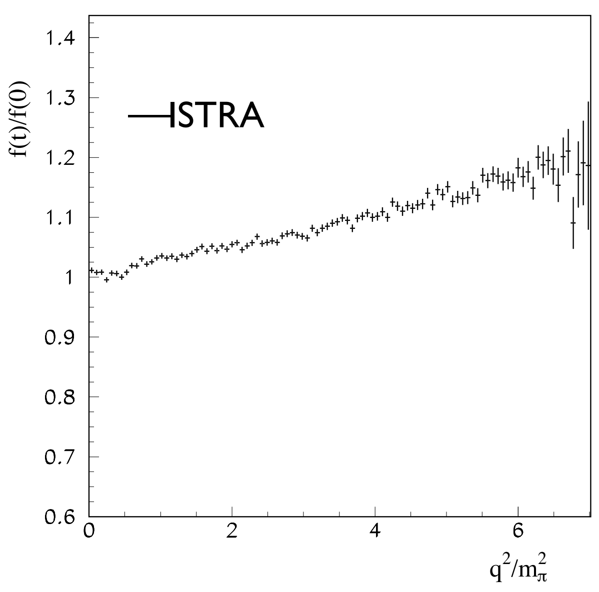
Unitarity and the heavy quark expansion in fits of semileptonic decay amplitudes on the lattice

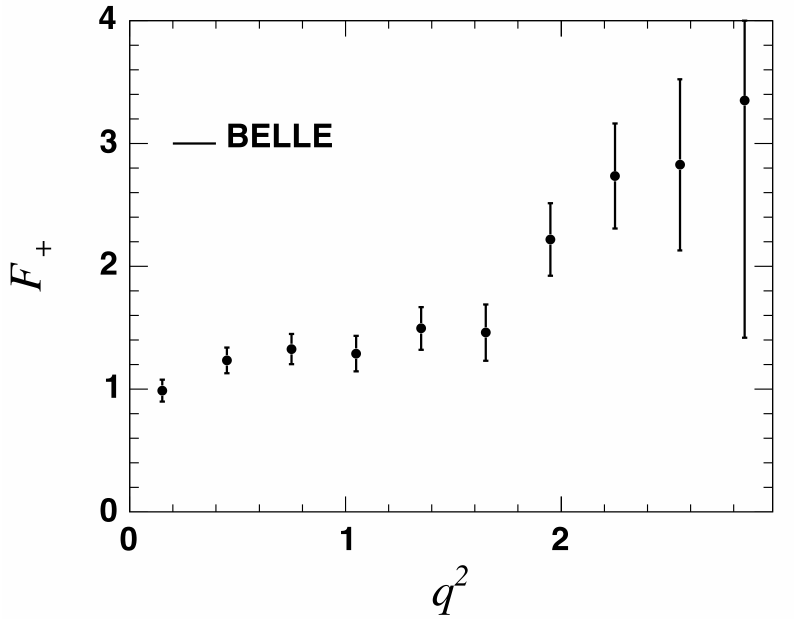
Paul Mackenzie
Fermilab
mackenzie@fnal.gov

4th ILFTN Workshop
湘南国際村
２００６年３月８－１１

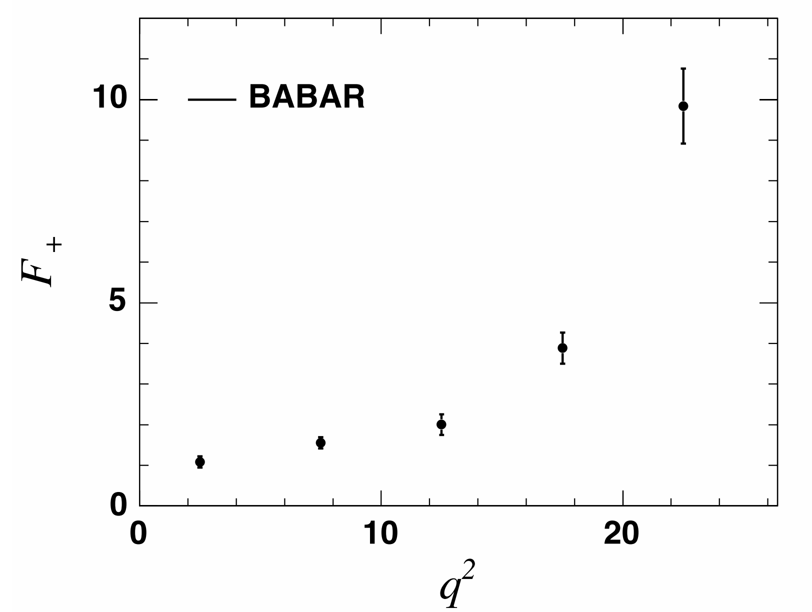
The problem



$K \to \pi \ell \nu$



$D \to \pi \ell \nu$



$B \to \pi \ell \nu$

- How much curvature is there in the semileptonic form factors?
- How many parameters are necessary to describe the data to a given accuracy (say 1%)?

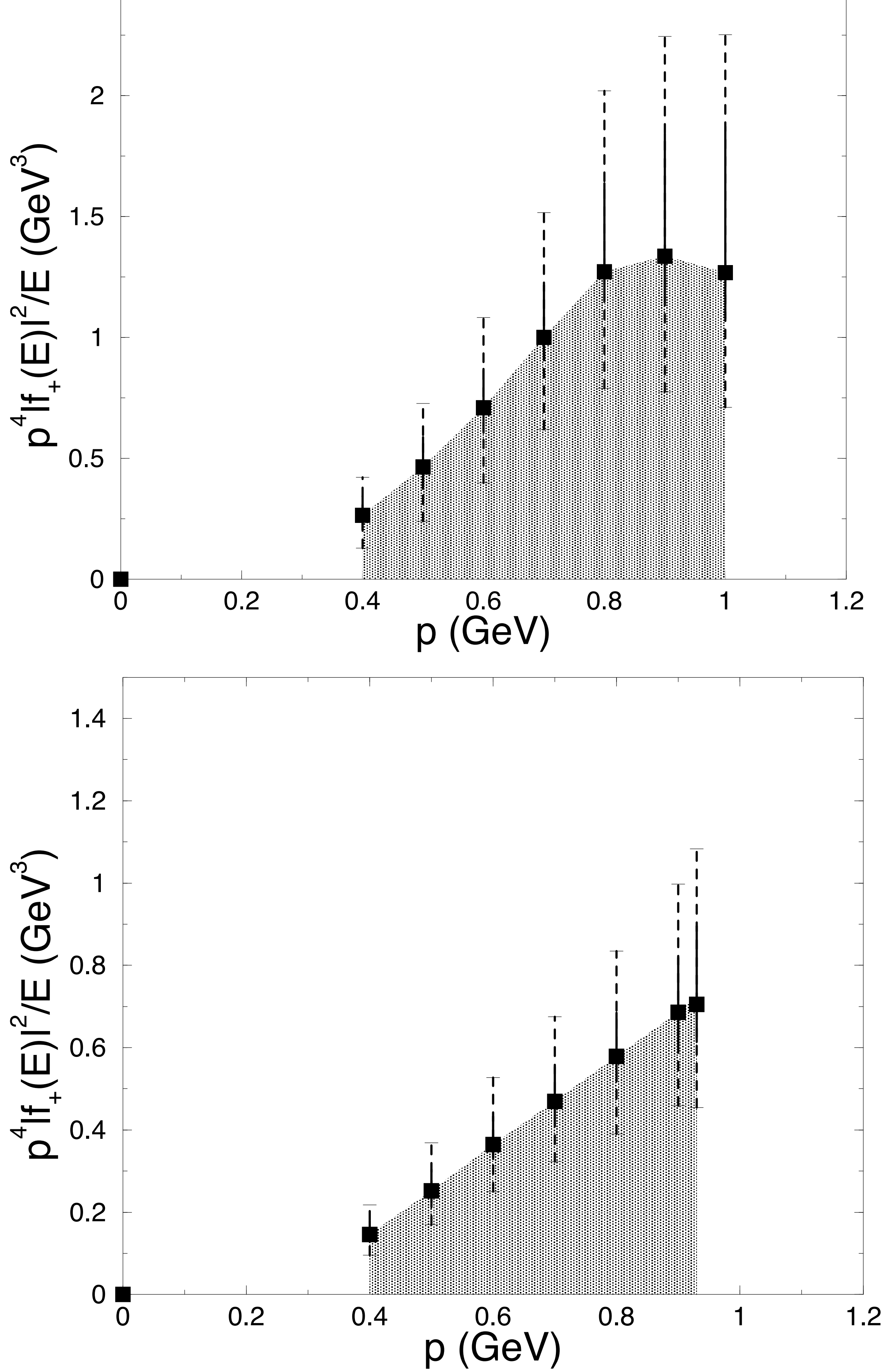


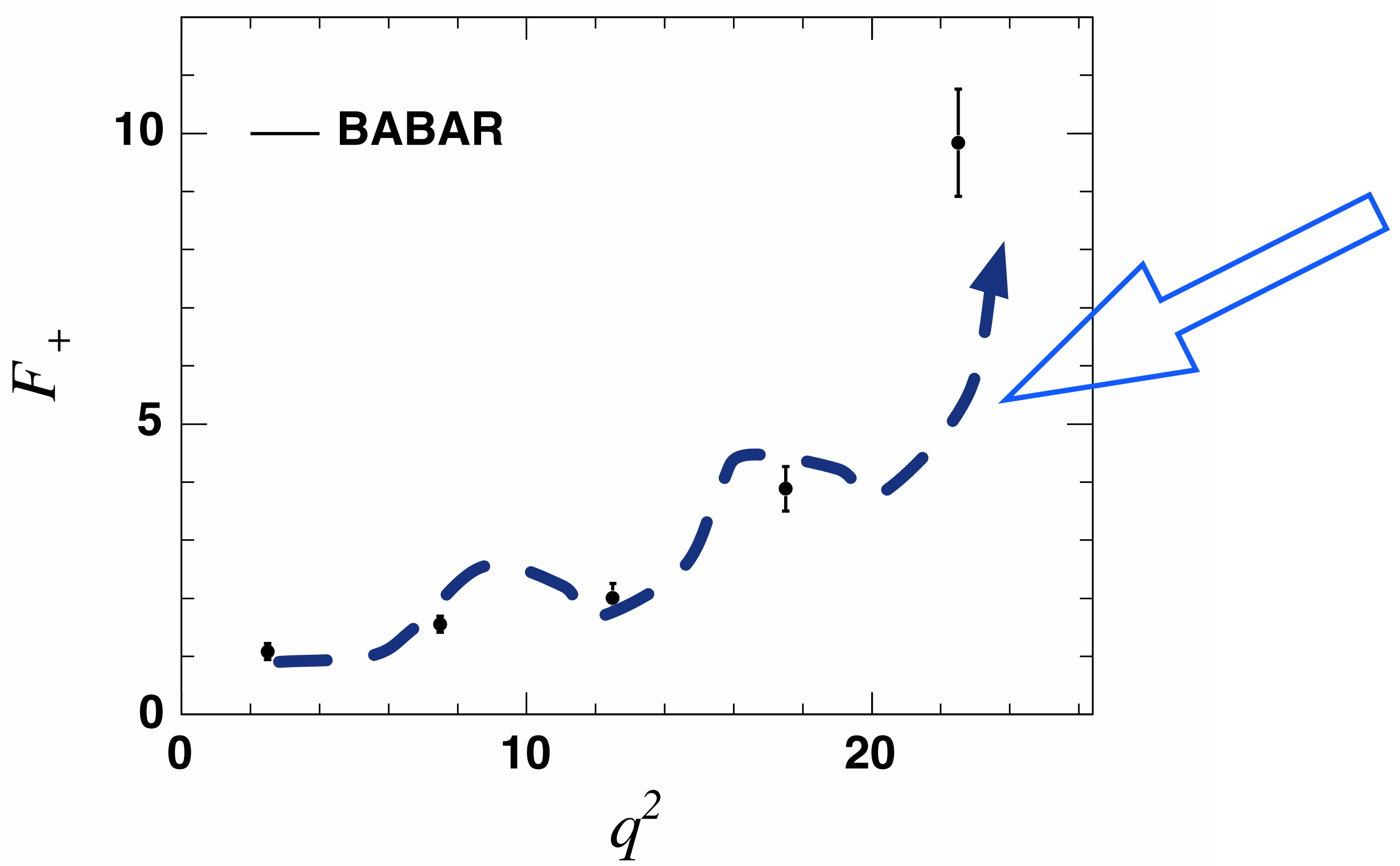
FIG. 1. The differential decay rate (without momentum-independent factors) as a function of $p = |\boldsymbol{p}_\pi|$, for (a) $B \to \pi l \nu$ and (b) $D \to \pi l \nu$. The solid error bars show the statistical uncertainty and the dotted ones show the sum in quadrature of statistical and systematic uncertainties.

