Heavy Flavor Physics with 2+1 Flavors of Improved Staggered Quarks

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work done with

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(HPQCD Collaboration)

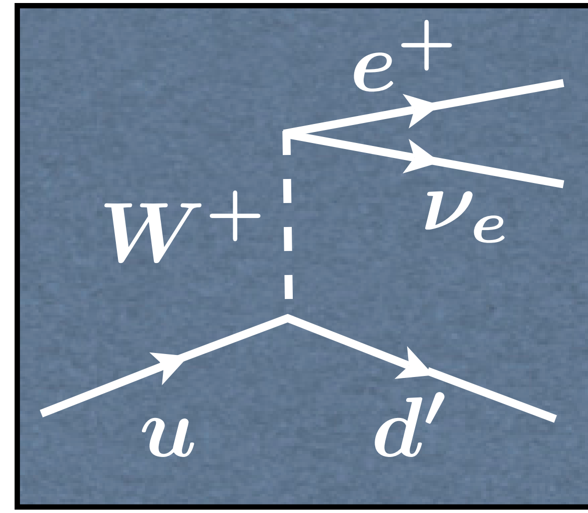
Outline

- Brief review: quark flavor physics
  - Search for beyond-the-Standard-Model physics
  - Lattice QCD input
- Relevant new ingredients
  - Improved staggered fermions
  - Heavy-light mesons (undoubled heavy + staggered light)
- Results and impact on phenomenology
- Current & future effort; conclusions

Quark flavor

Quark Flavor Mixing

- Standard Model (CKM mechanism)



- Only weak interactions can change quark flavor

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \begin{pmatrix} t \\ b' \end{pmatrix}$$

- Flavor mixing

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- *V* is the CKM matrix. Unitarity implies 4 free parameters

Wolfenstein Parameterization

Expansion based on empirical observation

$$|V_{us}| = 0.22 \ll 1$$

$$|V_{cb}| \approx |V_{us}|^2$$

$$|V_{ub}| \ll |V_{cb}|$$

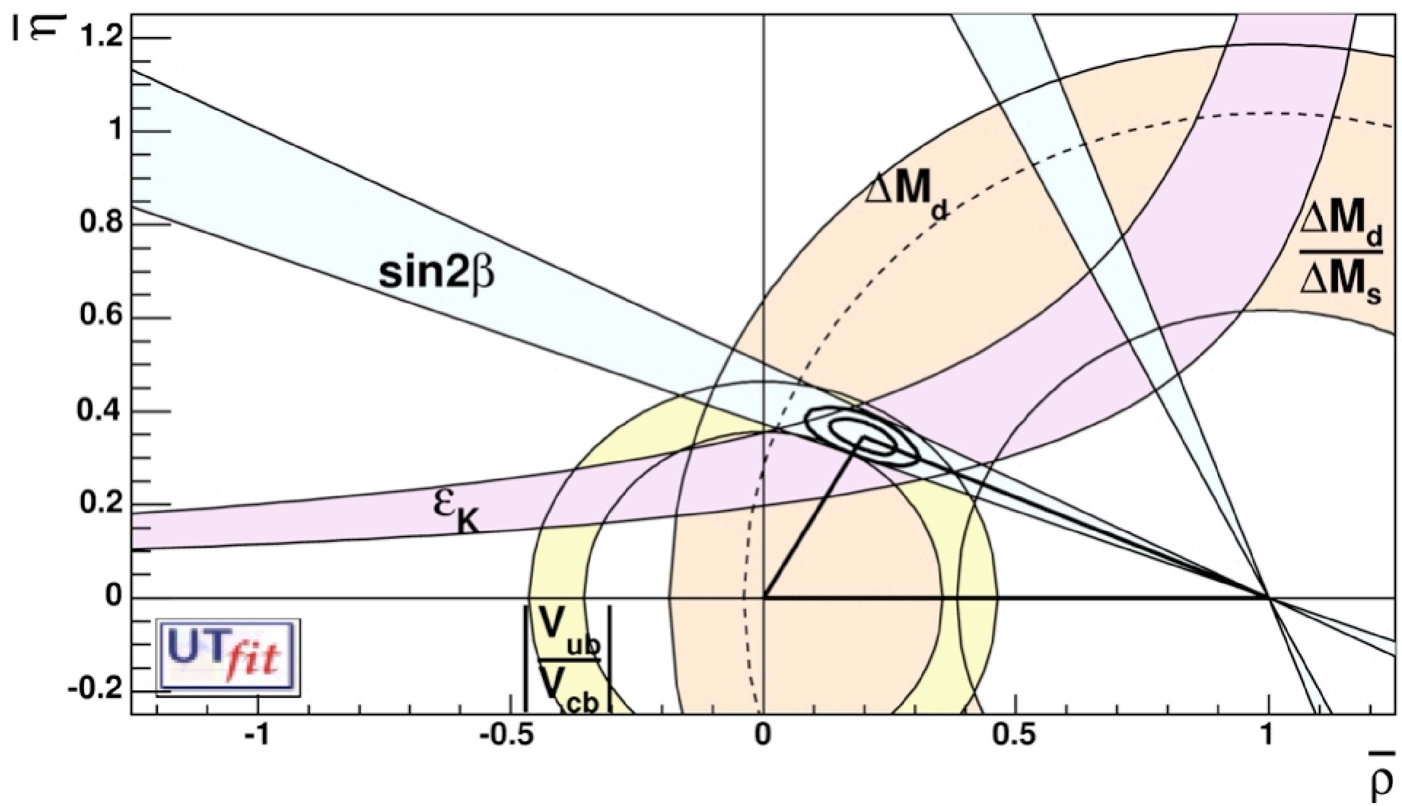
$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

In practice, go to next order

$$\bar{\rho} = \rho\left(1 - \frac{\lambda^2}{2}\right) \qquad \bar{\eta} = \eta\left(1 - \frac{\lambda^2}{2}\right)$$

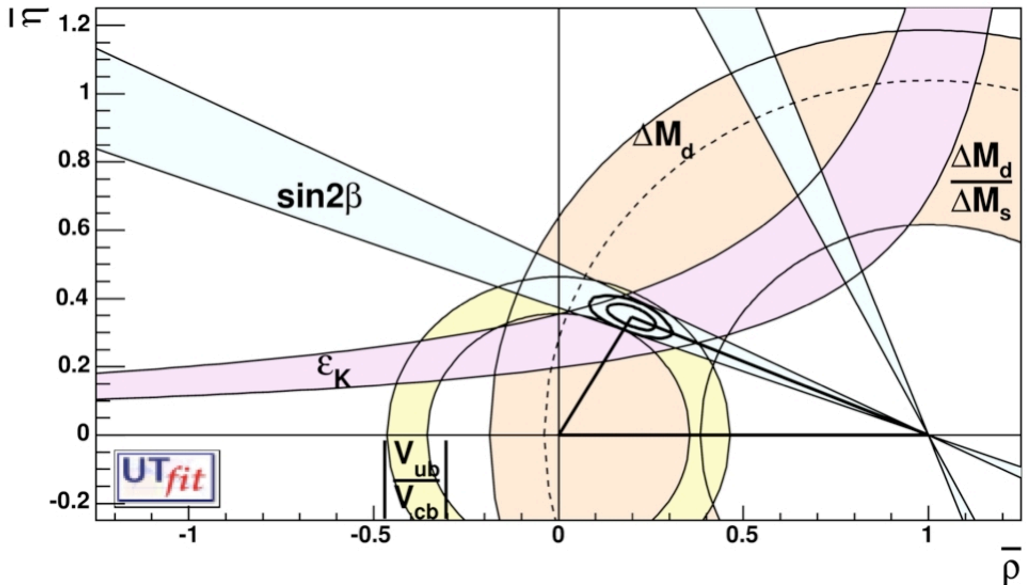
Wolfenstein Parameterization

$\lambda = 0.2205 \pm 0.0018 (0.8\%)$ $A = 0.824 \pm 0.075 (9\%)$



$\bar{\rho} = 0.196 \pm 0.045 (23\%)$ $\bar{\eta} = 0.347 \pm 0.025 (7\%)$

Experimental Constraints



- $\sin 2\beta$ from $B \to (J/\psi)K$
- ε_K from $K^0 \leftrightarrow \overline{K^0}$
- ΔM from $B^0 \leftrightarrow \overline{B^0}$
- V_{ub} and V_{cb} from semileptonic decays $B \to \pi\ell\nu$, $B \to D\ell\nu$

} need LQCD input

Meson Decays and Mixings

$$\begin{pmatrix} \boldsymbol{V_{ud}} & \boldsymbol{V_{us}} & \boldsymbol{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ * & K \to \pi\ell\nu & \\ \boldsymbol{V_{cd}} & \boldsymbol{V_{cs}} & \boldsymbol{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & \\ \boldsymbol{V_{td}} & \boldsymbol{V_{ts}} & \boldsymbol{V_{tb}} \\ B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s & * \end{pmatrix}$$

