönberg & Chandrasekhar [93]); *multi–fluid polytropes*, in which the different components interact only gravitationally, as in the case of a molecular cloud with an embedded star cluster; and *multi–pressure polytropes*, in which there is a single self–gravitating fluid with several pressure components,

$$P(r) = \Sigma P_i(r) = \Sigma K_{{\rm p}i} \rho^{\gamma_{{\rm p}i}}. \tag{70}$$

Lizano & Shu [52] developed the first multi–pressure polytropic model for molecular clouds. They treated the axisymmetric magnetic field exactly, and modeled the gas pressure as consisting of an isothermal component for the thermal pressure and a logatropic component for the turbulent pressure. Gehman et al. [38] studied the stability of logatropes in both planar and cylindrical geometries; they effectively assumed an isentropic equation of state, which does not allow for either the stiffness or the mobility of the Alfvén waves. McLaughlin & Pudritz [66] studied a variant of the logatrope in spherical geometry. To assess the stability of the clouds, they adopted the boundary condition proposed by Maloney [57] in which the central temperature is held constant. While this is plausible for the thermal pressure, no justification has been advanced for using it for the wave pressure.

An approximate alternative to the multi–pressure polytrope has been developed by Myers & Fuller [75] and Caselli & Myers [15]. In the "TNT" model, the density is assumed to obey

$$n \propto \left(\frac{r_0}{r}\right)^2 + \left(\frac{r_0}{r}\right)^p, \tag{71}$$

where the two terms represent the effect of thermal and nonthermal motions, respectively. This form for the density is inserted into the equation of hydrostatic equilibrium to determine the density at r_0 and the velocity dispersion as a function of r. In most cases it is possible to obtain a good fit to data on the line width as a function of r by fitting the characteristic length scale r_0 and the exponent p. Caselli & Myers find that "massive cores" have significantly smaller values of r_0 (0.01 pc vs. 0.3 pc) and larger mean extinctions ($A_V = 15$ mag vs. 3.6 mag) than “low mass cores” (although the values of the masses are not discussed).

A preliminary study of multi–pressure polytropes including thermal pressure, magnetic pressure, and Alfvén waves has been given in John Holliman's thesis [43]. An important result from this work is that when allowance is made for the layer of gas in which the C is atomic, which has a thickness of 0.7 mag [109], stable clouds can have a mean gas pressure in the CO up to about 8 times the gas pressure acting on the surface of the cloud. (Elmegreen [27] had previously considered the pressure due to the smaller layer of HI, and did not address the stability of his model.) Applying the virial theorem (eq. 24) to the CO, we have $\bar{P}(\mathrm{CO}) \simeq P_G(\mathrm{CO})$. Since $P_G(\mathrm{CO}) \simeq 1.4 \times 10^5 \bar{N}_{\mathrm{H}22}^2(\mathrm{CO})$ K cm^{-3} from equation (23), the mean column density associated with the CO is

$$\bar{N}_{\mathrm{H}22}(\mathrm{CO}) \simeq 1.0 \left(\frac{P_s}{2 \times 10^4 \text{ K cm}^{-3}} \right)^{1/2} . \tag{72}$$

Allowing for the somewhat higher ambient pressure in the inner Galaxy and in regions of active star formation, and for the atomic gas associated with the GMCs, we infer

$$0.4 \lesssim \bar{N}_{\mathrm{H}22} \lesssim 2, \tag{73}$$

where the lower limit is set by the condition that there be a significant column density of CO (§1.1). This argument, which builds on previous work by Chiéze [17] and Elmegreen [27], provides an explanation for Larson's third law: All molecular clouds have about the same column density since if the column is too low they are not molecular and if it is too high they are not stable. The argument near the end of §2.2 shows why Larson's law then applies to typical star–forming clumps in GMCs as well. OB star–forming clumps have far larger extinctions ($\bar{N}_{\mathrm{H}22} \sim 60$) [87], and so do not satisfy Larson's third law; their stability needs investigation.