In Fermilab calculation of *B→πlν,* we reported results in the region of recoil momentum (or equavalently, q2) where we understood our error bars.

We suggested comparing theory and experiment only in this region.

Was this conservatism necessary?

El-Khadra et al., Phys. Rev. D64:014502, 2001.



Physical intuition says that shapes with wild oscillations are implausible.

How can such intuition be made quantitative?

Analyticity and unitarity

have been used by many authors to investigate constraining form factors:

C. Bourrely et al., Nucl. Phys. B189, 157(1981).

C. G. Boyd, B. Grinstein and R. F. Lebed, Phys. Rev. Lett. 74, 4603(1995); Nucl. Phys. B461, 493(1996).

M. Fukunaga and T. Onogi, Phys. Rev. D71, 034506(2005).

L. Lellouch, Nucl. Phys. B479, 353 (1996).

C. G. Boyd and M. J. Savage, Phys. Rev. D56, 303 (1997).

M. C. Arnesen, B. Grinstein, I. Z. Rothstein and I. W. Stewart, Phys. Rev. Lett. 95, 071802 (2005) [hep-ph/0504209].

...

... and work dating back to early measurements of $K \to \pi l \nu$.

In this talk:

- Implement the formalism of Arnesen, Grinstein, Rothstein, and Stewart.
  - Particularly transparent implementation of the effects of unitarity.
  - Involves no additional work in fitting beyond that normally done.
- Supplement with results of heavy quark theory, following Becher and Hill (Phys. Lett. **B633:** 61-69, 2006; hep-ph/0509090).
  - Improve the bounds on form factors dramatically, explain the smallness of some parameters.

Outline

- Introduction
- Fermilab/MILC semileptonic review
- Becirevic-Kaidalov function
- Application of Arnesen et al. formalism to lattice data

Thanks Iain Stewart.

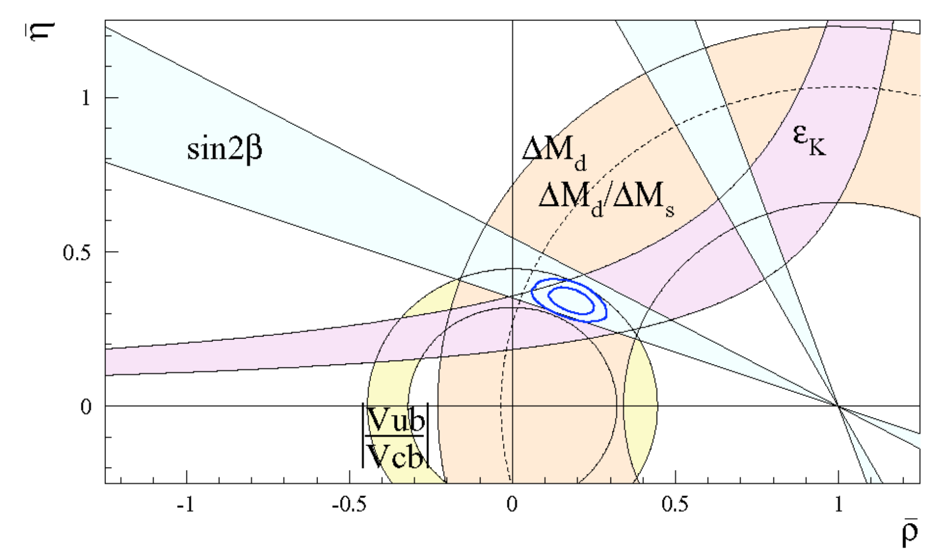
- Becher and Hill results from heavy quark physics

Thanks Richard Hill for conversations and figures.

CKM physics and the Fermilab/MILC heavy-light program

See the review of Masataka Okamoto, Lattice 2005, hep-lat/0510113.

- Because of their importance to CKM determinations, semileptonic decay amplitudes have attracted interest from many approaches:
  - Lattice
  - Sum Rules
  - HQET
  - SCET
  - ...



Important to combine efforts to obtain the most powerful constraints possible.

Fermilab/MILC heavy-light program

Staggered light quarks and Fermilab heavy quarks.
MILC staggered unquenched gauge configurations.

5 lattice spacings.
On each, $m_s >= m_l < m_s/8$.
On each, 500-600 configurations generated (or to be generated).

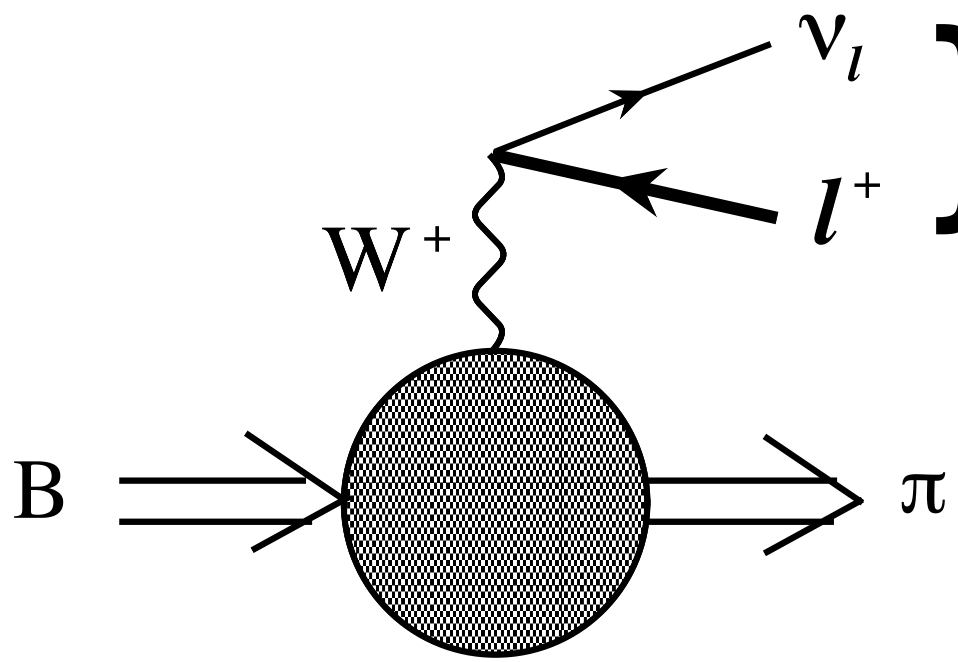
Smallness of the light quark mass allows smaller uncertainties from chiral extrapoltaions than before.

a (fm)		Status
0.18		Being analyzed
0.15		Ready to be analyzed.
0.125 (coarse)		Main published results.
0.09 (fine)		Being analyzed
0.06		Being generated

- Configurations generated many places: supercomputer centers, QCDOC, ...
- Analysis done (mostly) at Fermilab.
  - 2/3 teraflop installed last year.
  - 2 teraflops being installed this summer.

Completion of leptonic and semileptonic analysis on the “fine” (a=0.09 fm) lattices is currently consuming the bulk of our computer time.

Semileptonic Decay



$$q^2 = m_B^2 + m_\pi^2 - 2m_B E$$

$$\frac{d\Gamma}{dp} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{2m_B p^4 |f_+(E)|^2}{E}$$

$$\langle \pi(p_\pi) | \mathcal{V}^\mu | B(p_B) \rangle = f_+(E) \left[p_B + p_\pi - \frac{m_B^2 - m_\pi^2}{q^2} q \right]^\mu + f_0(E) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

Bercirevic-Kaidalov parameterization of shape

Parameterize other physics above the threshold.

Properly includes the B* pole

$$f_+(q^2) = \frac{f(0)}{(1-q^2/m^2_{D*})(1-\alpha q^2/m^2_{D*})}$$

$$f_0(q^2) = \frac{f(0)}{(1-q^2/m^2_{D*}/\beta)}$$

Helps simplify chiral extrapolation on the lattice.

We used BK to simplify the analysis. Amplitudes were interpolated to fiducial momentum values using BK. Then, the interpolated amplitudes were extrapolated to the chiral limit.