* most precise determinations for

$\boldsymbol{V_{ud}}$ from nuclear beta decays and neutron decay

$\boldsymbol{V_{tb}}$ **from** $\boldsymbol{t \to b\ell\nu}$

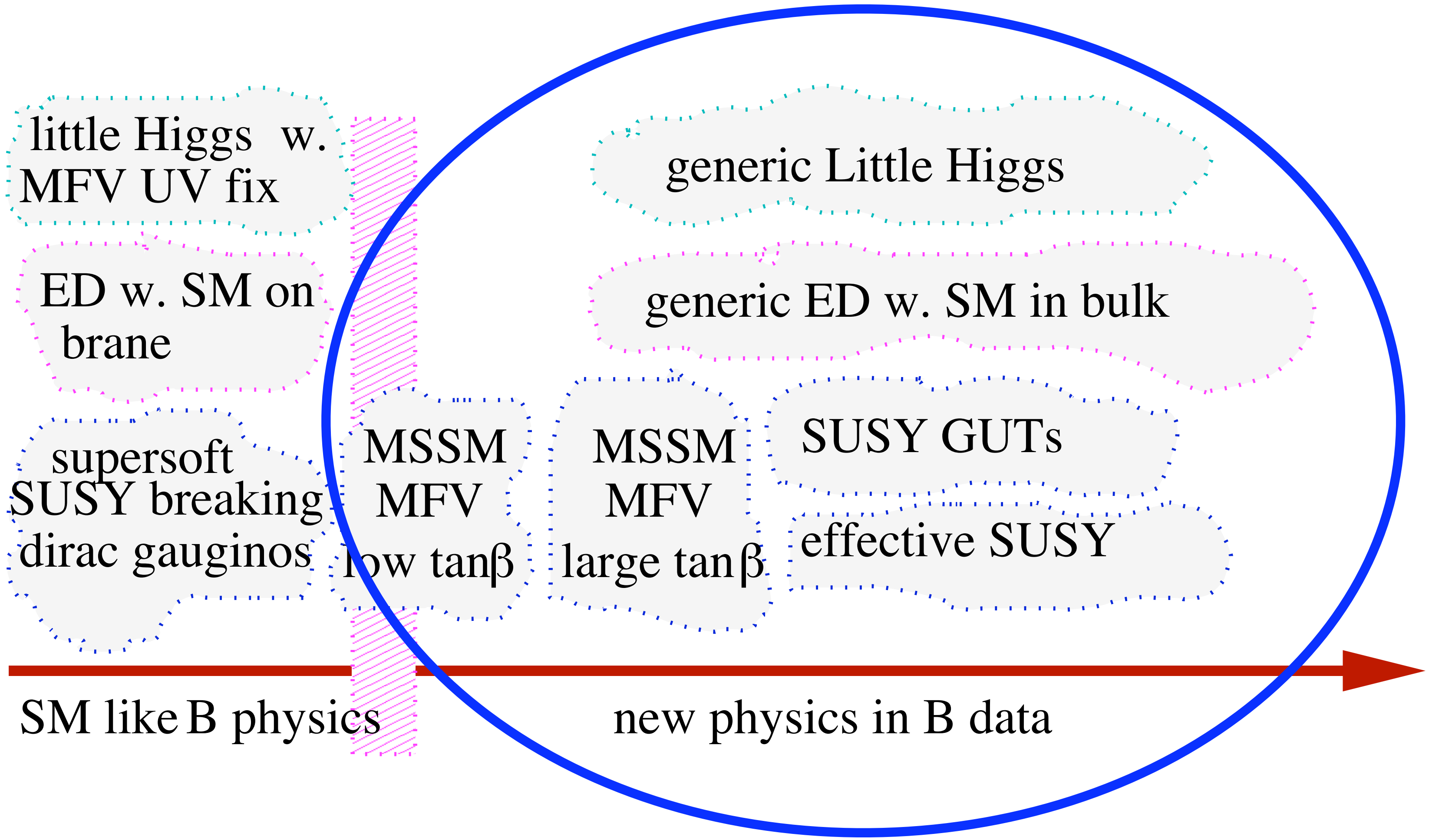
Why flavor physics is interesting

- CKM matrix elements are fundamental parameters of the Standard Model
- The Standard Model is only "natural" up to TeV energies
  - Higgs mediates EWSB at TeV scale, but its mass is fine tuned

$$\delta m^2 \sim \frac{\Lambda^2}{16\pi^2}$$

- New physics models generically introduce new sources of flavor changing interactions
- Impressive experimental results now! (*and* they care what we say)

New physics models & quark flavor



Discovery potential of flavor physics experiments **or**

Nondiscovery rules out/tightly constrains these models

Figure from G. Hiller, hep-ph/0207121

Flavor Physics Experiments

Lattice QCD

2 recent improvements

- **Improved staggered quarks**
  - Reduced unacceptable lattice artifacts
  - Made "unquenching" possible: Light quark masses with moderate lattice spacing

T. Blum, *et al.*, PR D55 (1997)
K. Orginos, *et al.*, PR D60 (1999)
G.P. Lepage, PR D59, 074502 (1999)

- **Staggered light quarks + standard heavy quarks**
  - Showed benefits of staggered light quarks applied to heavy-light mesons
  - Solved problems with extrapolating to physical light quark mass

M.W., *et al.*, PR D67 (2003)

Doublers, a matter of taste

Naive discretization

$$\mathcal{L} = \bar{\psi}(\gamma \cdot \nabla + m)\psi$$

$$\nabla_\mu \psi(x) = \frac{1}{2}\frac{1}{u_0}\left[U_\mu(x)\psi(x+a\hat{\mu}) - U_\mu^\dagger(x-a\hat{\mu})\psi(x-a\hat{\mu})\right]$$

Extra poles in free quark propagator

$$G^{(0)}(p)\, a = (i\gamma^\mu \sin p_\mu a + ma)^{-1}$$

Result of a lattice symmetry, e.g.

$$\psi(x) \rightarrow (i\gamma^5\gamma^1)\, e^{ix_1\pi/a}\, \psi(x)$$

$$\psi(p) \rightarrow (i\gamma^5\gamma^1)\; \psi\left(p + \tfrac{\pi\hat{x}}{a}\right)$$

Let's call extra states tastes

A Smattering of Staggering

Stagger the naive fermions by a spin diagonalization

$$\psi(x) \to \Omega(x)\tilde{\chi}(x) \qquad \bar{\psi}(x) \to \bar{\tilde{\chi}}(x)\Omega^\dagger(x)$$

$$\Omega(x) \equiv \prod_\mu (\gamma^\mu)^{x_\mu/a}$$

Resulting action is diagonal in spin

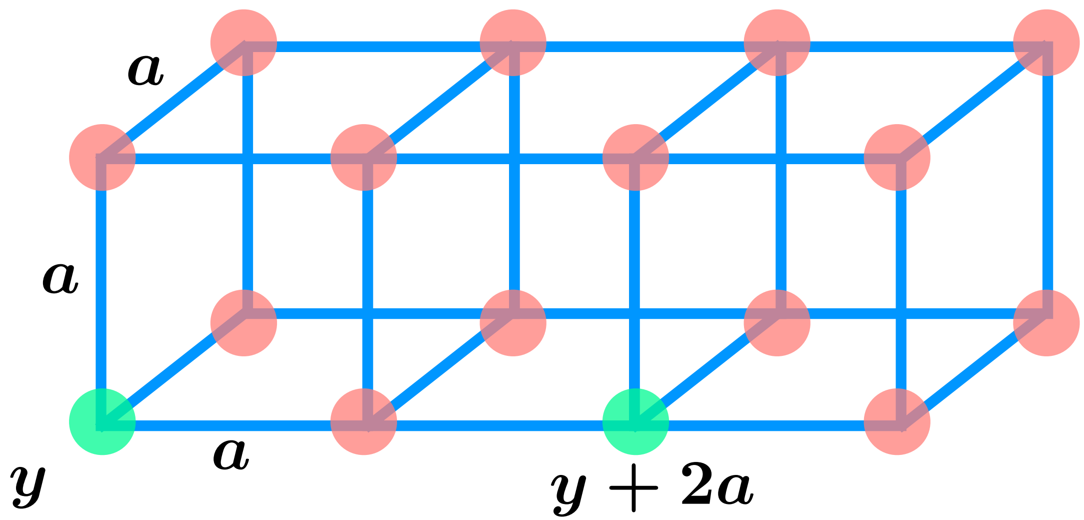
$$\bar{\tilde{\chi}}\,[(\eta\cdot\nabla + m)\times I_4]\,\tilde{\chi}$$

$$\eta^\mu(x) \equiv (-1)^{(x_0+x_1+\ldots+x_{\mu-1})/a}$$

Keep only 1 spin component fields $\chi, \bar{\chi}$

$$G_\psi(y,x) \equiv G_\chi(y,x) \otimes \Omega(y)\Omega^\dagger(x)$$

Hypercube interpretation



spin taste

$$q^{\alpha a}(y) \;=\; \frac{1}{8}\sum_{\epsilon}\Omega^{\alpha a}(\epsilon)\,\chi(y+\epsilon)$$

$$\epsilon \;=\; (0,0,0,0),\ldots,(a,a,a,a) \qquad \Omega(\epsilon) \;\equiv\; \prod_\mu (\gamma^\mu)^{\epsilon_\mu/a}$$

Large discretization errors explained by effective spacing = 2a

Momentum space interpretation

Direction from origin to points in elemental hypercube

$$g \in \{\emptyset, (0), (1), \ldots, (0,2), \ldots, (0,1,2,3)\}$$

Associated 4-momenta for the 16 corners of the Brillouin zone

$$(\pi_g)_\mu = \begin{cases} \frac{\pi}{a} & \text{if } \mu \in g \\ 0 & \text{otherwise} \end{cases}$$

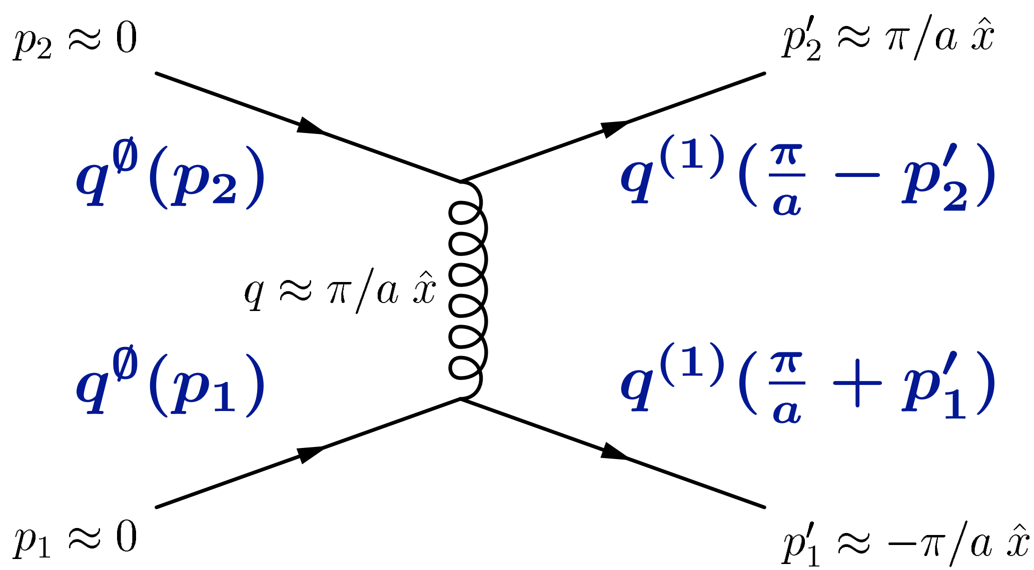
Associated product of Dirac matrices

$$M_g = \prod_{\mu \in g} (i\gamma_5\gamma_\mu)$$

Interpret naive field in each hexadecant of B.Z. as different taste

$$q^g(k) = M_g \psi(k + \pi_g)$$

Improving the staggered action



- Exchange of hard lattice-y gluons break taste and generate large scaling violations
- Large momentum modes are the offenders: can suppress them using perturbation theory
- Fat links - MILC
- AsqTad - Naik, MILC, Lepage

$$V_\mu(x) \;=\; \left[\prod_{\nu\neq\mu}\left(1 \;+\; \frac{a^2\nabla_\nu^{(2)}}{4}\right)\right]\Bigg|_{\mathrm{sym'd}} U_\mu(x)$$