As discussed in §1.2, any one of Larson's laws can be derived from the other two. To obtain the line width—size relation, we write the velocity dispersion in the cloud in terms of the virial parameter α (eq. 26),

$$\sigma = 0.55 \left(\alpha \bar{N}_{\mathrm{H}22} R_{\mathrm{pc}} \right)^{1/2} \quad \text{km s}^{-1}. \tag{74}$$

For clouds that are gravitationally bound ($\alpha \sim 1$), the fact that the mean column density is restricted to a fairly narrow range of values then leads to the observed line width—size relation, $\sigma \propto R^{1/2}$. The line width also depends on the magnetic field strength: Since $\bar{A}_V \propto \bar{B}/(M/M_\Phi)$ from equation (39), we see that $\sigma \propto \bar{B}^{1/2}$ [74] [76] provided M/M_Φ is about the same for all the clouds as indicated in §§2.4.4 and 2.4.5.

A limitation of the polytropic models described above is that they do not allow for the damping of MHD waves on small scales. Curry & McKee [22] have developed composite polytropic models to deal with this problem. A simple example of a composite polytrope is one in which a cold isothermal core is embedded in a hot isothermal envelope; in this case, it is possible to have a total pressure drop between the center of the cold core and the surface of the hot envelope of up to $14^2 = 196$. To model molecular cloud cores, they assume that the inner region is dominated by thermal motions whereas the outer region is dominated by nonthermal motions and a static field. By carefully allowing for projection effects, they are able to get good agreement with the observed line width profiles.

2.7. MODELING TURBULENCE IN MOLECULAR CLOUDS

The most important recent development in the theoretical study of molecular clouds is the advent of sophisticated simulations of turbulent, magnetic, self–gravitating clouds [37], [55], [56], [83], [84], [85], [104]. Many of the issues associated with turbulence in molecular clouds and the results from these simulations have been reviewed by Vazquez–Semadeni et al. [110]. Here I shall mention only two results that are of particular relevance to our discussion.

First, the simulations show that when the turbulent velocities are supersonic, the gas becomes clumped. As yet, there is no agreement on exactly how the clumping factor depends on the physical conditions. Vazquez–Semadeni et al [110] summarize the existing multi–dimensional, isothermal calculations as finding that the mean mass–averaged value of $\log(\rho/\bar{\rho})$ scales with the logarithm of the sonic Mach number. Since the Mach number increases with scale, this result suggests that the clumping factor in clumps is smaller than in GMCs as a whole. To date, there is insufficient dynamical range in space to study hierarchical structure in GMCs (cores in star–forming clumps in GMCs), or in time to study how the initial conditions in the interstellar medium might affect the clump structure of GMCs.

The second and more important result is that the waves damp remarkably quickly [55], [56], [104]. For example, Stone et al. [104] find that the energy dissipation time for forced MHD turbulence is about $0.75L/v_{\rm rms}$, where L is the scale on which the turbulence is supplied and $v_{\rm rms} = \sqrt{3}\sigma_{\rm nt}$ is the rms velocity of the turbulence. None of the groups find a significant difference between magnetized and unmagnetized turbulence. This is completely contrary to the expectation that magnetic fields would reduce dissipation by "cushioning" the flow. Furthermore, since circularly polarized Alfvén waves of arbitrary amplitude can propagate without dissipation in a uniform medium, it had been thought that they could survive more than an eddy turnover time in a non–uniform medium. Stone et al estimate that the energy dissipation rate per unit mass is $\dot{\epsilon}_{\rm diss} \simeq {v_{\rm rms}}^3/L$. The motions observed in molecular clouds must therefore be continually rejuvenated, presumably by energy injection from newly formed stars [82].