Bercirevic-Kaidalov parameterization of shape

$$f_+(q^2) = \frac{f(0)}{(1-q^2/m_{D*}^2)(1-\alpha q^2/m_{D*}^2)}$$

$$f_0(q^2) = \frac{f(0)}{(1-q^2/m_{D*}^2/\beta)}$$

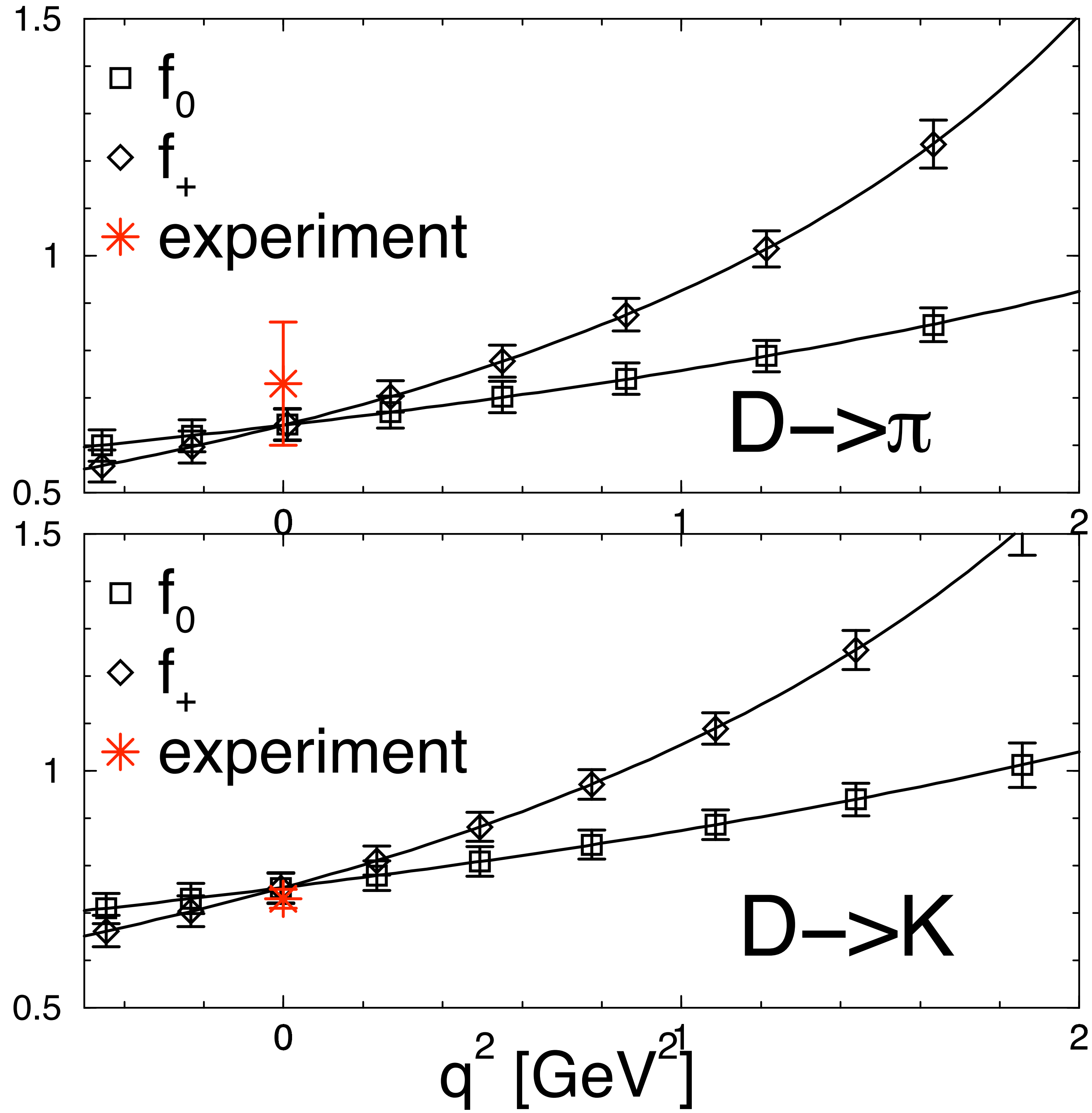
BK includes the leading pole properly, but
α and β are simply parameterizations of higher poles and cuts.

-) Introduces hard to estimate model dependence.
-) Inconsistent with the known high-recoil dependence of the form factors? (Richard Hill.)

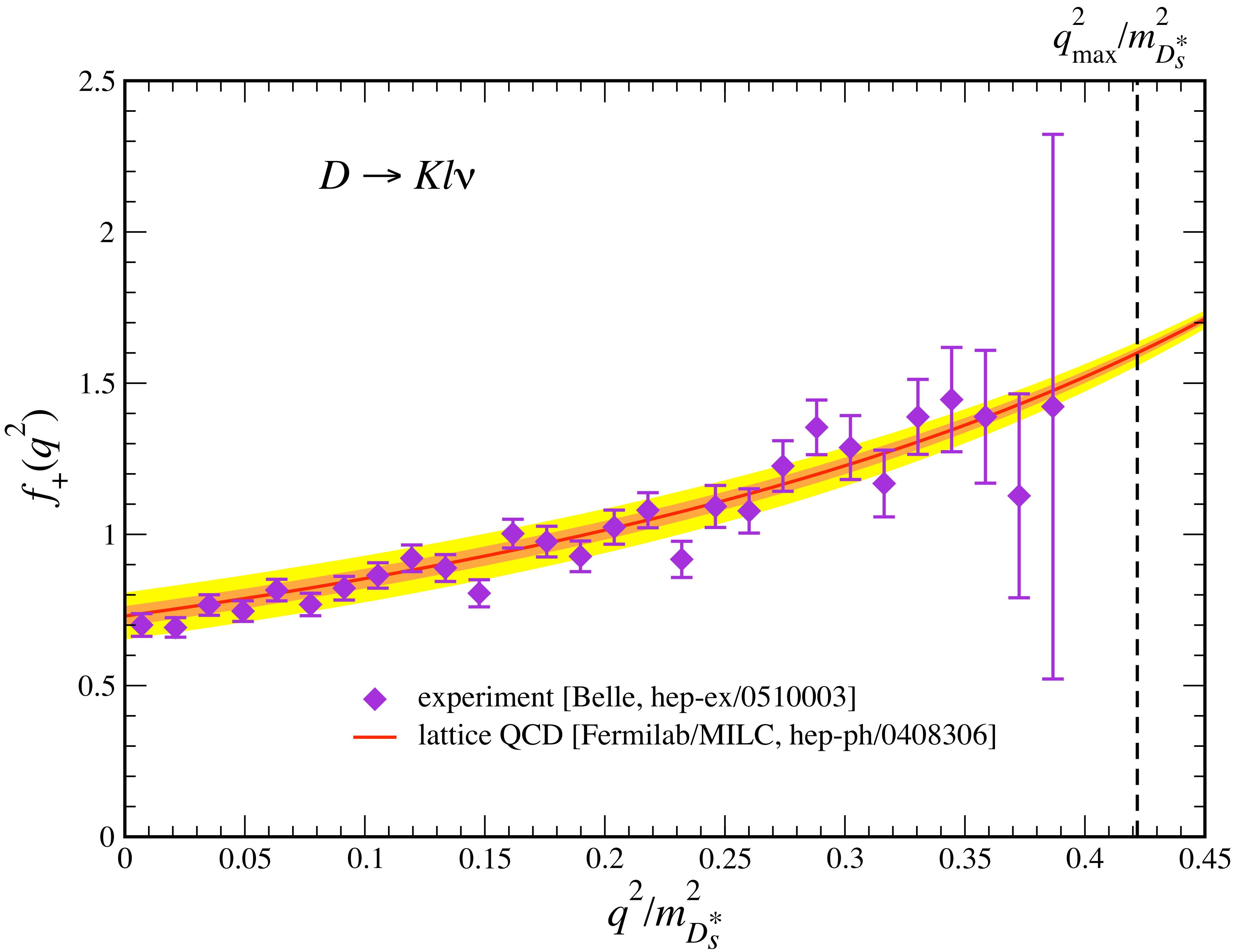
Results

Assumes standard CKM values.

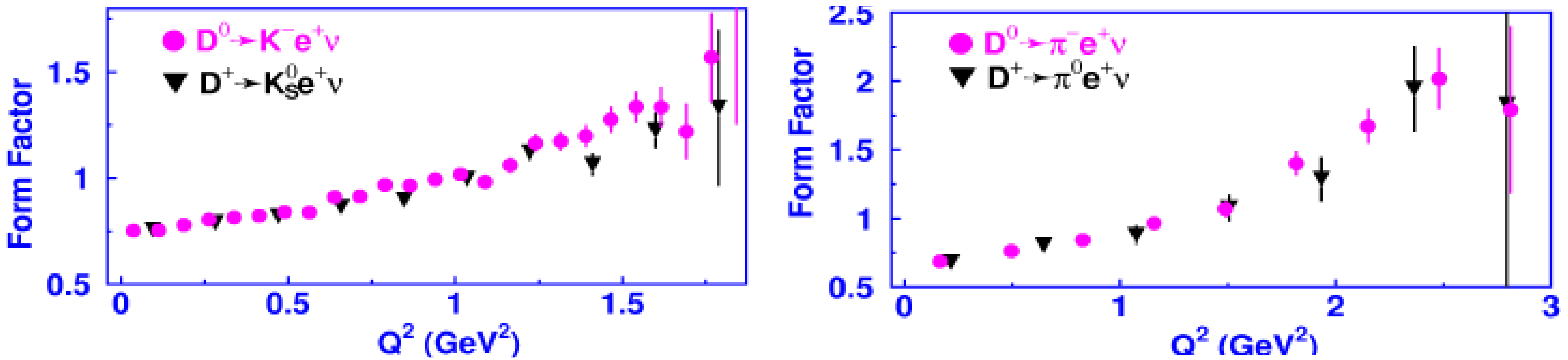
Focusing on D physics at first because of its timeliness with the new CLEO results.



Comparison of form factor shape with previous experiment



CLEO will measure *D* meson form factor shapes to much higher precision than previously.

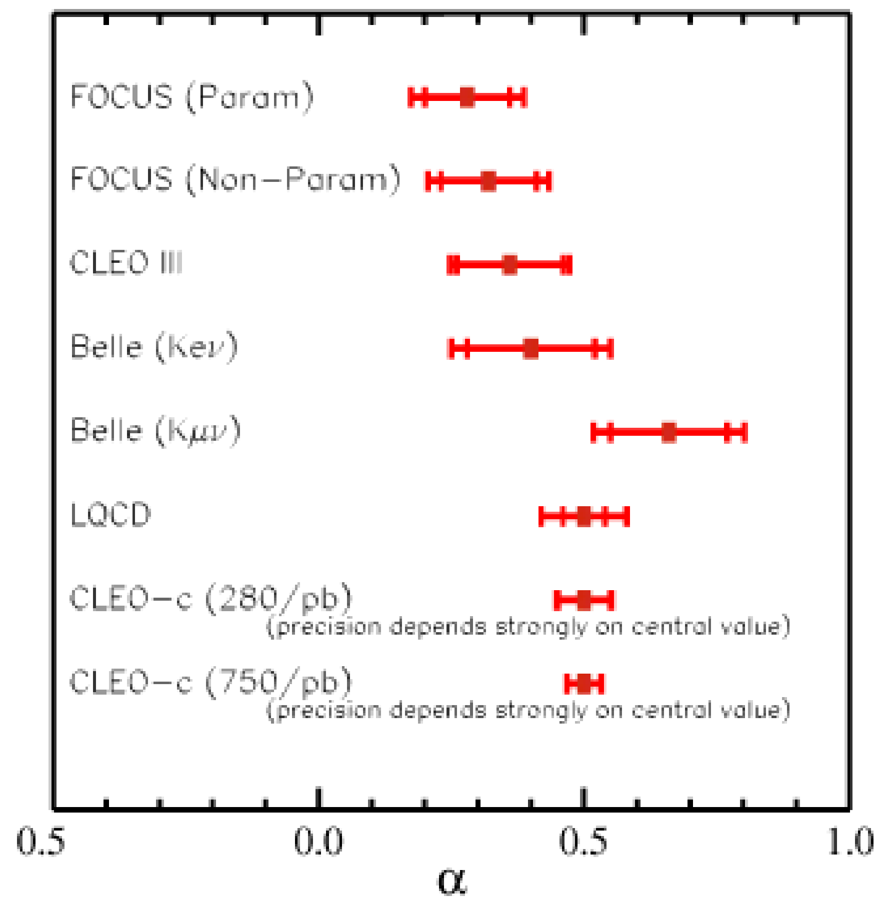


Analysis due “soon”.