2 improvements

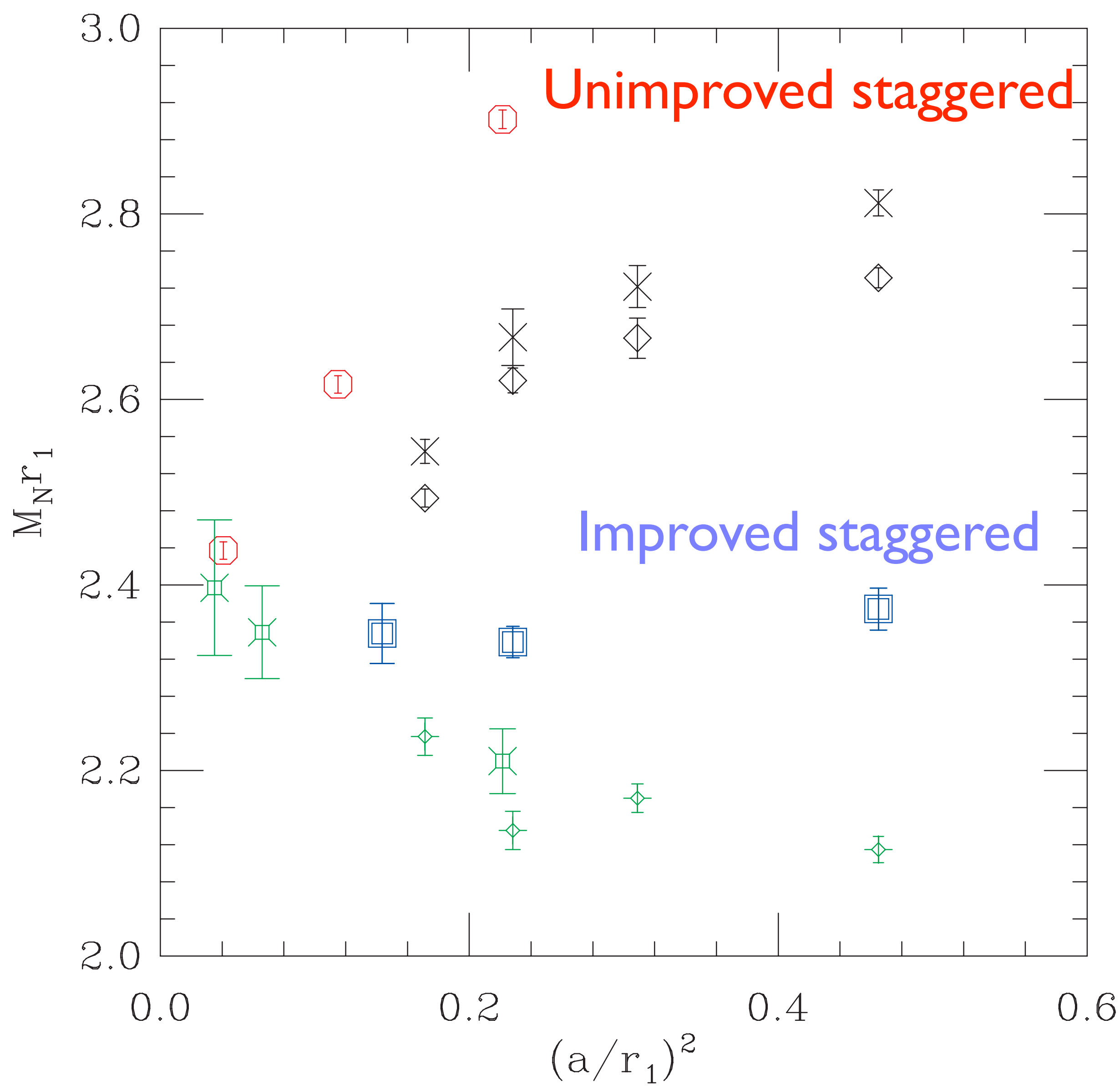
Improved staggered quarks

- Staggered fermions are computationally cheap
- Remnant chiral symmetry : protects mixing under renormalization and forbids largest lattice artifacts
- Symanzik improvement explains source of large $\mathcal{O}(a^2)$ errors and removes them -- decent scaling at $a \approx 0.13\ \mathrm{fm}$
- Realistic simulations with 3 flavors of dynamical quarks possible on present resources
- MILC collaboration gauge field configurations : many sea quark masses, 2-3 lattice spacings
- Fourth-root hypothesis has open theoretical questions -- empirical tests

2 improvements

Improved staggered quarks

- AsqTad action
- Quenched tests
- Heavy nucleon mass vs. lattice spacing (r_1 = 0.32-0.35 fm)
- Much improved scaling



MILC, hep-lat/9912018

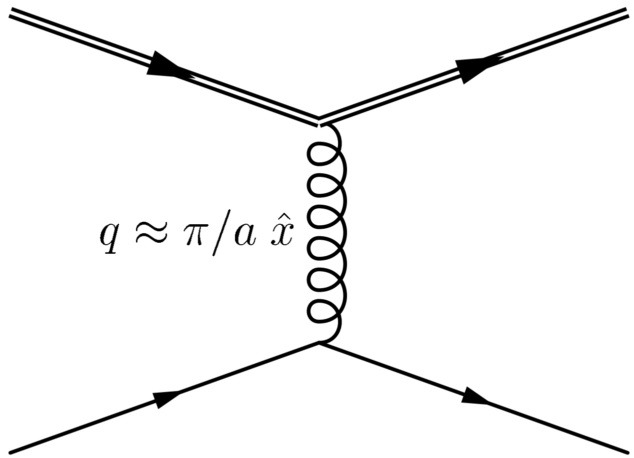
2 improvements

Staggered quarks in heavy-light mesons

M.W. *et al.*, PRD 67, 054505 (2003)

- Contrary to expectations, heavy-light mesons (NRQCD-stagg) much simpler than staggered mesons (stagg-stagg)
- Good properties of staggered quarks still pay off : fewer degrees-of-freedom, remnant chiral symmetry
- Best way to push to chiral limit and reduce uncertainty
- Now Fermilab+MILC is joining with FNAL-staggered studies -- *D* meson decays

Heavy-staggered mesons



$$\left\langle \overline{\Psi}(x)\gamma_5\psi(x)\ \overline{\psi}(0)\gamma_5\Psi(0) \right\rangle$$

$$G_\psi(x,y) = g_\chi(x,y)\ \prod_\mu \gamma_\mu^{x_\mu}\ \gamma_{4-\mu}^{y_{4-\mu}}$$

- NRQCD/FNAL quarks not doubled
- High momentum heavy quarks contribute little to corr'n functions
- Compute corr'n f'ns using naive propagator
- Gory detail and quenched tests in M.W., *et al.*, PRD 67, 054505 (2003)

(Axial)Vector Currents

Temporal components

$$\Gamma_\mu \equiv \begin{cases} \gamma_\mu & \text{for } V_\mu \\ \gamma_\mu\gamma_5 & \text{for } A_\mu \end{cases}$$

$$\begin{aligned}
J_0^{(0)}(x) &= \bar{q}(x)\,\Gamma_0\,Q(x), \\
J_0^{(1)}(x) &= -\frac{1}{2M_0}\bar{q}(x)\,\Gamma_0\,\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\,Q(x), \\
J_0^{(2)}(x) &= -\frac{1}{2M_0}\bar{q}(x)\,\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\,\Gamma_0\,Q(x).
\end{aligned}$$

Spatial components

$$\begin{aligned}
J_k^{(0)}(x) &= \bar{q}(x)\,\Gamma_k\,Q(x), \\
J_k^{(1)}(x) &= -\frac{1}{2M_0}\bar{q}(x)\,\Gamma_k\,\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\,Q(x), \\
J_k^{(2)}(x) &= -\frac{1}{2M_0}\bar{q}(x)\,\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\,\Gamma_k\,Q(x), \\
J_k^{(3)}(x) &= -\frac{1}{2M_0}\bar{q}(x)\,\nabla_k\,Q(x) \\
J_k^{(4)}(x) &= \frac{1}{2M_0}\bar{q}(x)\,\overleftarrow{\nabla}_k\,Q(x),
\end{aligned}$$

Operator Matching

For example,

$$\langle A_0 \rangle_{\rm QCD} = (1 + \alpha_s \tilde{\rho}_0)\, \langle J_0^{(0)} \rangle + (1 + \alpha_s \rho_1)\, \langle J_0^{(1),sub} \rangle + \alpha_s \rho_2 \langle J_0^{(2),sub} \rangle$$

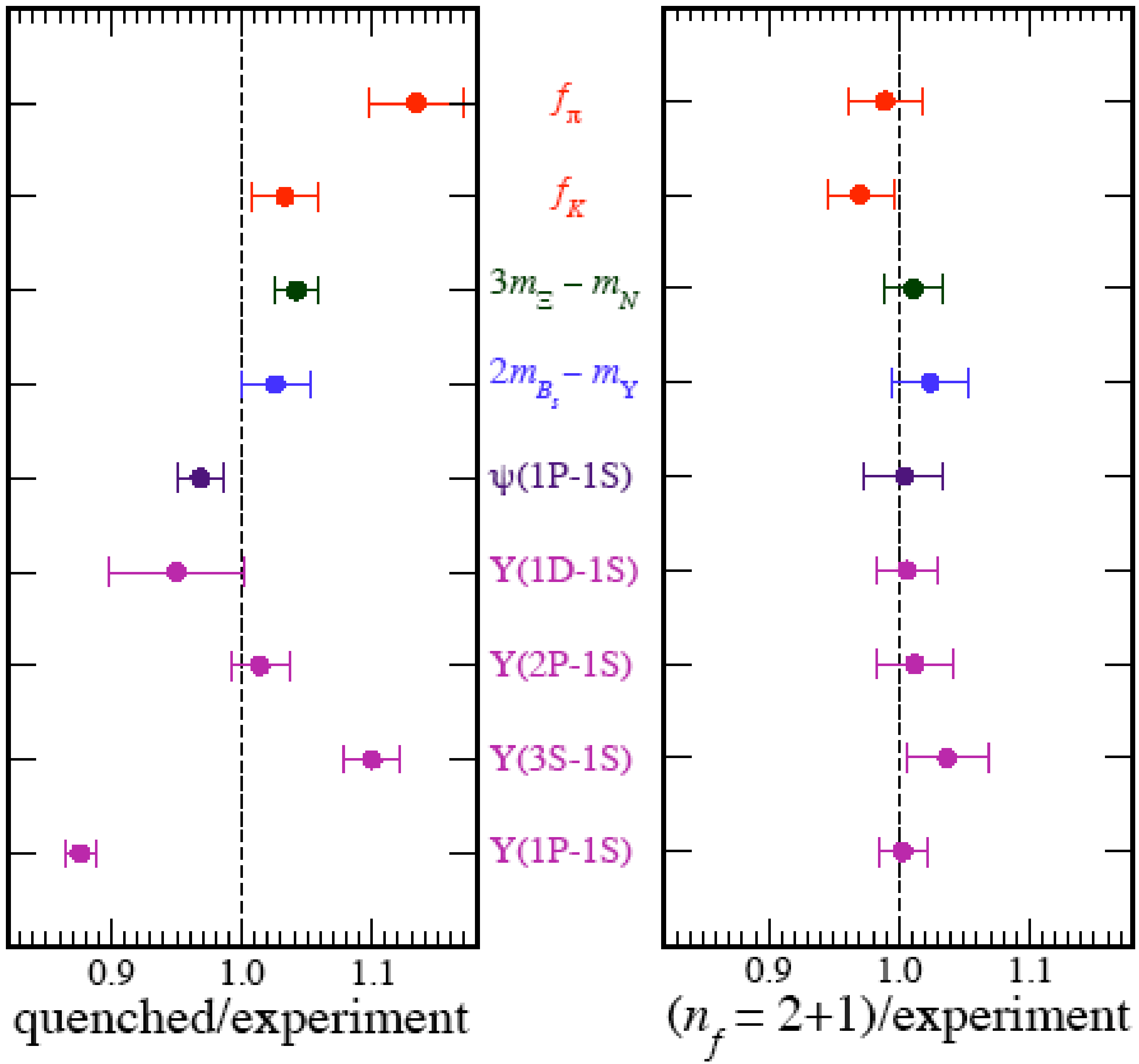
$$J^{(i),sub} = J^{(i)} - \alpha_s \zeta_{10} J^{(0)}$$

Turns out to be leading uncertainty

Perturbative coefficients computed in

E. Gulez, J. Shigemitsu, M.W., PRD 69, 074501 (2004)

Light sea quark effects are important!