Although several groups have independently found that the waves damp extremely rapidly, this result must be treated with caution. From a technical standpoint, the numerical resolution is as yet inadequate to resolve the waves that occur in the clumps that form within the simulation volume; this could be important since such waves could resist compression of the clumps. To date, simulations have been done for gas that fills a rectangular volume; it has not been possible to simulate an isolated GMC. More importantly, there are several problems from an observational standpoint. It is very difficult to understand how a GMC such as G216-2.5, which has no visible star formation, can have a level of turbulence that exceeds that in the Rosette molecular cloud, which has an embedded OB association [114]. On a smaller scale, cores are observed to have comparable levels of non-thermal motions whether there is an embedded star or not (e.g., [15]). One of the striking results of the simulations is that the rate of dissipation appears to be insensitive to whether the turbulence is super–Alfvénic or not, yet observations show that the Alfvén Mach number is of order unity [20]. Although there is considerable scatter in the properties of molecular clouds, the existence of regularities such as those found by Larson [48] is difficult to understand if the turbulence decays on the dynamical time scale whereas the stars that are supposed to support the clouds form on a considerably slower time scale, as discussed in §1.3.

3. Evolution of Molecular Clouds and Star Formation

3.1. FORMATION OF GMCS

The formation of GMCs is a rich and complex topic that has been reviewed by Elmegreen [29]. As in the case of galaxy formation, there are two countervailing views: in the "bottom–up" picture, small objects coagulate to form large ones, whereas in the "top–down" picture, large objects form first and fragment into small objects. In the case of GMCs, the bottom–up scenario corresponds to the growth of clouds by collisions. In order to generate the observed power–law mass distribution, a number of generations of collisions are necessary. However, as Elmegreen [29] points out, the short lifetime of molecular clouds ([10] [115]) makes this very difficult.

In the top–down scenario of GMC formation, clouds form in spiral arms, where the low shear and high densities allow gas to accumulate along the arm [30]. The volume from which the gas in a GMC is accumulated is quite large [67]. If we assume that the accumulation volume prior to compression by the spiral arm is an ellipsoid with a axial diameter L_0 and a radius R_0, that the mean density in this volume is initially n_0, and that the magnetic field (measured in μG) is initially $B_{0\mu}$, then one finds [65]

$$\begin{aligned} L_0 &= 96\left(\frac{B_{0\mu}}{n_0}\right)\frac{M}{M_\Phi} \quad \text{pc}, \qquad (75)\\ R_0 &= 380\left(\frac{M_6}{B_{0\mu}}\right)^{1/2}\left(\frac{M}{M_\Phi}\right)^{-1/2} \quad \text{pc}. \qquad (76)\end{aligned}$$

For typical conditions ($n_0 \sim 1$ cm^{-3}, $B_{0\mu} \sim 3$ and $M \simeq 2M_\Phi$, as discussed in §2.4.4), this gives $L_0 \sim 600$ pc and $R_0 \sim 150 M_6^{1/2}$ pc. This is large enough to contain many diffuse clouds, which could subsequently become clumps in the GMC [26].

3.2. DYNAMICAL EVOLUTION OF GMCS

Once a GMC has formed, it will be supported against gravity by a combination of static magnetic fields and turbulent pressure; since the observed motions are highly supersonic, thermal pressure is relatively unimportant. Because the waves are damped (§2.7), the cloud will contract. The contraction of the cloud will adiabatically compress the waves, tending to counteract the damping. In order to stop the contraction, energy injection is required, and this is provided by protostellar outflows [82].

The resulting evolution of the cloud can be described with the "cloud energy equation" [60]. Let $\epsilon \equiv E/M$ be the total cloud energy per unit mass. Since the motions observed in GMCs are highly supersonic, we shall assume that the energy in internal degrees of freedom is negligible, as in §2.2. The rate of change of ϵ can be written symbolically as

$$\frac{d\epsilon}{dt} = \mathcal{G} - \mathcal{L}, \qquad (77)$$

where $\mathcal{G}$ is the rate of energy gains per unit mass and $\mathcal{L}$ is the rate of energy losses per unit mass.