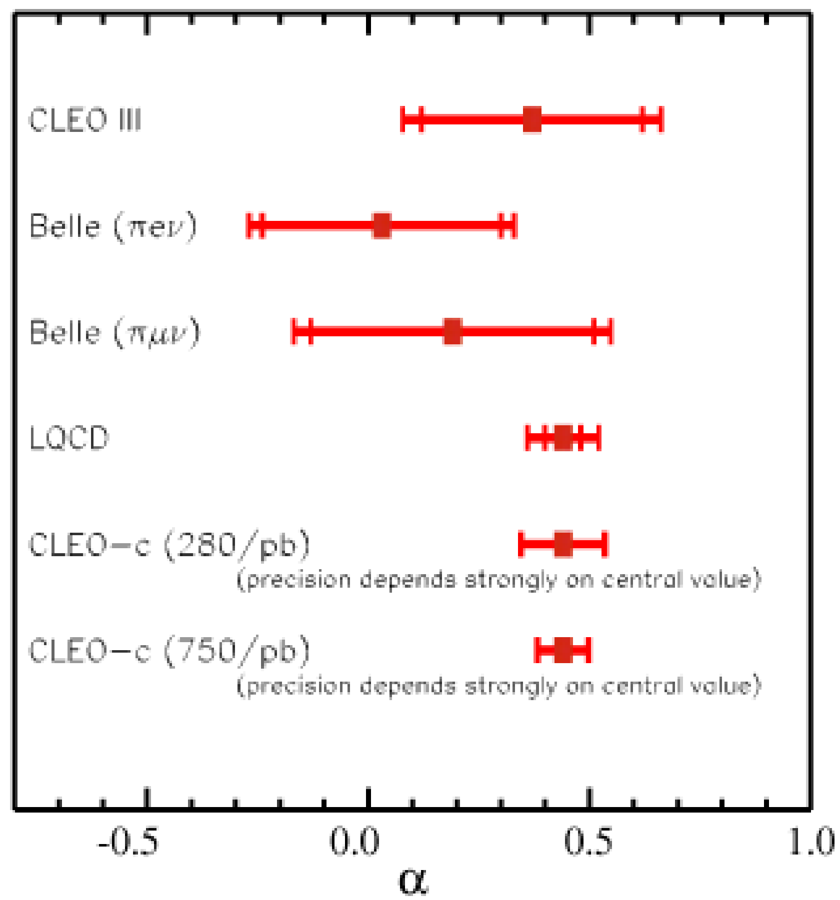
Expected CLEO-c precision

They currently report in terms of the BK shape parameter α (not the best choice).

$D \to Ke\nu$



$D \to \pi e\nu$

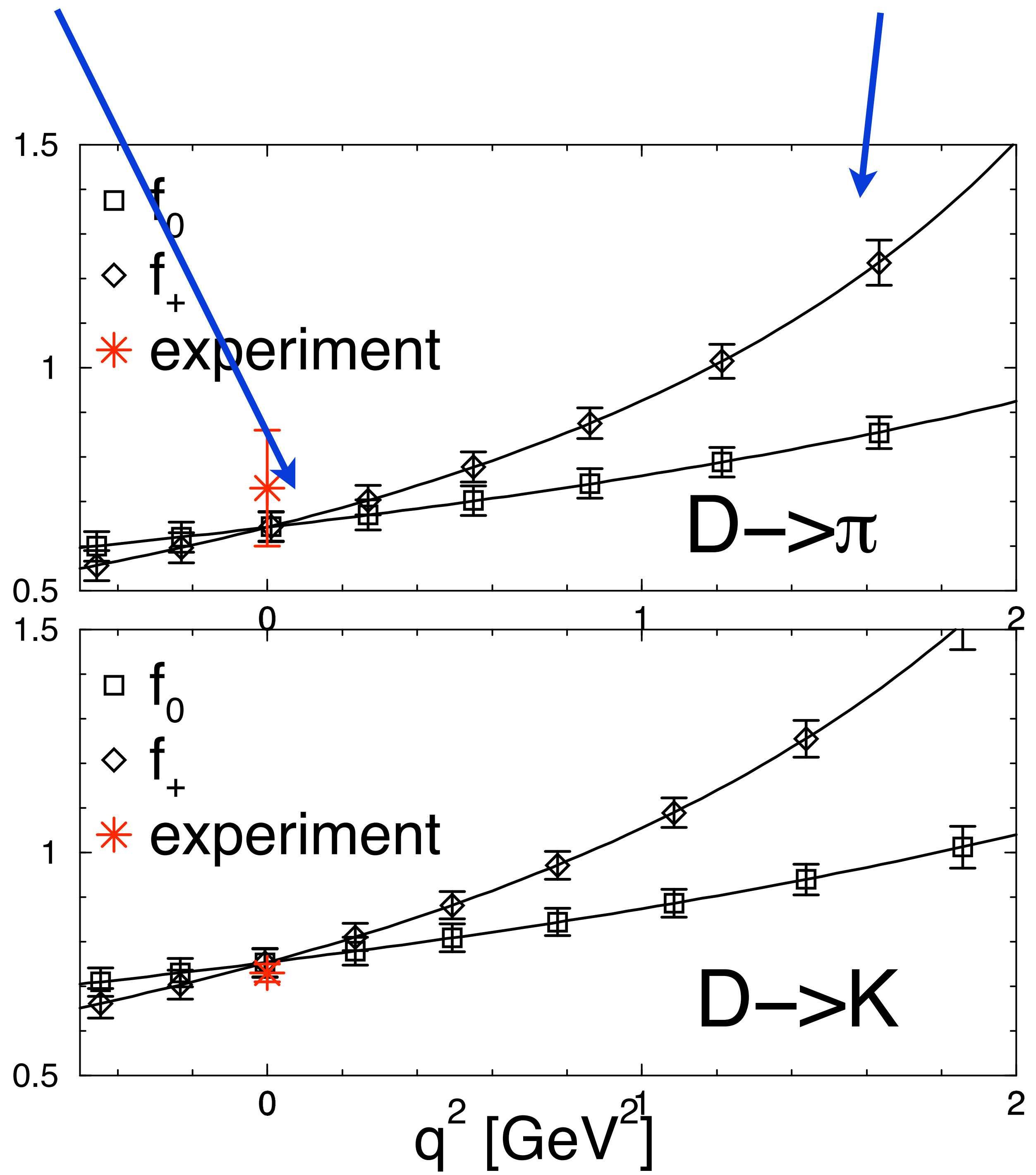


A good challenge for the lattice.

Our current understanding of the shape is model dependent.

Uncertainty should be larger at
high pion momentum than at low pion momentum.

The BK model thinks it understands the shape.



Unitarity bounds for form factors

Following the discussion of Arnesen et al., but the ideas go back to many others.

Kaon semileptonic form factors are very well described by a normalization and a slope. It's been known for a long time that this is implied by unitarity.

What about other meson semileptonic decays.

Consider a remapping of the semileptonic decay variable $t=q^2$ into a new variable z in the complex plane:

$$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

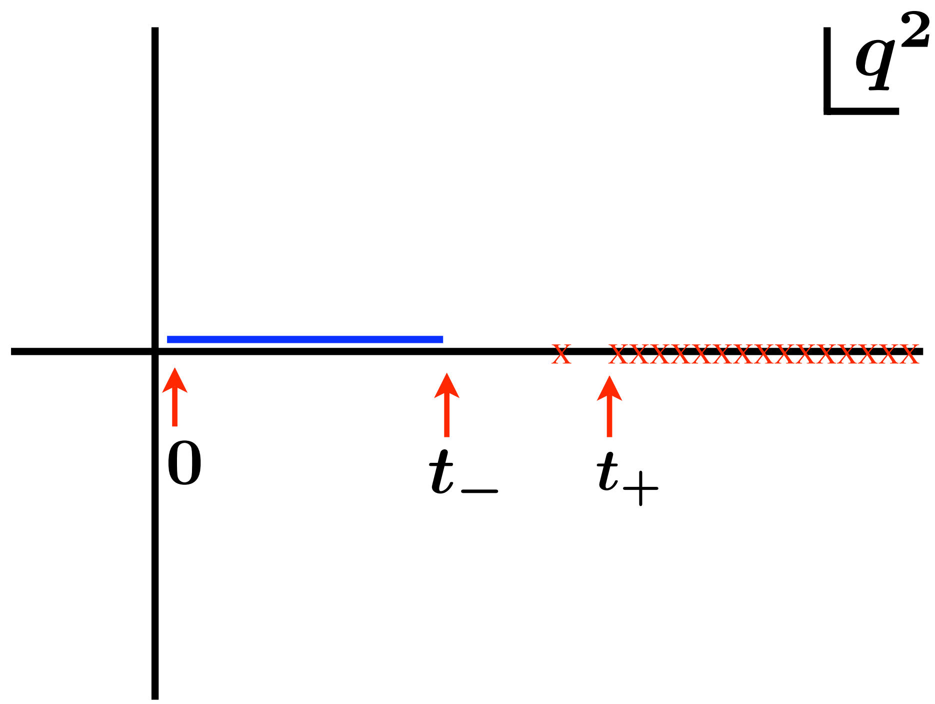
z maps $q^2=t>t_+$ onto $|z|=1.$, and
$t<t_+$ onto [-1,1] in the complex plane.

($t = (p_H - p_L)^2$, $t_+ = (m_H+m_L)^2$, $t_- = (m_H-m_L)^2$).

t_0, taken as 0.65 t_- here, is a fudge factor adjusted to center the physical region on $z\sim0$.