C. Davies, *et al.*, PRL 92 (2004)

Logic of 4th Root Hypothesis

- Hypothesize that 4th root procedure is QCD in continuum limit
  - A testable hypothesis
  - Comparable to hypotheses of quark mass extrapolation from outside the chiral regime
- Empirical tests
  - So far so good
- Skeptics welcome
  - Also invited to look hard at non-lattice CKM uncertainties
- All approaches should be pushed hard

Some Results

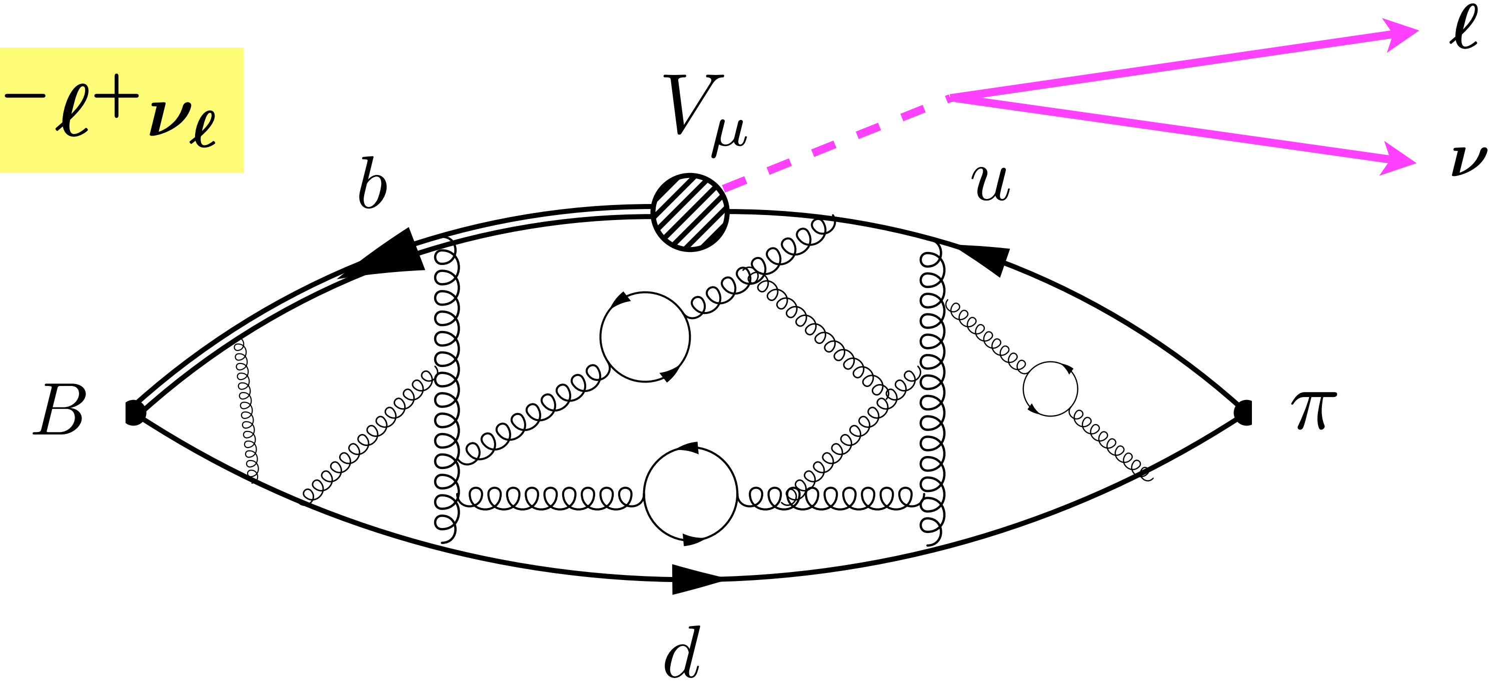
M.W. with : C. Davies, A. Gray,
E. Gulez, G.P. Lepage, J. Shigemitsu

Overview of simulation parameters

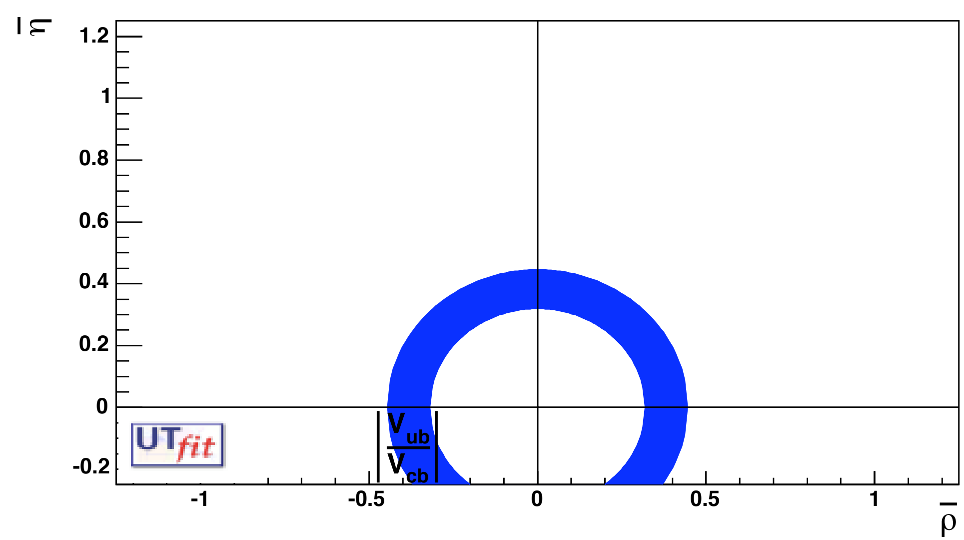
- MILC collaboration's 2+1 flavor configurations
- "coarse" $a = 0.13 \text{ fm}$ and "fine" $a = 0.09 \text{ fm}$
- Spatial volume $(2.5 \text{ fm})^3$
- Lightest up/down mass $m_s/8$
- We compute at both unquenched and partially quenched masses
- NRQCD action for bottom, correct through $O(\Lambda^2_{\text{QCD}}/m^2_Q)$

Semileptonic Decays

$B^0 \to \pi^- \ell^+ \nu_\ell$



$B^0 \to D^- \ell^+ \nu_\ell$



Semileptonic Decays

$B^0 \to \pi^- \ell^+ \nu_\ell$

$$\frac{1}{|V_{ub}|^2}\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3}|\vec{p'}|^3|f_+(q^2)|^2$$

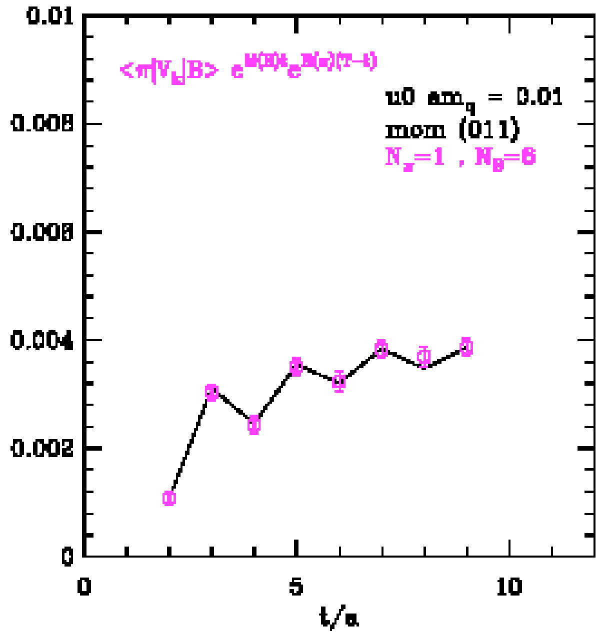
$$\langle \pi(p')|V^\mu|B(p)\rangle = f_+(q^2)(p^\mu + p'^\mu) + f_-(q^2)(p^\mu - p'^\mu)$$

$B^0 \to D^- \ell^+ \nu_\ell$

$$\frac{1}{|V_{cb}|^2}\frac{d\Gamma}{dw^2} = \frac{G_F^2}{48\pi^3}(m_B + m_D)^2 m_D^3 (w^2-1)^{3/2} \times \left[h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w)\right]$$

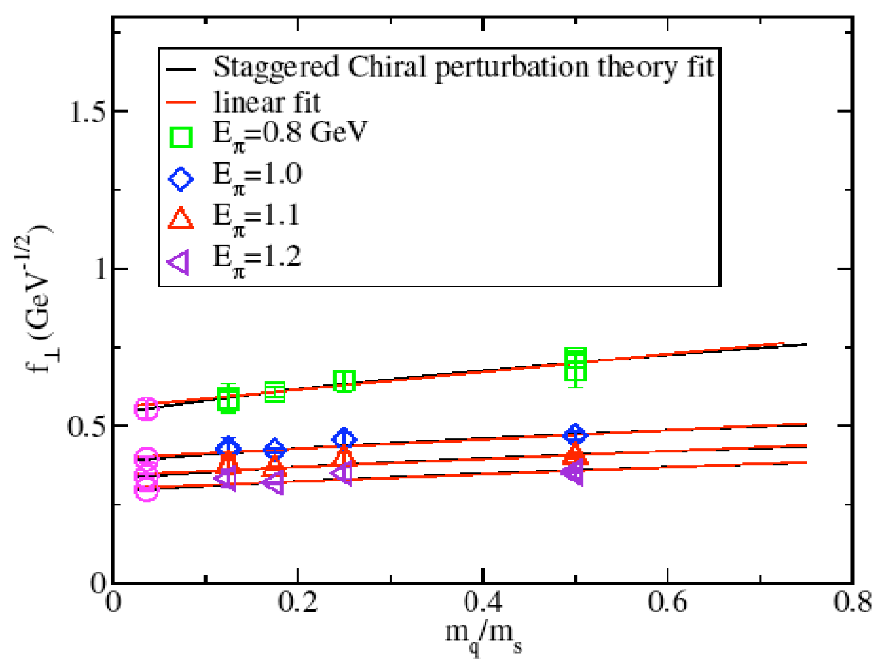
$$\langle D(p')|V^\mu|B(p)\rangle = \frac{h_+(w^2)(v^\mu + v'^\mu) + h_-(w^2)(v^\mu - v'^\mu)}{\sqrt{m_B m_D}}$$

Fits, fits, fits, and ...