Energy is lost by wave damping, $\mathcal{L} = -\dot{E}_w/M$, where the wave energy E_w includes the energy in both motions and in fluctuating fields. For motions coupled to the field, the fluctuating field energy is in equipartition with the kinetic energy [118]. If we assume that the motions are isotropic, then equipartition applies to 2 of the 3 directions and $E_w = (5/3)\mathcal{T}$. In fact, Stone et al. [104] find $E_w \simeq 1.6\mathcal{T}$ when the gas is strongly magnetized ($\beta \equiv P_{\rm th}/P_B$ in the range 0.01-0.1). Let η be the ratio of the damping time to the free fall time $t_{\rm ff}$; we then have

$$\mathcal{L} \simeq 1.6\,\frac{\mathcal{T}/M}{\eta t_{\rm ff}} = 1.6\,\frac{{v_{\rm rms}}^2/2}{\eta t_{\rm ff}}. \qquad (78)$$

The simulations discussed in §2.7 suggest that $\eta \sim 1$, but for the reasons discussed there $\eta \sim$ a few seems more reasonable.

As pointed out by Norman & Silk [82], energy injection by newly formed stars is the dominant source of kinetic energy for molecular clouds. The rate at which protostellar outflows inject energy can be written as

$$\mathcal{G} = \epsilon_{\rm in}\left(\frac{\dot{M}_*}{M}\right) \equiv \frac{\epsilon_{\rm in}}{t_{g*}}, \tag{79}$$

where $\epsilon_{\rm in}$ is the energy injected per unit stellar mass and t_{g*} is the *star formation time scale*, defined as the time to convert the gas entirely into stars. An outflow drives a shock into the surrounding medium and a reverse shock is driven back into the outflow. Under the assumption that both shocks are radiative, momentum conservation implies $m_w v_w = M_{sw} v_{sw}$, where m_w and v_w are the mass and velocity of the outflowing wind, and M_{sw} and v_{sw} are the mass and velocity of the swept up cloud. The swept up material merges with the ambient cloud when v_{sw} drops to about the effective sound speed $(P_{\rm tot}/\rho)^{1/2}$, which is of order $v_{\rm rms}$ in a highly turbulent cloud. Introducing a factor $\phi_w \sim 1$ that allows for the uncertainty in our estimate of the energy injection per outflow, we then have

$$m_* \epsilon_{\rm in} = 1.6\phi_w \times \frac{1}{2} M_{sw} {v_{\rm rms}}^2 = 1.6\left(\frac{\phi_w m_w v_w}{v_{\rm rms}}\right)\frac{{v_{\rm rms}}^2}{2}, \tag{80}$$

where we have included a factor 1.6 for the energy stored in fluctuating magnetic fields. Magnetic energy stored in the wind could do work on the ambient medium, which would make ϕ_w greater than unity; on the other hand, the protostellar jets could escape from the cloud, which would reduce ϕ_w. Inserting this result into equation (79) and using equations (77) and (78), we find that the cloud energy equation becomes

$$\frac{d\epsilon}{dt} = 1.6\left[\left(\frac{\phi_w m_w v_w}{m_* v_{\rm rms}}\right)\frac{1}{t_{g*}} - \frac{1}{\eta t_{\rm ff}}\right]\frac{{v_{\rm rms}}^2}{2}. \tag{81}$$

This equation shows that the cloud will contract if the star formation rate is too small (for a bound cloud, ϵ will become more negative) and expand if the rate is too large. If the cloud contracts quasistatically (so that $\ddot{I}$ is negligible), and if the cloud is strongly bound ($P_s \ll \bar{P}$, which may be difficult to achieve in practice), then the virial theorem implies $E = \mathcal{T} + \mathcal{M} + \mathcal{W} = -\mathcal{T}$, or $\epsilon = -{v_{\rm rms}}^2/2$. Thus, contraction of a bound cloud leads to an *increase* in the velocity dispersion of the cloud. This "virialization" might account for the motions observed in clouds that are not actively forming stars [60].