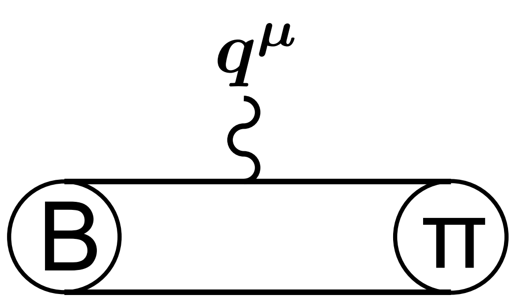
Analyticity

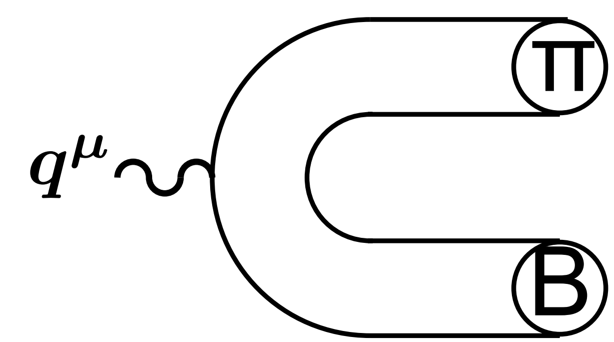
$F(q^2)$ analytic except when $q^2=m^2$ of physical state:



semileptonic region
(B→π decay)

production region
(Bπ production)

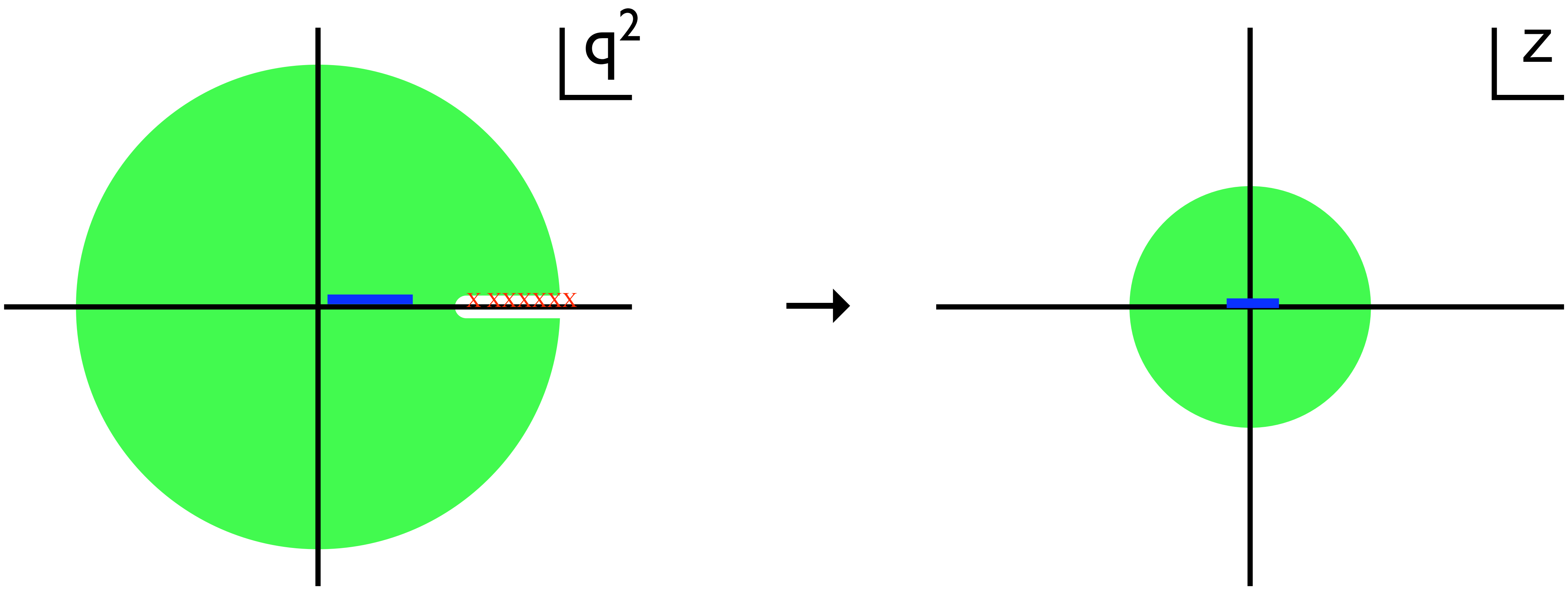




$$0 < q^2 < t_- \equiv (m_B - m_\pi)^2$$

$$q^2 > t_+ \equiv (m_B + m_\pi)^2$$

z maps the physical region into a region centered at 0.



$$z = \frac{\sqrt{1 - q^2/t_+} - \sqrt{1 - t_0/t_+}}{\sqrt{1 - q^2/t_+} + \sqrt{1 - t_0/t_+}}$$

What are the physical regions of z for the various decays?

For the various semileptonic decays, the physical region, $0<t<t_-$, is mapped into

B->π *l* ν: -0.34<*z*<0.22,
D->π *l* ν : -0.17<*z*<0.16,
D->*K* *l* ν : -0.04<*z*<0.06,
B->*D* *l* ν : -0.02<*z*<0.04.

When unitarity is taken into account, the smallness of the physical range of *z* will constitute a small parameter that limits the number of parameters required to describe the form factors to a given accuracy.

A power series expansion of the form factors in z can be written in the form:

$$f(t) = \frac{1}{P(t)\,\phi(t, t_0)} \sum_{k=0}^{\infty} a_k(t_0)\; z(t, t_0)^k$$

Function that has unit norm at *z=1*., and that vanishes at the poles of *f*, e.g., at the *B** pole.

Arbitrary function calculated in perturbation theory to produce a simple form for the a_k.

P and φ contain most of the complexity of the form factors.

A power series expansion of the form factors in z can be written in the form:

$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$

(for f+) $P(t) = z(t; m_{B^*}^2)$ Builds in effects of B* pole without spoiling unitarity constraints.

$$\phi(t,t_0) = \sqrt{\frac{n_I}{K\chi_J^{(0)}}}\left(\sqrt{t_+-t}+\sqrt{t_+-t_0}\right)\frac{(t_+-t)^{(a+1)/4}}{(t_+-t_0)^{1/4}} \times\left(\sqrt{t_+-t}+\sqrt{t_+}\right)^{-(b+3)}\left(\sqrt{t_+-t}+\sqrt{t_+-t_-}\right)^{a/2},$$

Obtained from PT to give the a's a simple form.

nI=1.5, a=3, b=2
K=48π^2

$$\chi_{f_+}^{(0)} = \frac{3\left[1+1.140\,\alpha_s(m_b)\right]}{32\pi^2 m_b^2} - \frac{\overline{m}_b\,\langle\bar{u}u\rangle}{m_b^6} - \frac{\langle\alpha_s G^2\rangle}{12\pi m_b^6},$$

By calculating the current-current correlation function in perturbation theory and using the $J^\mu B\pi$ amplitude,

$$\mathrm{Im}\,\Pi^{\mu\nu} = \int [\mathrm{p.s.}]\,\delta(q - p_{B\pi}) \langle 0|J^{\dagger\nu}|\bar{B}\pi\rangle\langle\bar{B}\pi|J^\mu|0\rangle + \ldots$$

with crossing symmetry and analyticity, one obtains a simple constraint on the a_ks in the equation

$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$

It is simply $\displaystyle\sum_{k=0}^{n_A} a_k^2 \leq 1$

Return later to the question of

$\langle 0|J^{\dagger\nu}|\bar{B}\pi\rangle\langle\bar{B}\pi|J^\mu|0\rangle$ < Sum vs.

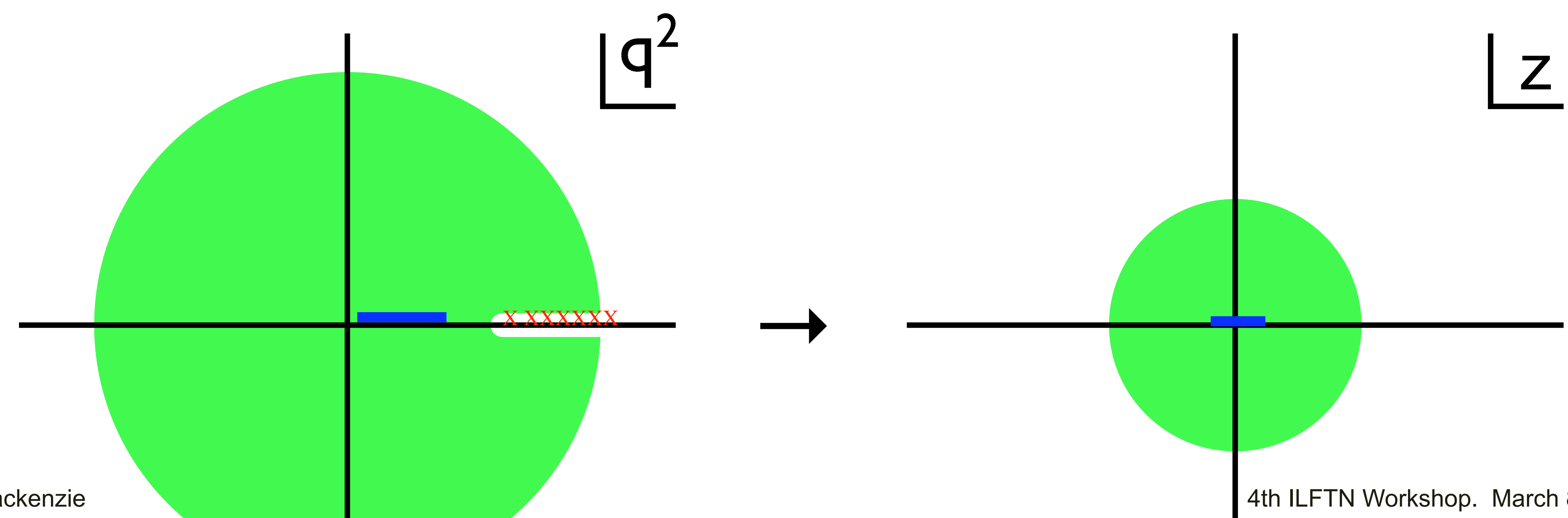
$\langle 0|J^{\dagger\nu}|\bar{B}\pi\rangle\langle\bar{B}\pi|J^\mu|0\rangle$ << Sum.

With $\sum_{k=0}^{n_A} a_k^2 \leq 1$,

the significance of the small range of the *z* variables becomes apparent. To obtain the form factors to high accuracy, say 1%, only a small number of parameters is needed, only 5 or 6 even in the case of *B*->π *l* ν.

B->π *l* ν: -0.34<*z*<0.22,
D->π *l* ν : -0.17<*z*<0.16,
D->*K* *l* ν : -0.04<*z*<0.06,
B->*D* *l* ν : -0.02<*z*<0.04.