1) Fit 3-point correlators
2) Interpolate to fixed E_π
3) Extrapolate in quark mass

$$\langle \pi | V^0 | B \rangle \; = \; \sqrt{2 m_B}\, f_{\|}$$
$$\langle \pi | V^k | B \rangle \; = \; \sqrt{2 m_B}\, p_\pi^k\, f_{\perp}$$

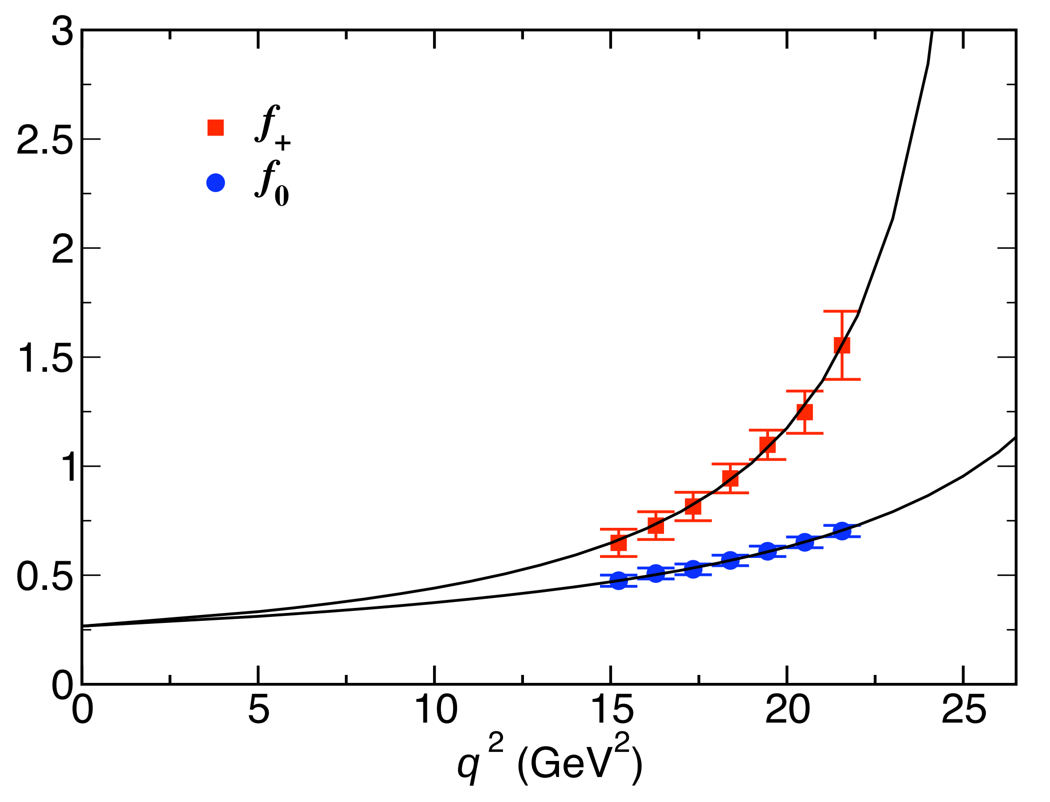


Form factor shape

Ball-Zwicky, 4-parameter Becirevic-Kaidalov

$$f_+(q^2) = \frac{f_+(0)}{1-\tilde{q}^2} + \frac{r\tilde{q}^2}{(1-\tilde{q}^2)(1-\alpha\tilde{q}^2)} \qquad \tilde{q}^2 \equiv q^2/m^2_{B^*}$$

$$f_0(q^2) = \frac{f_+(0)}{1-\tilde{q}^2/\beta}$$



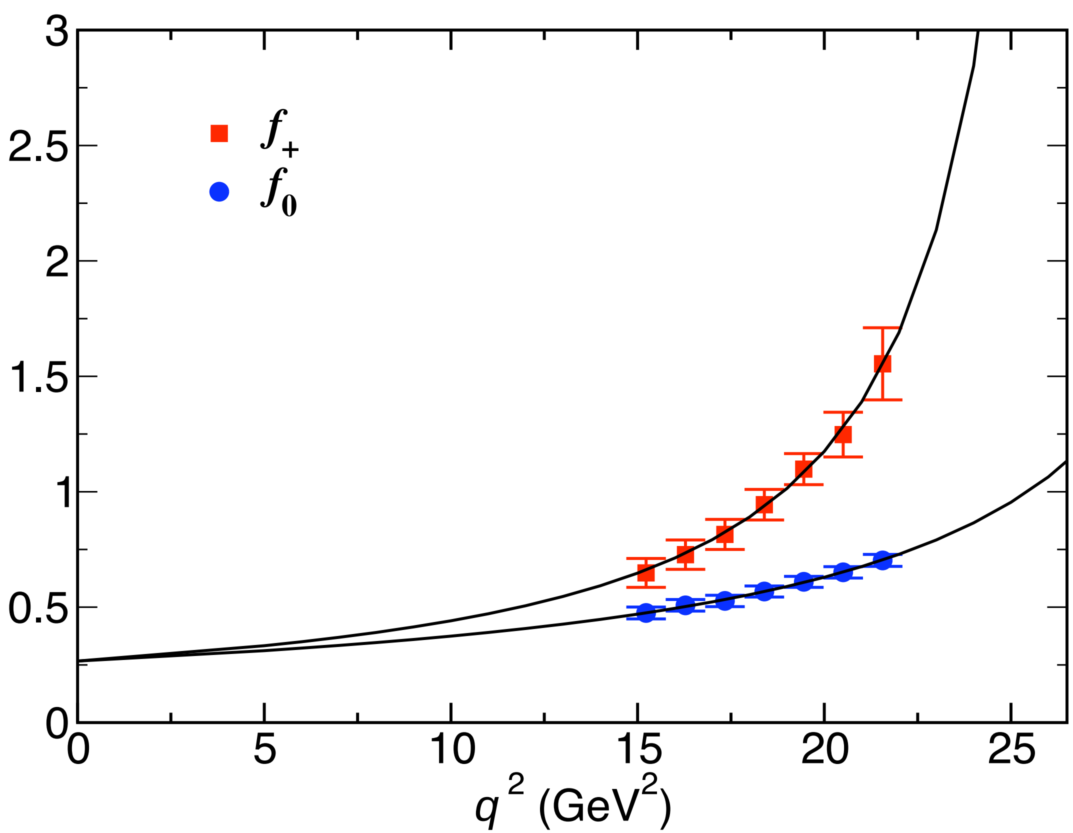
Talk by P. Mackenzie

Experiment + Lattice QCD

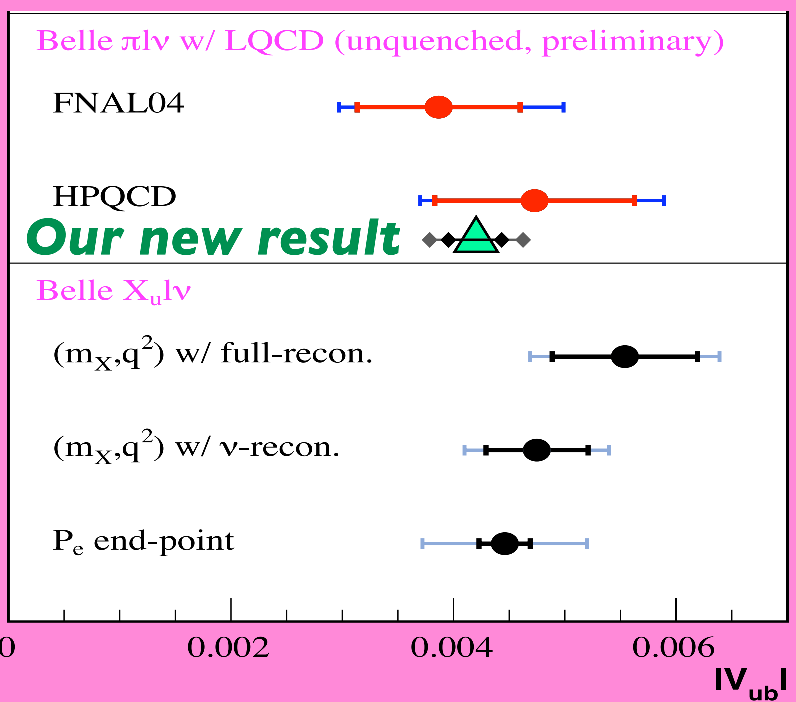
$$\frac{1}{|V_{ub}|^2}\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3}|\vec{p'}|^3|f_+(q^2)|^2$$