As the density of the contracting cloud rises, the star formation rate is expected to rise as well. If the rate of energy injection by newly formed stars balances the damping rate, the star formation is said to be *dynamically regulated* [7]:

$$\frac{t_{g*\mathrm{DR}}}{t_{\mathrm{ff}}} = \left(\frac{m_w v_w}{m_* v_{\mathrm{rms}}}\right) \phi_w \eta. \tag{82}$$

Theoretically, we expect the momentum per unit stellar mass to be

$$\left(\frac{m_w}{m_*}\right) v_w \simeq \frac{1}{3} \times 200\,\mathrm{km\ s^{-1}} \simeq 70\,\mathrm{km\ s^{-1}} \tag{83}$$

[78]. Observations suggest a somewhat lower value of about 40 km s^{-1} [7]. With the latter value, and taking $v_{\mathrm{rms}} \sim 2$ km s^{-1}, we have $t_{g*\mathrm{DR}}/t_{\mathrm{ff}} \simeq 20\phi_w\eta \gg 1$. Thus, dynamically regulated star formation is inefficient, consistent with observation.

To obtain a more precise comparison between the dynamically regulated star formation rate and observation, we write the dynamically regulated star formation rate as

$$\dot{M}_{*\mathrm{DR}} = \frac{M}{t_{g*\mathrm{DR}}} = \left(\frac{m_* v_{\mathrm{rms}}}{m_w v_w}\right) \frac{M}{\phi_w \eta t_{\mathrm{ff}}}. \tag{84}$$

Noting that $v_{\mathrm{rms}}/t_{\mathrm{ff}} \propto \alpha^{1/2}\bar{N}_{\mathrm{H}}$ and using 40 km s^{-1} for the momentum injection per unit stellar mass, we find

$$\dot{M}_{*\mathrm{DR}} \simeq 2.7 \times 10^{-8} \left(\frac{\alpha^{1/2}\bar{N}_{\mathrm{H22}} M}{\phi_w \eta}\right) \quad M_\odot\ \mathrm{yr}^{-1}. \tag{85}$$

First, apply this equation to the entire Galaxy. The mass of H_2 inside the solar circle is $1.0 \times 10^9\ M_\odot$ [115], of which about half is actively forming stars [99]. With an average column density $\bar{N}_{\mathrm{H22}} \simeq 1.5$ [100] and with $\alpha \simeq 1$ [5], we have $\dot{M}_* \simeq 20/(\phi_w\eta)\ M_\odot\ \mathrm{yr}^{-1}$. This is consistent with the observed value of 3 $M_\odot$ yr^{-1} [62] for $\phi_w\eta \simeq 7$. McKee [60] adopted a momentum per unit stellar mass of 70 km s^{-1} instead of 40 km s^{-1}(which is equivalent to $\phi_w = 1.7$) and took $\eta = 5$, and so concluded that energy injection by low mass stars could supply the turbulent energy needed to support GMCs. On the other hand, if $\eta \sim 1$ as suggested by the simulations discussed in §2.7, then either winds are much more efficient at energizing clouds ($\phi_w \gg 1$) or most GMCs are energized by some other source, such as massive stars.