$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$

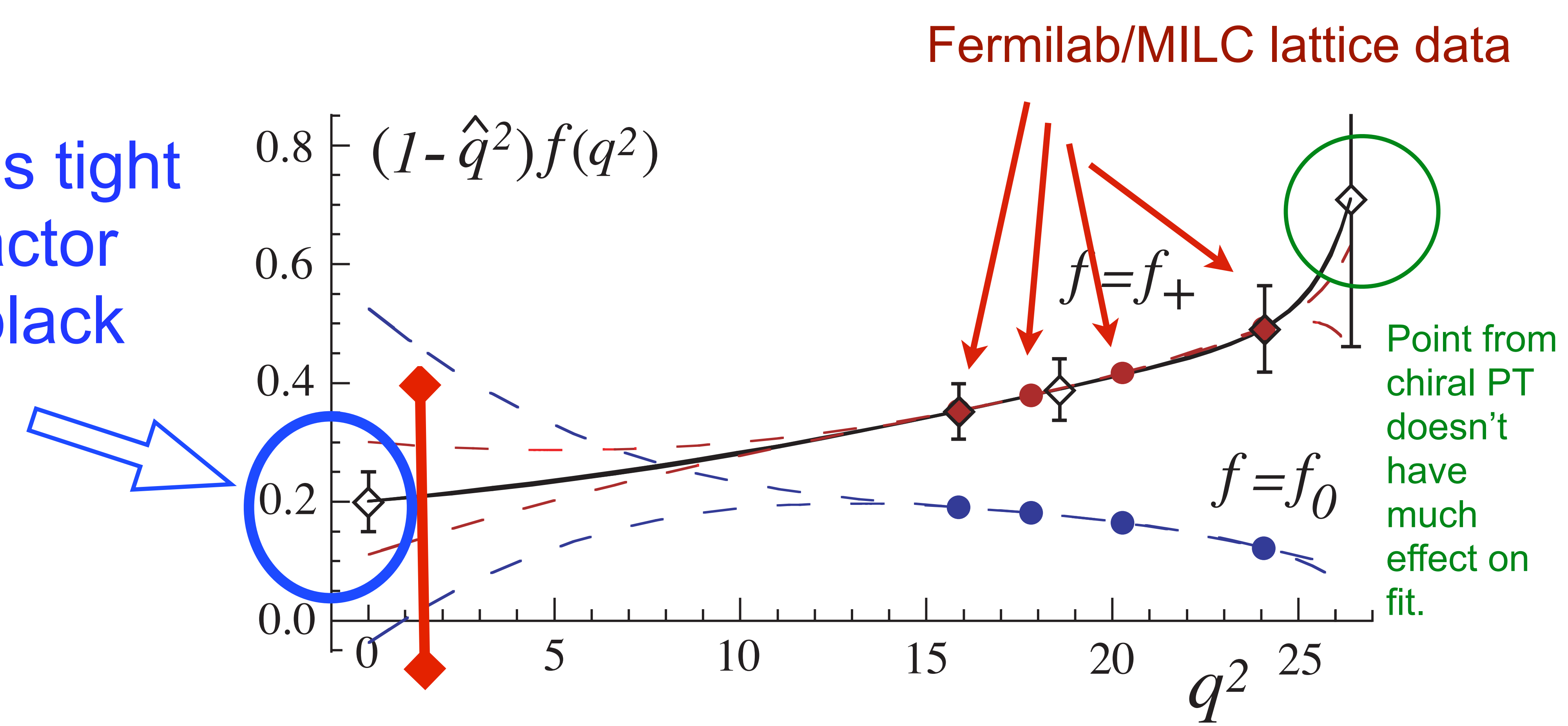


Arnesen, Grinstein, Rothstein, and Stewart

use this fact to obtain strict bound on the form factors from a combination of SCET and lattice data.

As we shall see, lattice data by themselves don't completely constrain the form factors.

From SCET.
With lattice, yields tight bound on form factor (invisible within black line).



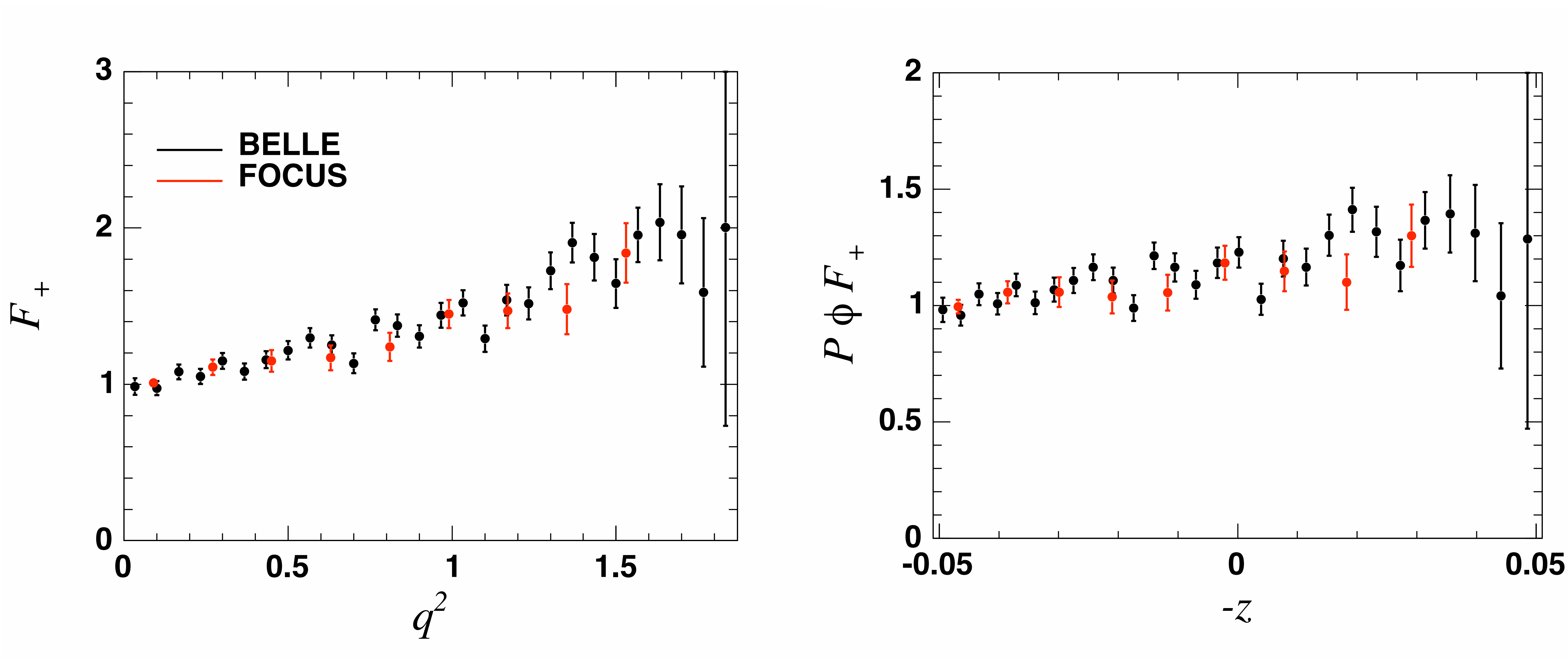
True lattice errors in extrapolation.
(Need to do more work than AGRS to reduce.

Questions

- What can lattice do by itself?
  - (For lattice gauge theorists.)
- How can we get the best bound on V_{ub} by combining information from all methods?
  - (For experimentalists and all phenomenologists.)