$$|V_{ub}| = (4.22 \pm 0.30 \pm 0.51) \times 10^{-4}$$

expt. LQCD

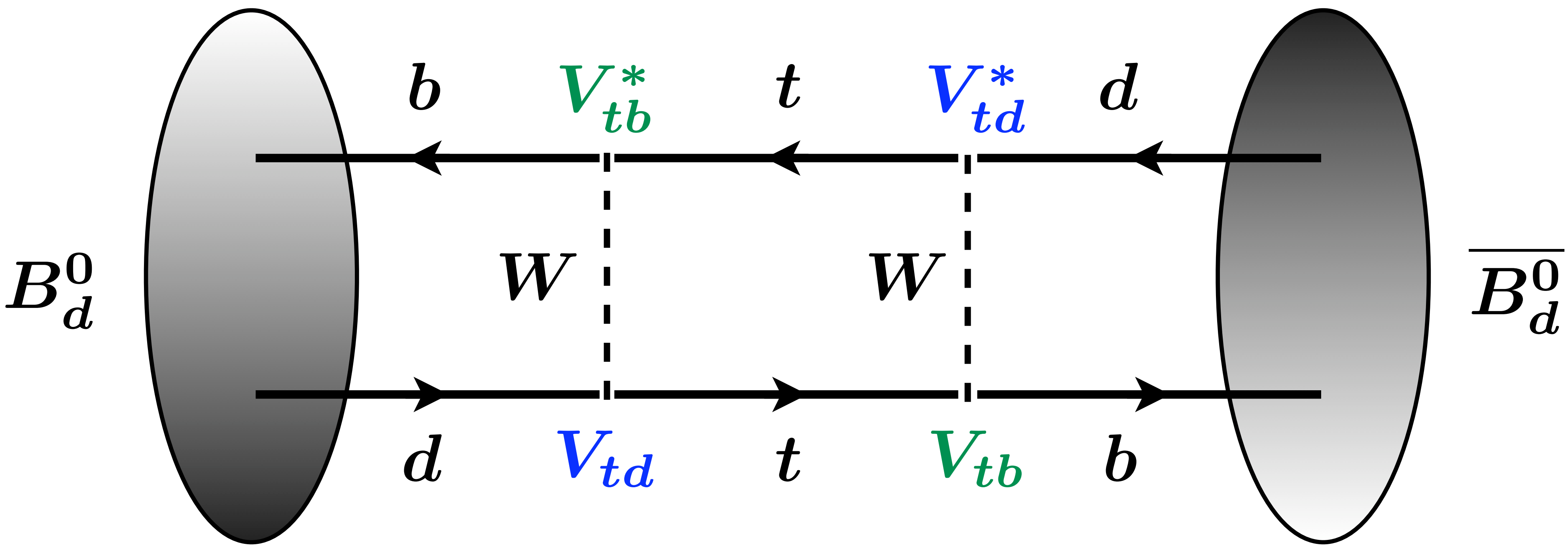


Belle, ICHEP 2004
hep-ex/0408145

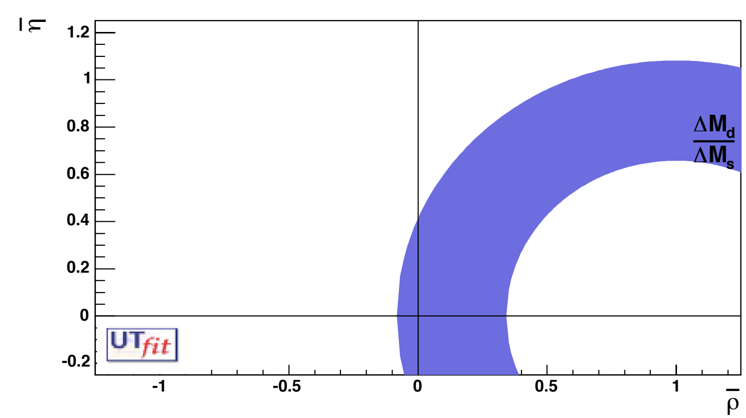


E. Gulez, A. Gray, M.W. *et al.*, hep-lat/0601021

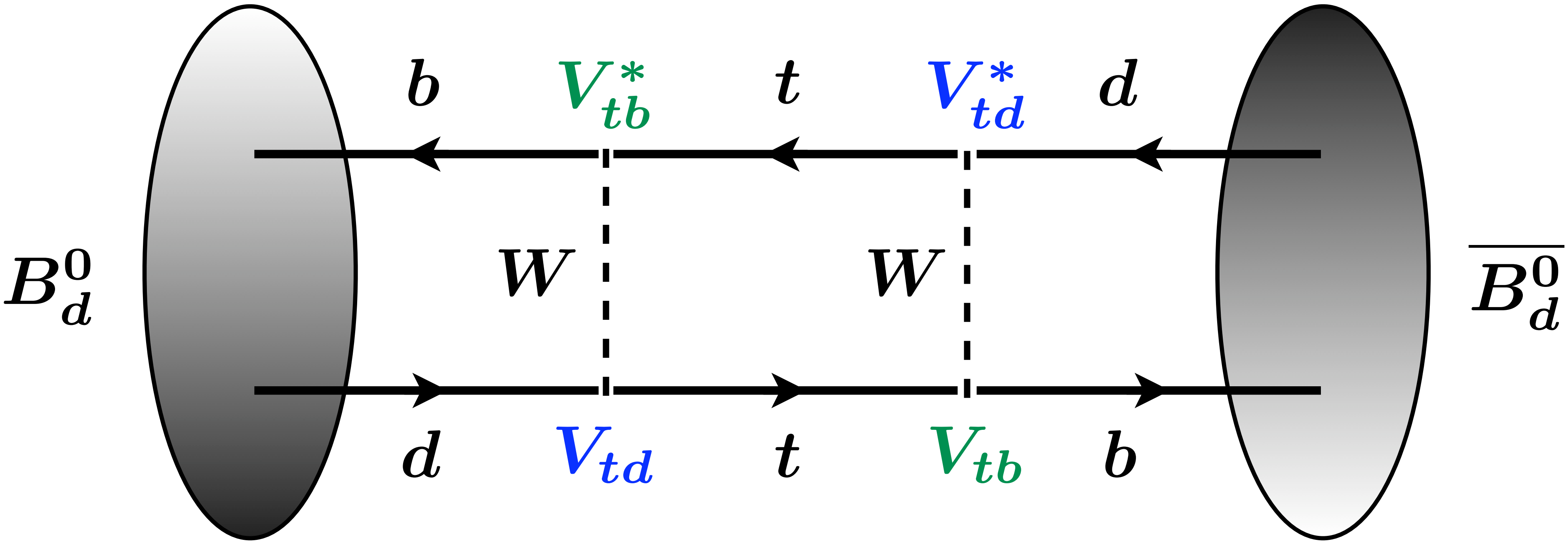
$B^0 - \overline{B^0}$ Mixing

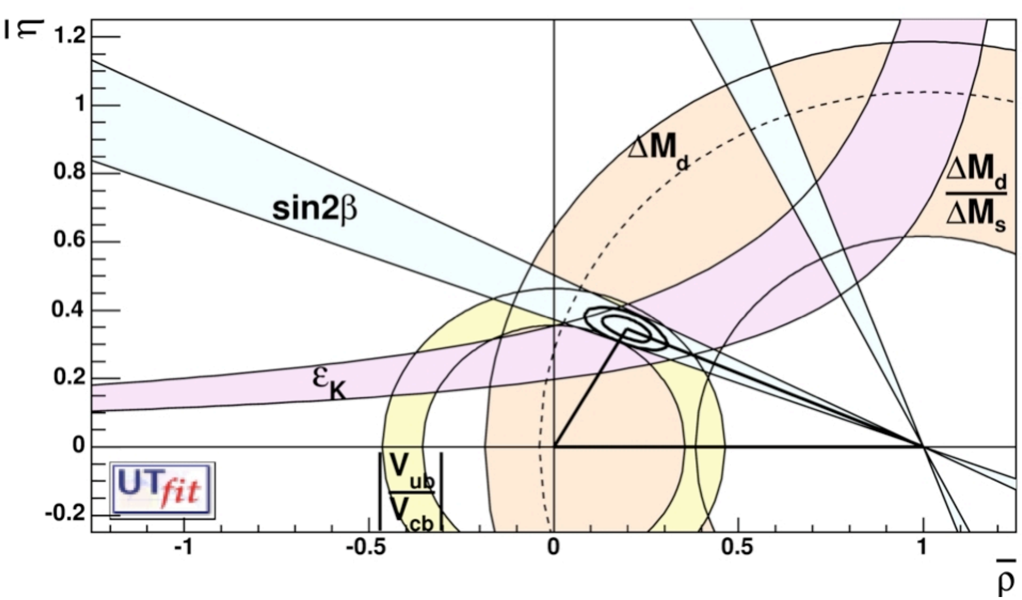


$$|V_{td}|^2 \propto [(1-\bar{\rho})^2 + \bar{\eta}^2]$$



$B^0 - \overline{B^0}$ Mixing





- Constraint complements $\sin 2\beta$ & $|V_{ub}|/|V_{cb}|$
- More likely ***New Physics*** contributions

$B^0 - \overline{B^0}$ Mixing

Only $B_d^0 - \overline{B_d^0}$

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_W^2 \eta_B S(x_t) m_{B_d} f_{B_d}^2 B_{B_d} |V_{td} V_{tb}^*|^2$$

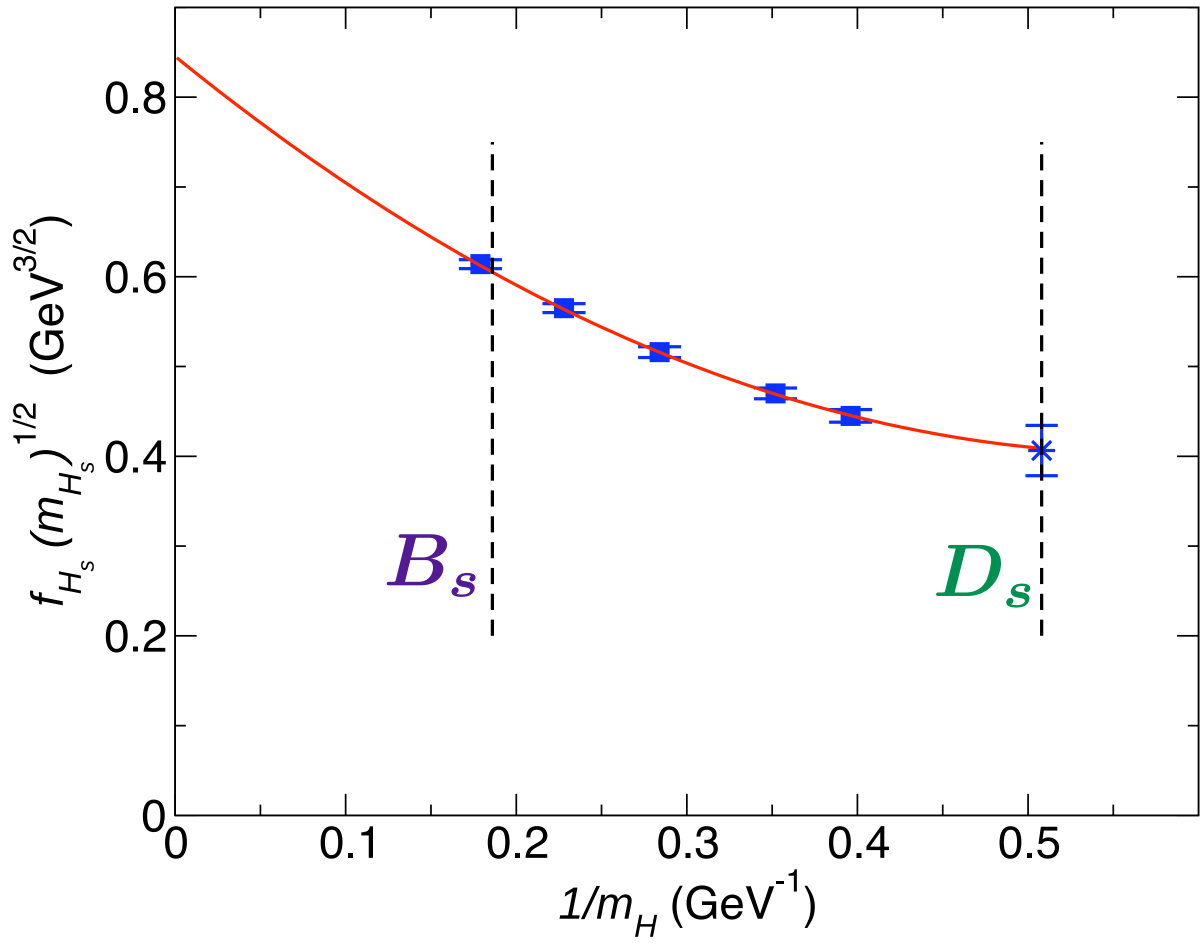
Including $B_s^0 - \overline{B_s^0}$

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \left| \frac{V_{ts}}{V_{td}} \right|^2$$