The concept of dynamically regulated star formation can be applied to individual star–forming clumps as well. Consider the star–forming clumps associated with the young star clusters NGC 2023, 2024, 2068, 2071 in Orion B [46] [47] [45]. The clumps all have masses of order 400 $M_\odot$, although the number of young stars associated with the clumps ranges from 21 in NGC 2023 to 309 in NGC 2024. With a mean stellar mass of 0.5 $M_\odot$ [91], $\alpha \sim 1$, $\bar{N}_{\rm H22} \sim 1.5$, the dynamically regulated star formation rate in one of these clouds is $\dot{\mathcal{N}}_{*\rm DR} \simeq 32/(\phi_w \eta)$ stars Myr^{-1}. Because of the small masses of the clumps involved, large fluctuations in the star formation rate may be expected. Furthermore, ϕ_w might be significantly less than unity if the outflows can escape from the clumps [58], as has been observed in some cases [89]. Thus, the dynamically regulated rate appears to be within the range observed in these clumps assuming that the star formation has been occurring for the past 1-3 Myr. The OB star–forming clumps observed by Plume et al [87] are an order of magnitude more massive than these, and it is not known if their properties are also consistent with dynamically regulated star formation.

3.3. PHOTOIONIZATION–REGULATED STAR FORMATION

To this point, we have discussed the star formation rate in terms of how many stars must form in order to provide adequate energy input to prevent molecular clouds from collapsing, but we have not discussed the physical mechanism that actually determines the rate at which stars form. For low mass stars, this time scale is believed to be controlled by ambipolar diffusion [68] [72] [95]. As we have seen in §2.4.1, the magnetic critical mass for clumps in molecular clouds is larger than the typical mass of a star. In order for gravity to overcome the force due to the magnetic field, gas must either accumulate along the field lines or across the field; the latter can occur only via ambipolar diffusion. If the gas is to accumulate along the field, then for $M/M_\Phi \simeq 2$ (§2.4.3) about half the mass along a given flux tube would have to be concentrated into a single star. A model in which the accumulation of gas along flux tubes is regulated by wave damping has been proposed recently by Myers & Lazarian [77]. Here we shall focus on the role of ambipolar diffusion in low mass star formation.

Most of the gas in a molecular cloud is neutral, and does not interact directly with the magnetic field. On the other hand, at typical densities in molecular clouds, the charged particles are well coupled to the field. As a result, if the neutral gas attempts to collapse under its own self gravity, it will be restrained by collisions with the charged particles. Balancing the force of gravity against that of friction gives

$$\frac{GM\rho}{R^2} \simeq n_i \langle \sigma v \rangle \rho v_{\rm AD}, \tag{86}$$

where $v_{\rm AD}$, the ambipolar diffusion velocity, is the relative velocity between the ions and the neutrals. The time scale for ambipolar diffusion then depends only on the ionization of the gas [102],

$$t_{\rm AD} \simeq \frac{R}{v_{\rm AD}} = \left(\frac{3\langle\sigma v\rangle}{4\pi G\mu_{\rm H}}\right) x_e. \tag{87}$$

The numerical coefficient in the relation between $t_{\rm AD}$ and x_e depends on the geometry of the cloud and the nature of the non–magnetic forces. To be specific, we identify $t_{\rm AD}$ as the time for ambipolar diffusion to initiate the formation of a very dense core. Fiedler & Mouschovias [35] simulated an axisymmetric cloud in which the thermal pressure slowed the ambipolar diffusion significantly; their results give $t_{\rm AD}/x_e \simeq 0.8\times 10^{14}$ yr. Ciolek & Mouschovias [18] found a similar result for the case of a thin disk, $t_{\rm AD}/x_e \simeq 1.0\times 10^{14}$ yr. We adopt

$$t_{\rm AD} = 1.0\times 10^{14}\phi_{\rm AD} x_e \quad \text{yr}, \tag{88}$$

where $\phi_{\rm AD}$ is a constant of order unity that allows for deviations from the typical value. In gas that is shielded from FUV radiation, the ionization is due to cosmic rays (eq. 2), which implies

$$\frac{t_{\rm AD}}{t_{\rm ff}} = 23\phi_{\rm AD}\left(\frac{C_i}{10^{-5}\ {\rm cm}^{-3/2}}\right). \tag{89}$$

For $C_i \sim 0.7\times 10^{-5}$ (§1.1), $t_{\rm AD} \simeq 15 t_{\rm ff}$, so that ambipolar diffusion is quite slow compared to gravitational collapse.