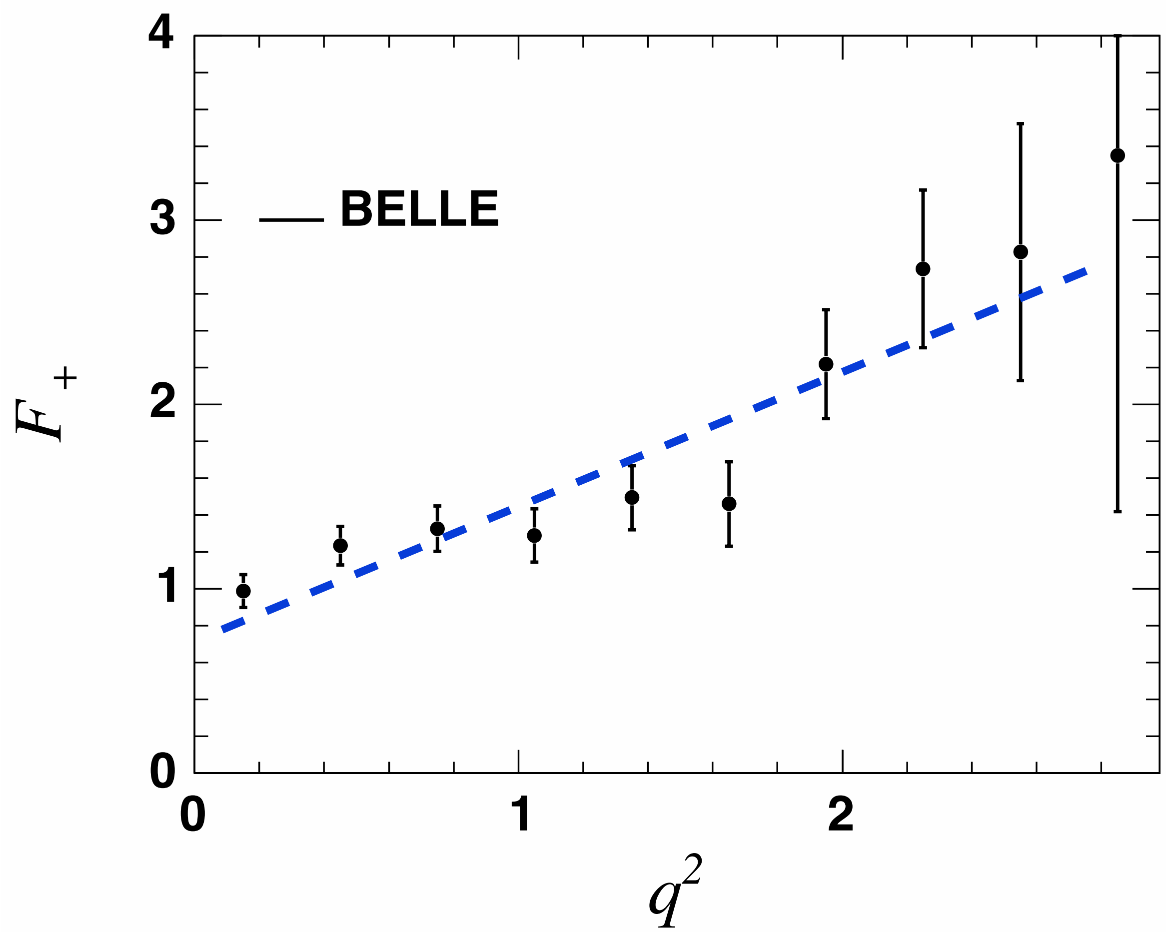
Effect of mapping on data

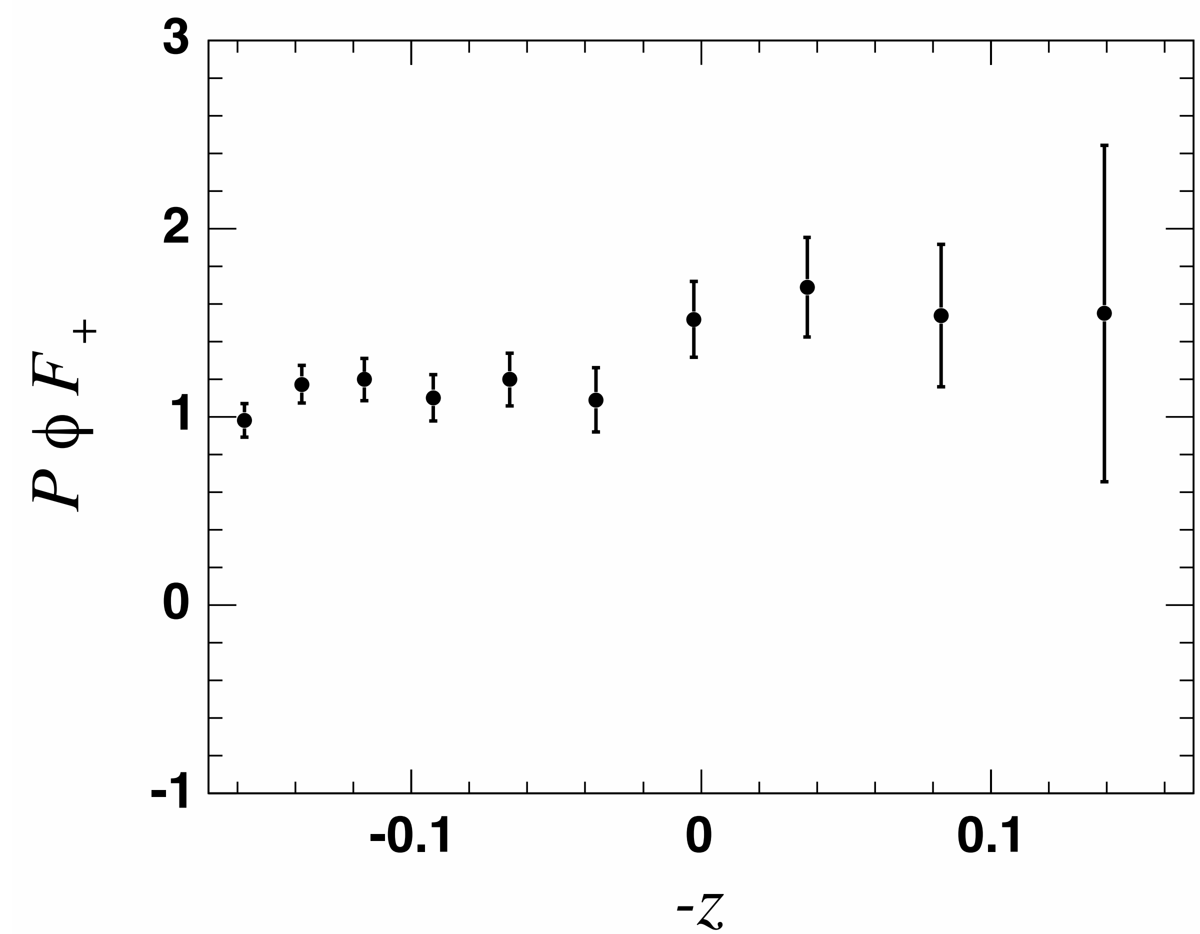
D→K



Not much curvature is seen in $D \rightarrow K l \nu$ with or without remapping.

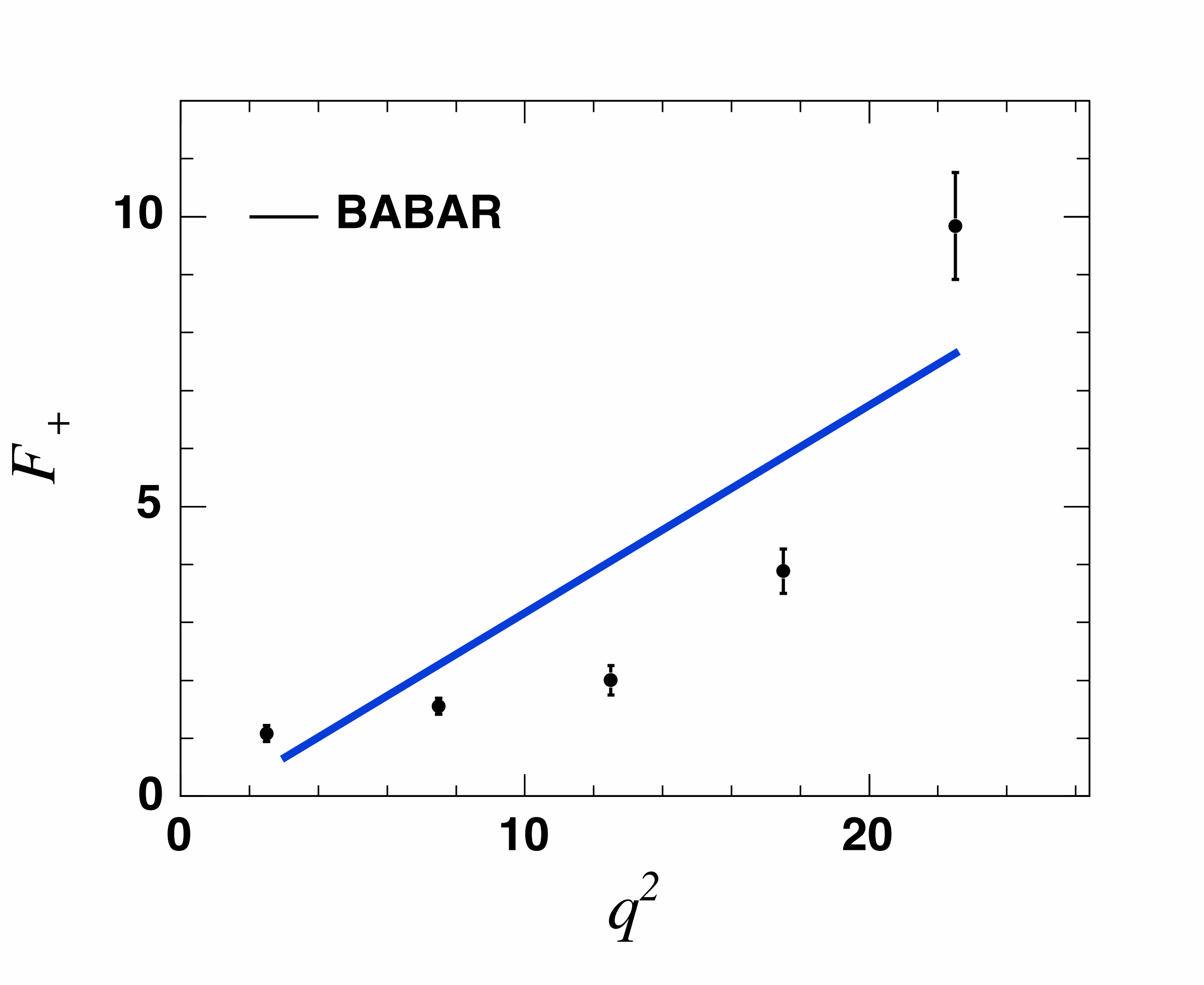
$D \to \pi$

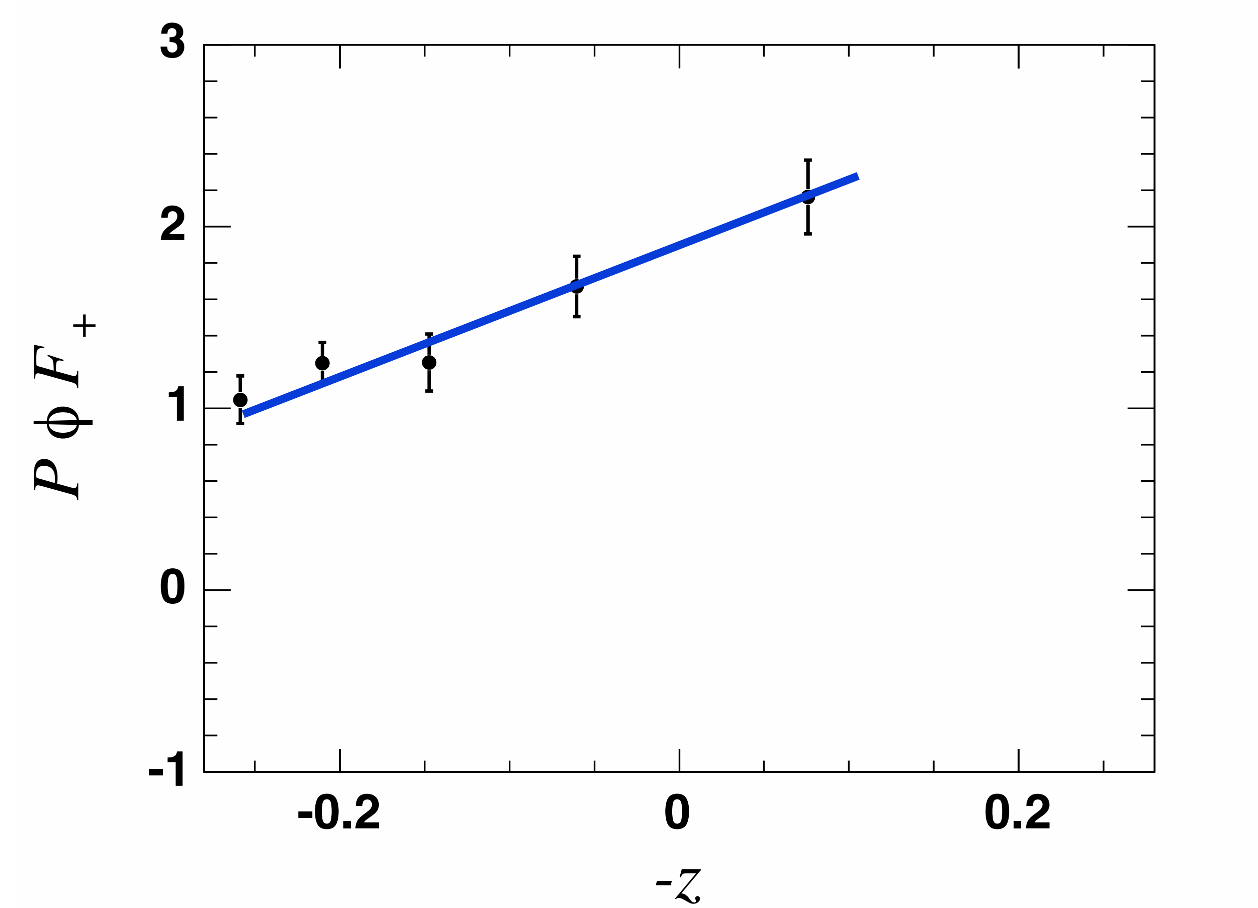




Same for $D \to \pi l \nu$

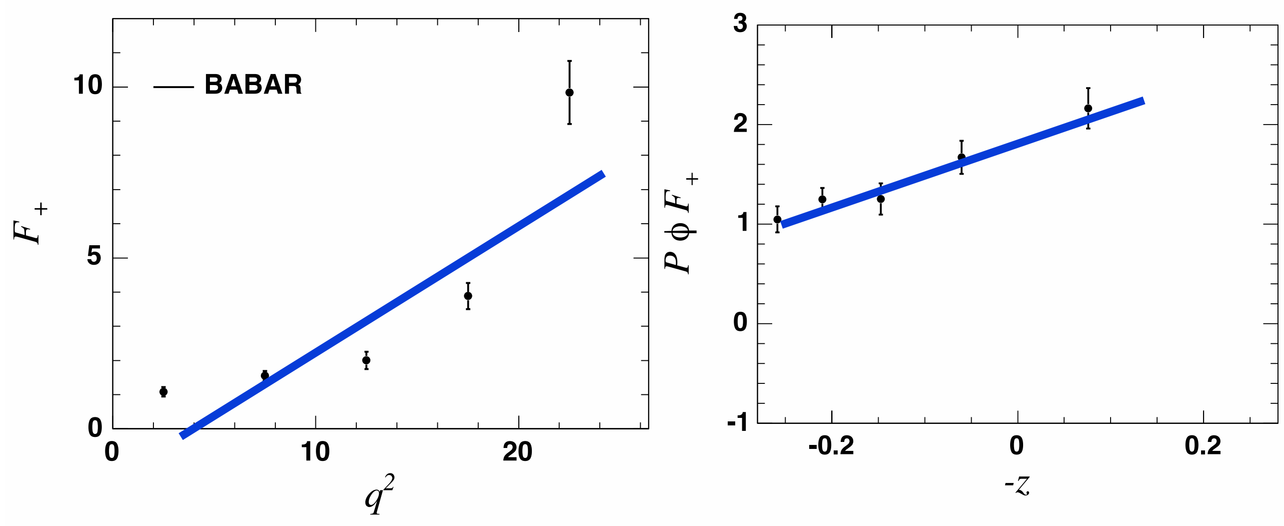
$B \to \pi$





Significant curvature in $B \to \pi l \nu$ data ...

is due to calculable perturbative effects. No visible curvature remains when these are factored out.