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

Most of the mass dependence, & simplest to compute on lattice

B_s decay constant

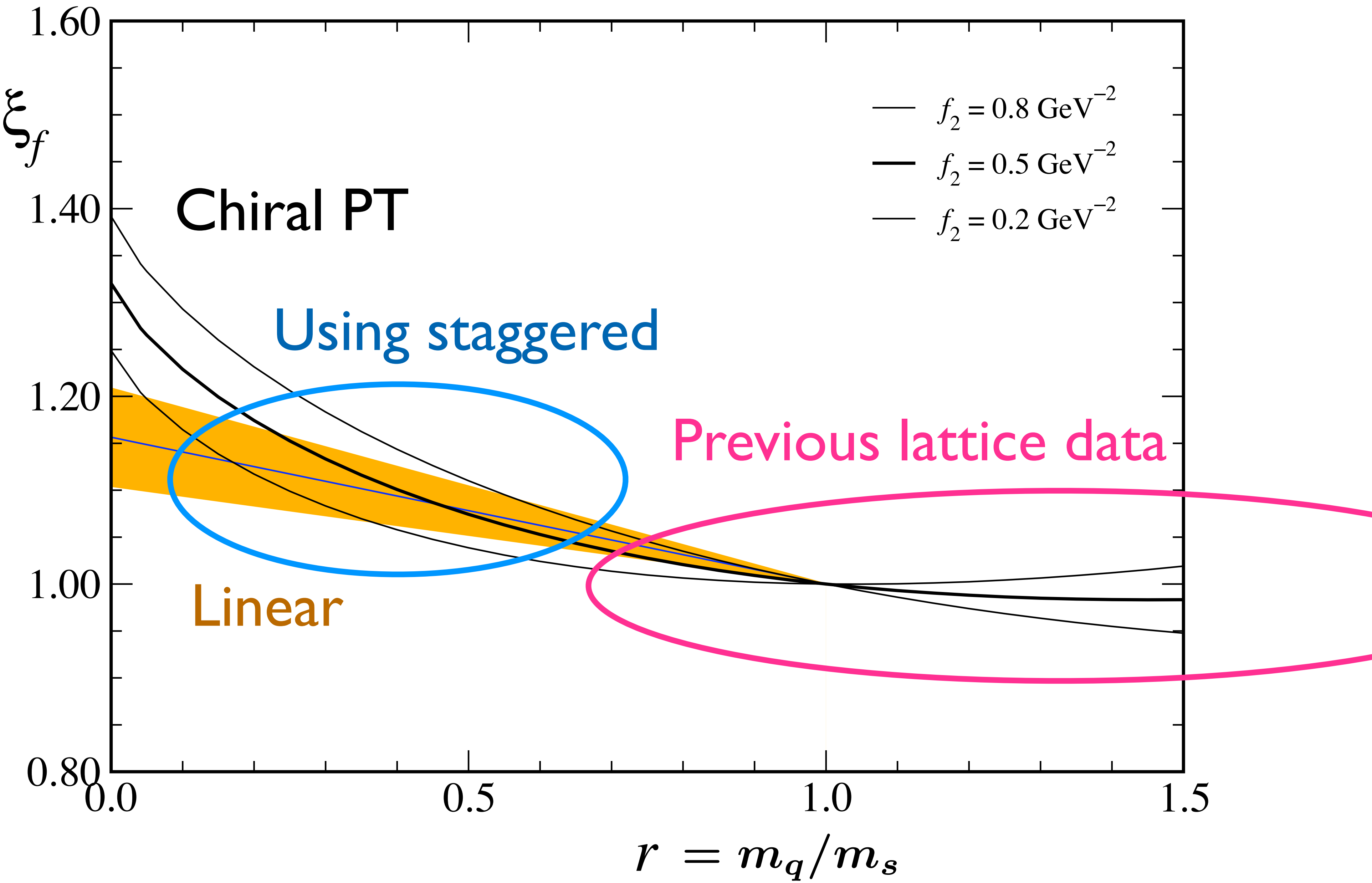


$$f_{B_s} = 260 \pm 7 \pm 26 \pm 8 \pm 5 \ \text{MeV}$$

stat perturb hq discr

M.W., *et al.* PRL 92, 162001 (2004)

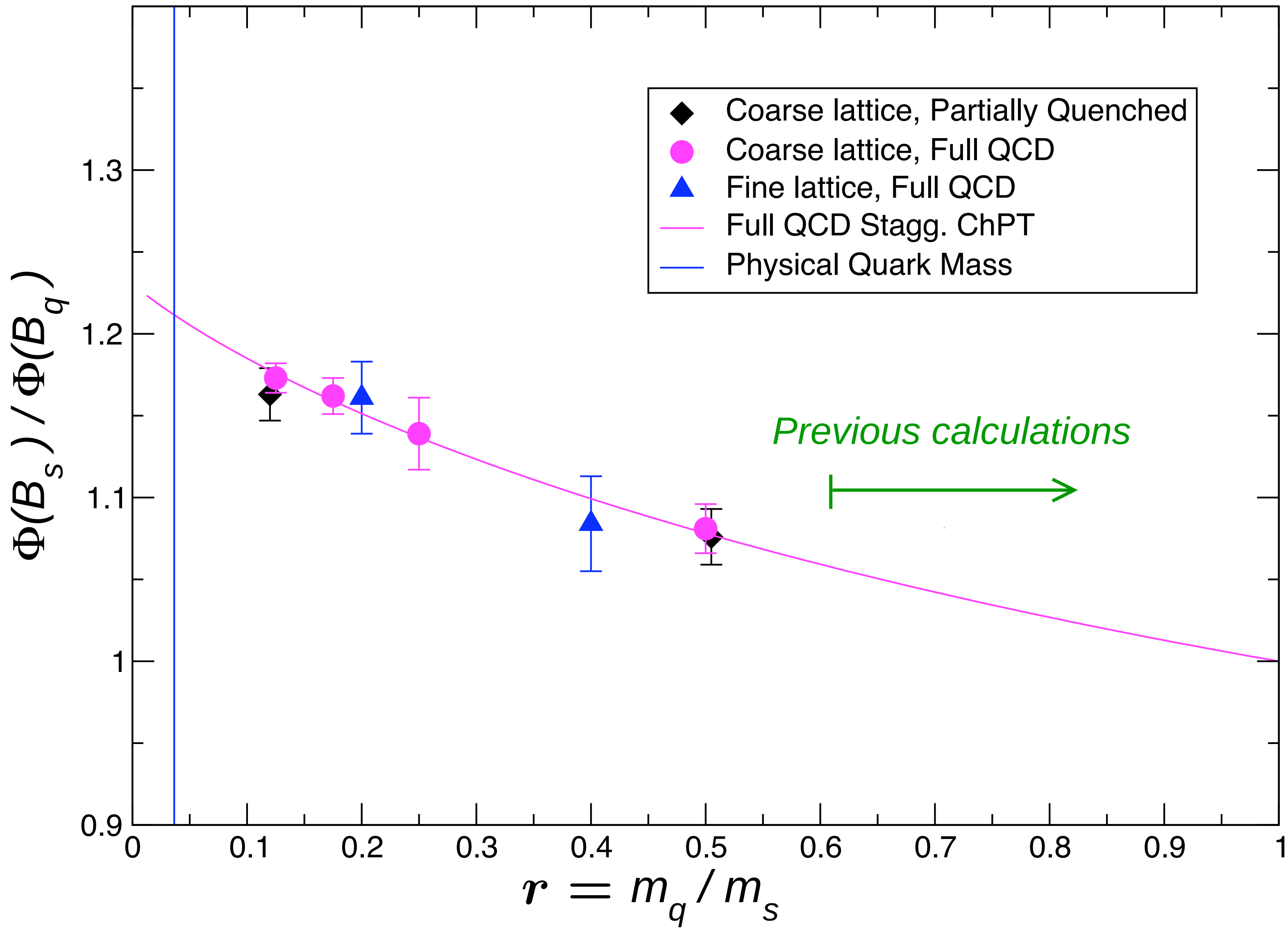
$B^0 - \overline{B^0}$ Mixing 2002



Kronfeld & Ryan, (also JLQCD)

$B^0 - \overline{B^0}$ Mixing 2005

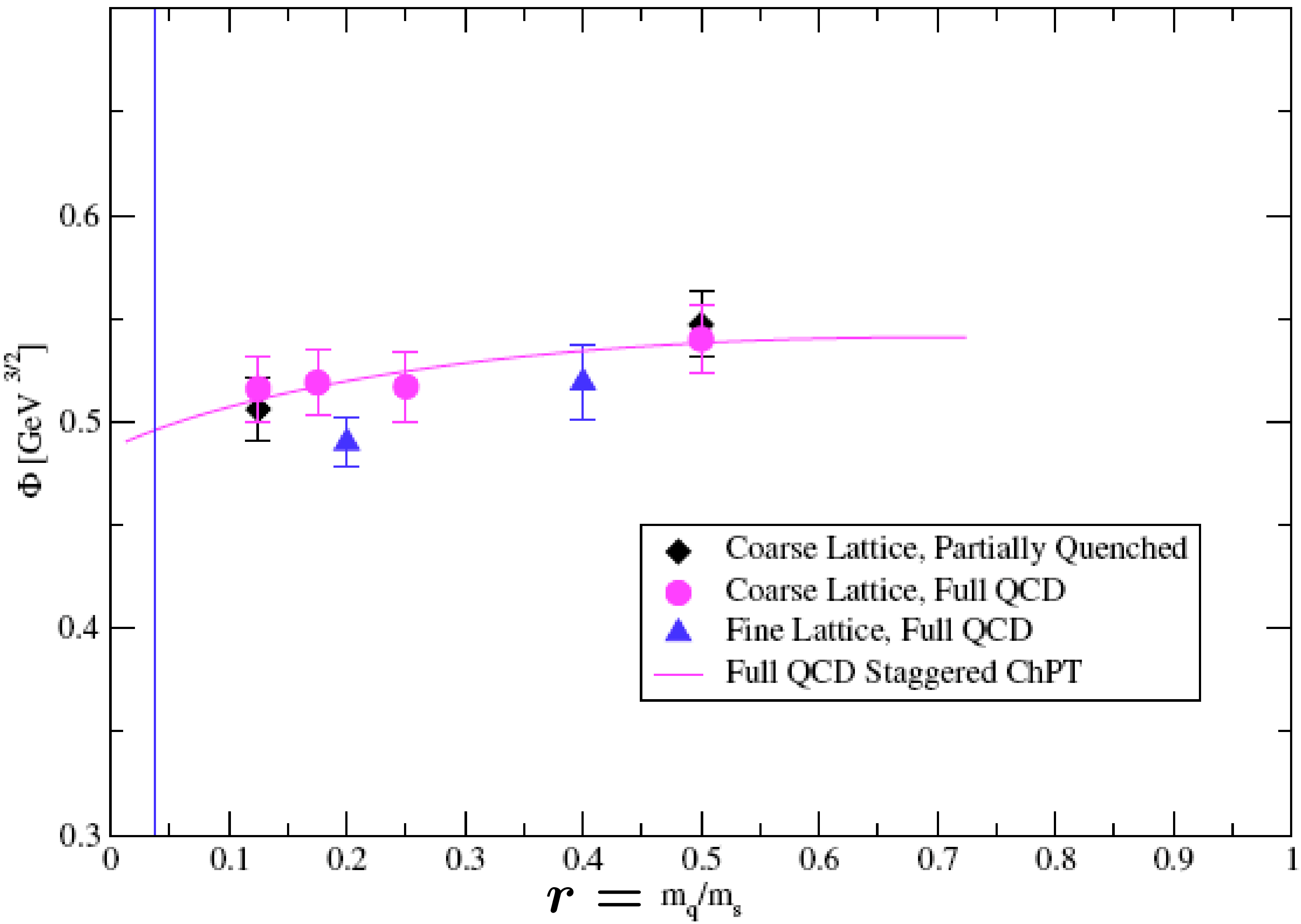
A. Gray, M.W. *et al* (HPQCD) PRL 95, 212001 (2005)