When the magnetic critical mass M_B significantly exceeds the typical stellar mass, as appears to be the case in Galactic molecular clouds (§2.4.1), then the rate at which stars form is set by the average rate of ambipolar diffusion in the cloud [60]:

$$\dot{M}_* = \frac{M}{t_{g*}} \simeq \int \frac{dM}{t_{\rm AD}(x_e)}. \tag{90}$$

The ionization in the outer parts of molecular clouds is relatively high due to photoionization of the metals. Regions in the clouds that are shielded by an extinction $A_V > A_V(\mathrm{CR}) \simeq 4$ are ionized primarily by cosmic rays; as a result, they have a lower ionization (eq. 2) and correspondingly lower ambipolar diffusion time. As a result, the star formation rate in a molecular cloud is approximately that in the cosmic ray ionized region,

$$\dot{M}_* \simeq \frac{M(A_V > A_{\rm CR})}{t_{\rm AD}(x_{e\rm CR})}. \tag{91}$$

Since the rate at which stars form is governed by the mass that is shielded from photoionizing FUV radiation, McKee termed this *photoionization–regulated star formation* [60]. This process is naturally inefficient, both because only a fraction of the cloud is in the cosmic ray ionized region (models give a typical value of about 10% [60]) and because the ambipolar diffusion time is much greater than the free fall time (eq. 89).

How does this rate compare with that observed in the Galaxy? If we adopt $n_{\rm H} \simeq 3000$ cm^{-3} from §1.2, we find $t_{\rm AD} \simeq 2.4 \times 10^7$ yr. For 10% of the mass in cosmic ray ionized regions, this gives $t_{g*} \simeq 2.4 \times 10^8$ yr. Since the total mass of molecular gas inside the solar circle is about $1.0 \times 10^9\ M_\odot$ [115], the predicted rate of photoionization regulated star formation there is $\dot{M}_* = 10^9 M_\odot / 2.4 \times 10^8$ yr $\simeq 4\ M_\odot$ yr^{-1}, quite close to the observed value of 3 $M_\odot$ yr^{-1}.

One of the key predictions of this theory is that star formation is restricted to regions of relatively high extinction, $A_V \gtrsim A_{\rm CR} \simeq 4$ mag. This has been tested in a study of the L1630 region of the Orion molecular cloud, and indeed all the star formation was found to be concentrated in regions in which the extinction was greater than this. McKee [60] also predicted that molecular clouds in the Magellanic Clouds would have comparable extinctions, and therefore higher column densities, than Galactic molecular clouds in order that star formation be able to provide the energy needed to prevent the molecular clouds from collapsing. Pak et al [86] have found that the column densities of molecular clouds in the LMC and SMC do in fact scale approximately inversely with the metallicity, consistent with an approximately constant extinction.

4. Conclusion

We have seen that it is possible to understand a number of the observed properties of molecular clouds in terms of a model in which GMCs and the clumps within them are modeled as clouds in approximate hydrostatic equilibrium, with the turbulence treated as a separate pressure component. In particular, we have seen why the clouds are generally gravitationally bound; why they have approximately constant column densities; why they have line widths that increase with size; why, along with the star–forming clumps and cores within them, they are somewhat magnetically supercritical; why they are the sites of star formation in the Galaxy; and why star formation is inefficient. Of course, this model is a drastically oversimplified picture of the real situation. Other researchers, looking at the same data, have developed completely different models: For example, Elmegreen & Falgarone [31] have developed a fractal model for structure in the interstellar medium in which the concept of pressure plays no role, and Ballesteros–Paredes et al [3] have argued that pressure balance is irrelevant in a turbulent medium such as the ISM. By the time of the next Crete meeting, there may be some synthesis between these differing viewpoints or perhaps an entirely new idea. The challenge facing us is formidible, for we must attempt to extend our understanding to regions of OB star formation as well.

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