$\Phi(B) \equiv f_B \sqrt{m_B}$

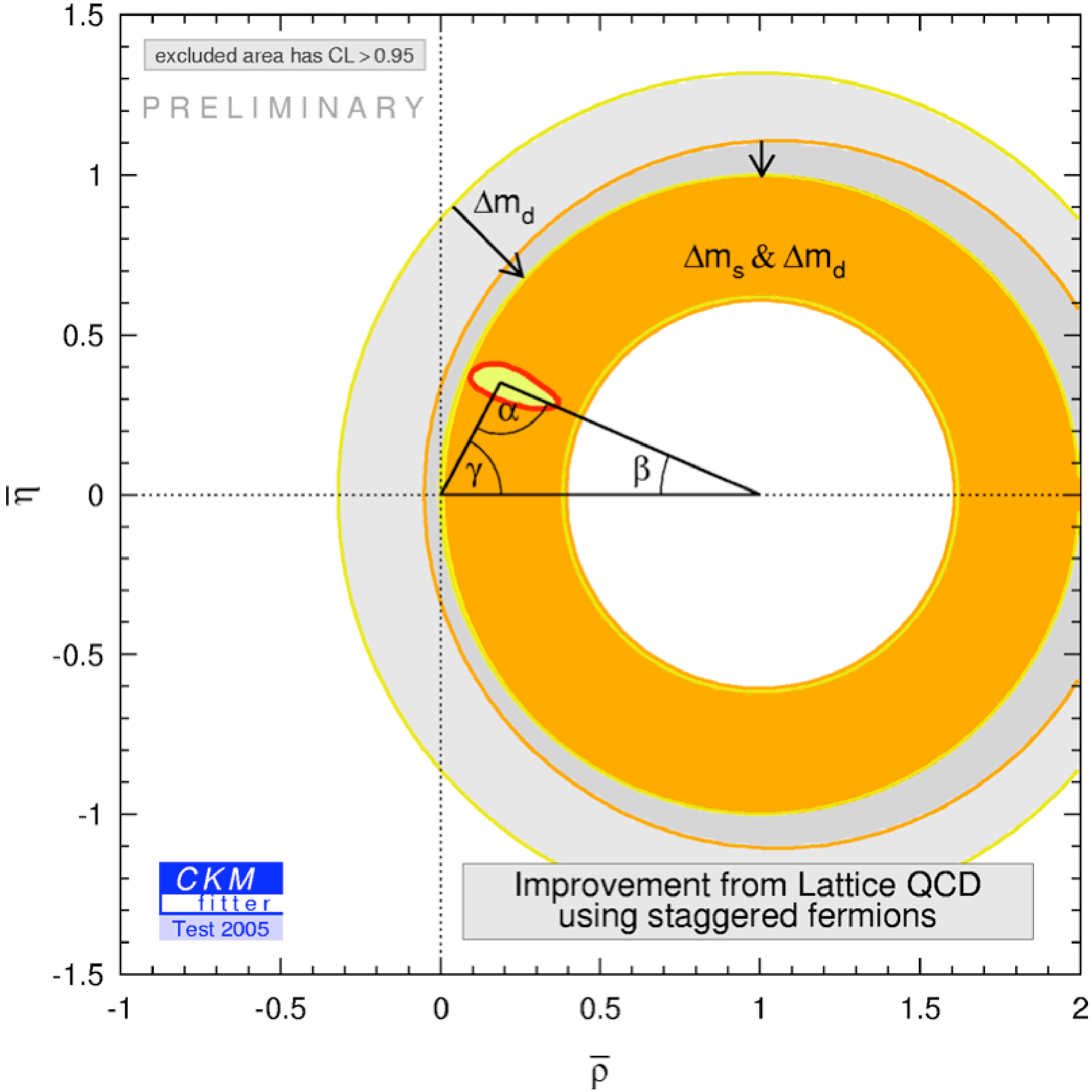
$B^0 - \overline{B^0}$ Mixing 2005

A. Gray, M.W. *et al* (HPQCD) PRL 95, 212001 (2005)



$\Phi(B) \equiv f_B \sqrt{m_B}$

Improvement in CKM constraints



2004

$$f_{B_d} = (200 \pm 28 \pm 9)\ \text{MeV}$$

$$\xi = 1.21 \pm 0.04 \pm 0.05$$

(Gaussian)(uniform)

CKMfitter, hep-ph/0406184

2005

$$f_{B_d} = (216 \pm 9 \pm 21)\ \text{MeV}$$

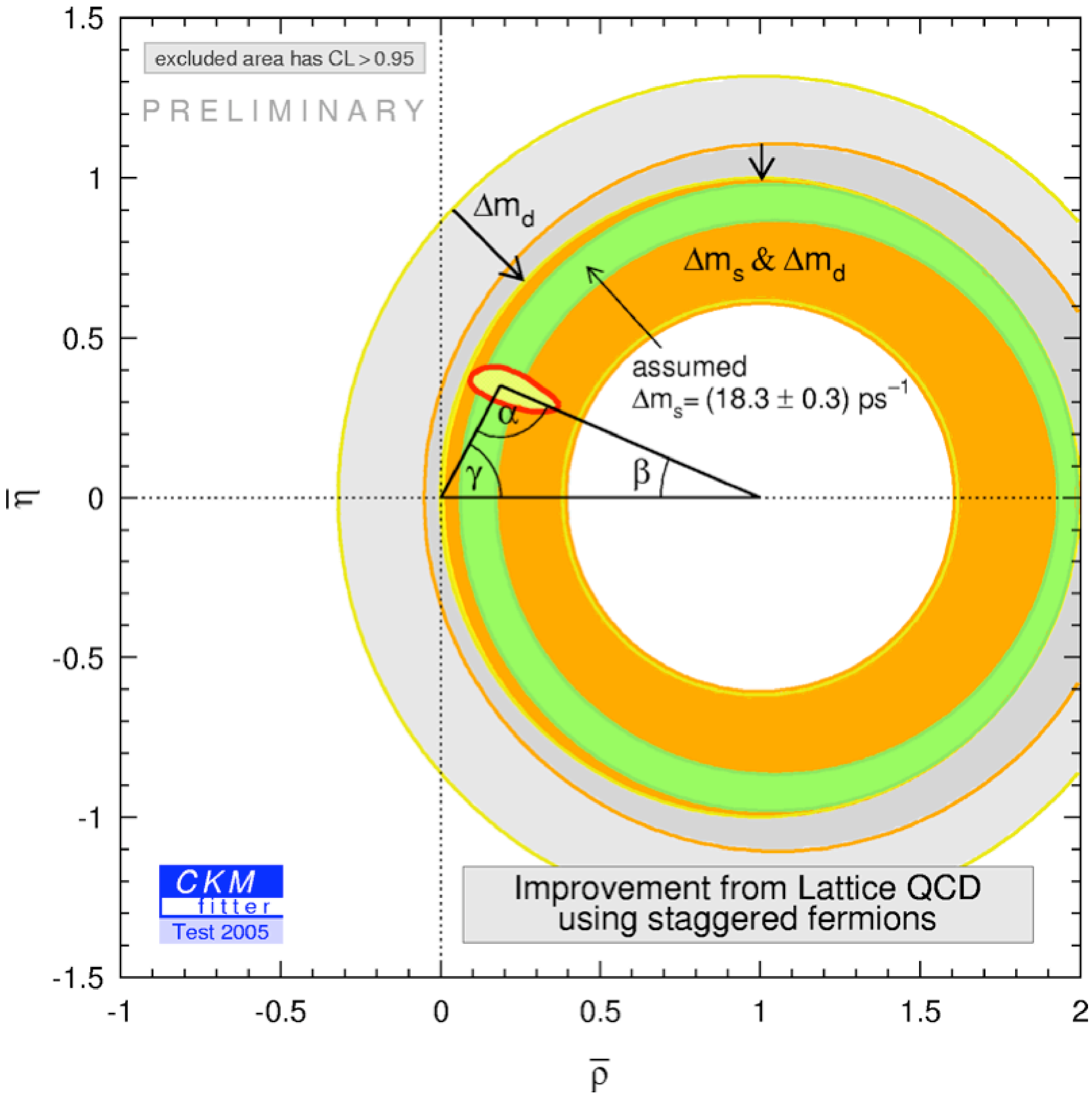
$$\xi = 1.21 \pm 0.03 \pm 0.01$$

(stat $+\chi$) (sys)

A. Gray, M.W. *et al* (HPQCD) PRL 95, 212001 (2005)

Plot from A. Hoecker (CKMfitter)

When $B_s^0 - \overline{B_s^0}$ mixing is observed



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CKMfitter, hep-ph/0406184

2005

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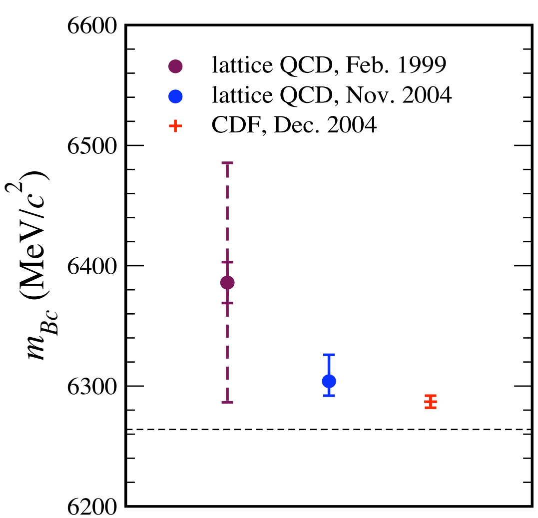
(stat $+\chi$) (sys)

A. Gray, M.W. *et al* (HPQCD)
PRL 95, 212001 (2005)

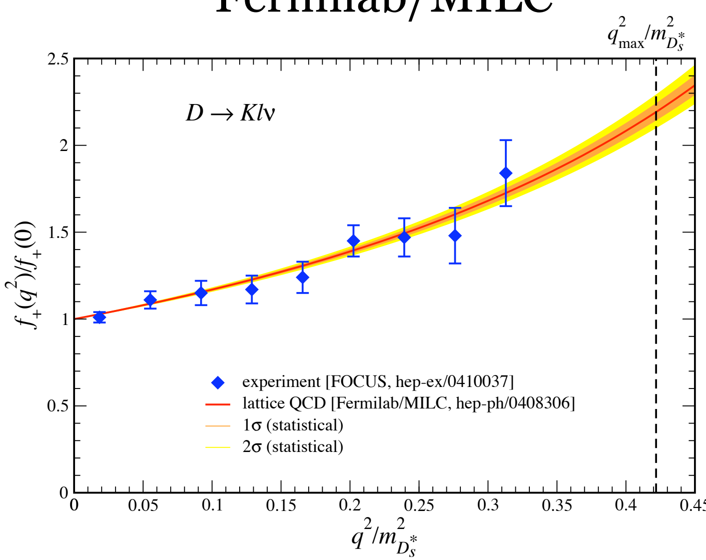
Other checks & predictions

- ☑ B_c meson mass (Fermilab/HPQCD LQCD; CDF expt)
- ☑ D meson decay constant (Fermilab/MILC LQCD; CLEO-c expt)
- ☑ $D \to K\ell\nu$ form factor (Fermilab/MILC LQCD; BES, FOCUS expt's)
- QCD coupling and quark masses

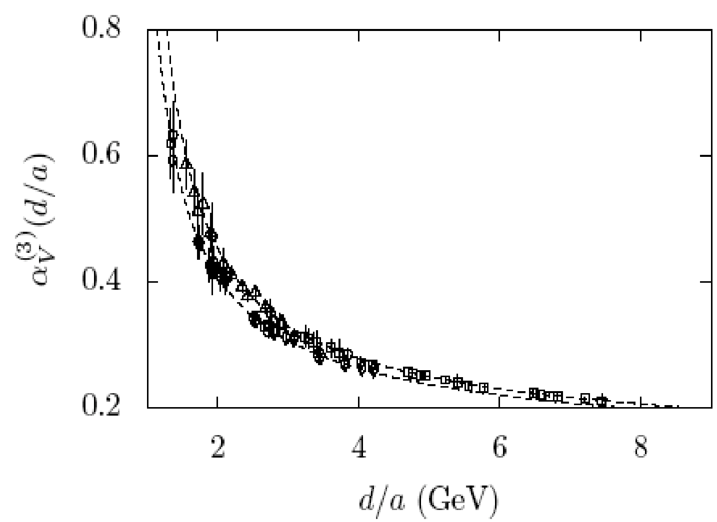
Fermilab/HPQCD



Fermilab/MILC



HPQCD



Current and future effort

- More masses on finer lattice
- Complete set of $\Delta B = 2$ matrix elements
- Extend perturbative matching to 2 loops (automation)
- Moving NRQCD to work at lower momentum transfer
  $B \to \pi \ell \nu \qquad B \to V \gamma$

Closing Remarks

- Recent improvements allow us to get precise results now.
- Heavy meson results are having an impact in flavor physics.
- Hard work ahead to further reduce uncertainties.
- Renormalization factors often leading uncertainty.
- Chiral extrapolation under much better control.