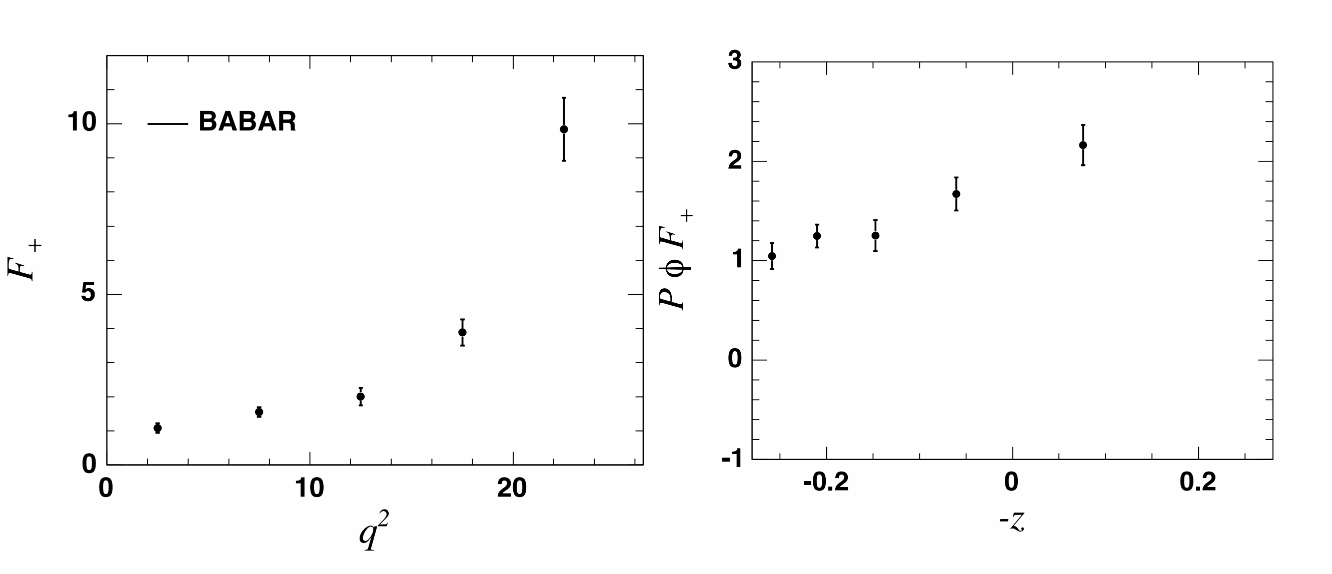


$B \to \pi$

$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$

P and φ are responsible for the curvature seen so far in the Bπ form factor. Task of lattice and experiment is to bound or measure the curvature in the power series.

B→πlν actually like *B→Dlν.*
Data is well described by a normalization and a slope.
Experiment gives the slope.
Taking the normalization from the lattice yields V_{ub}.



$B \to \pi$

$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$

Program for lattice and experiment:

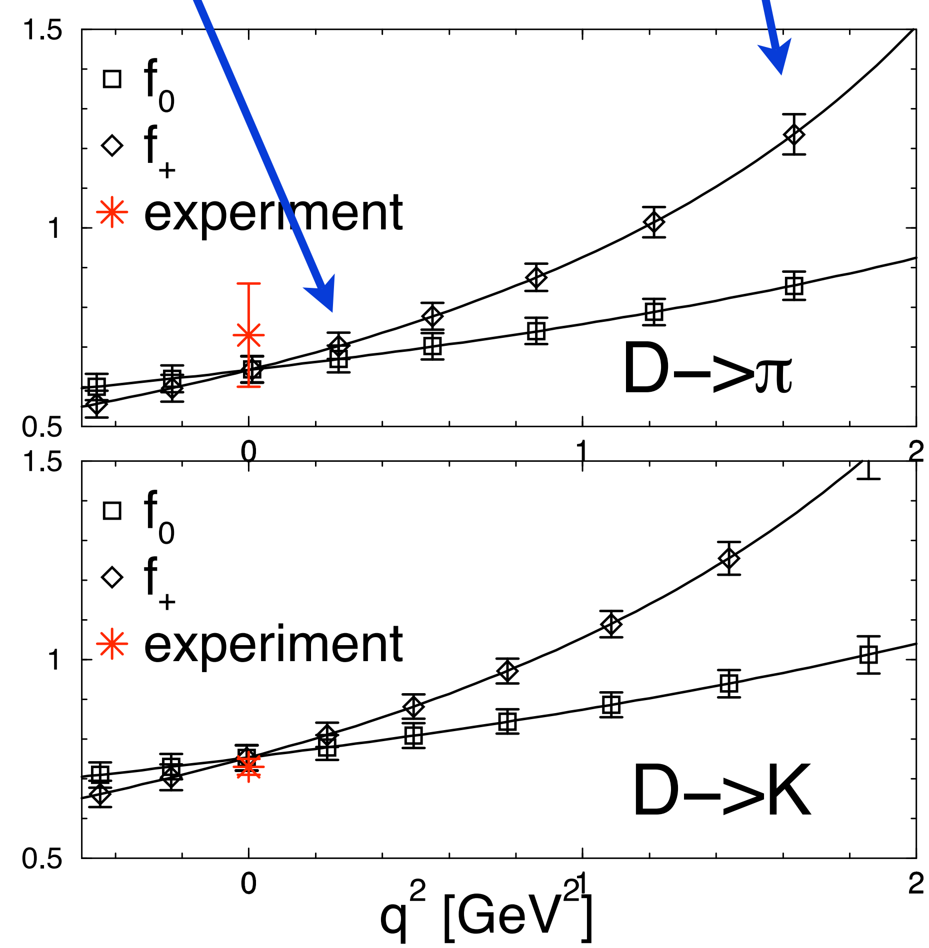
1. Determine and compare slopes in *z*.
2. Compare normalizations, ➞ $|V_{ub}|$.
3. Search for curvature.

Application to lattice data

Current method fitting with BK is convenient, but seems to have introduced model dependence.

The unitarity based parameterization formulas offer a simple way of limiting the number of parameters without introducing model dependence.

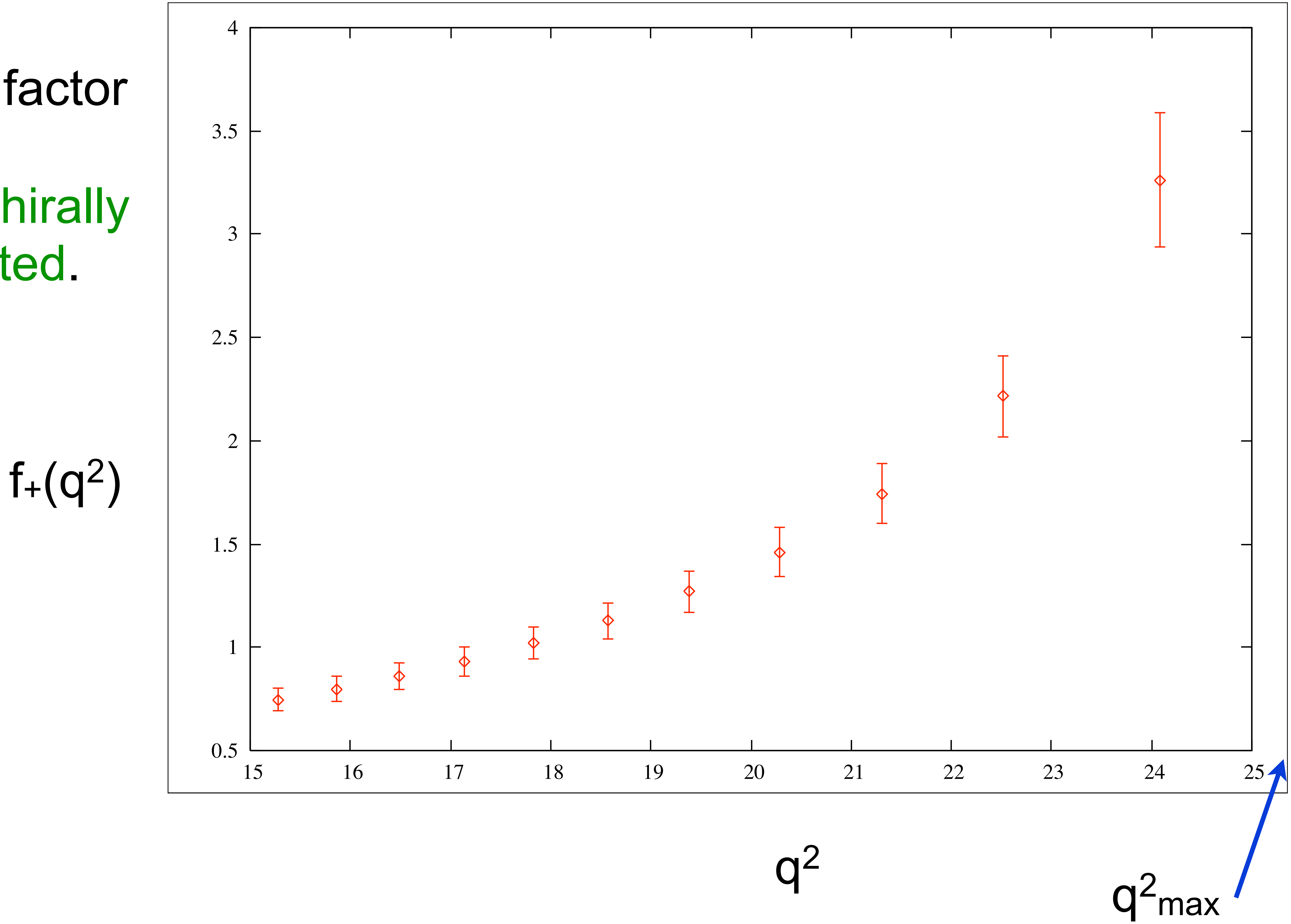
Statistical errors are smaller at high momentum than at low momentum.

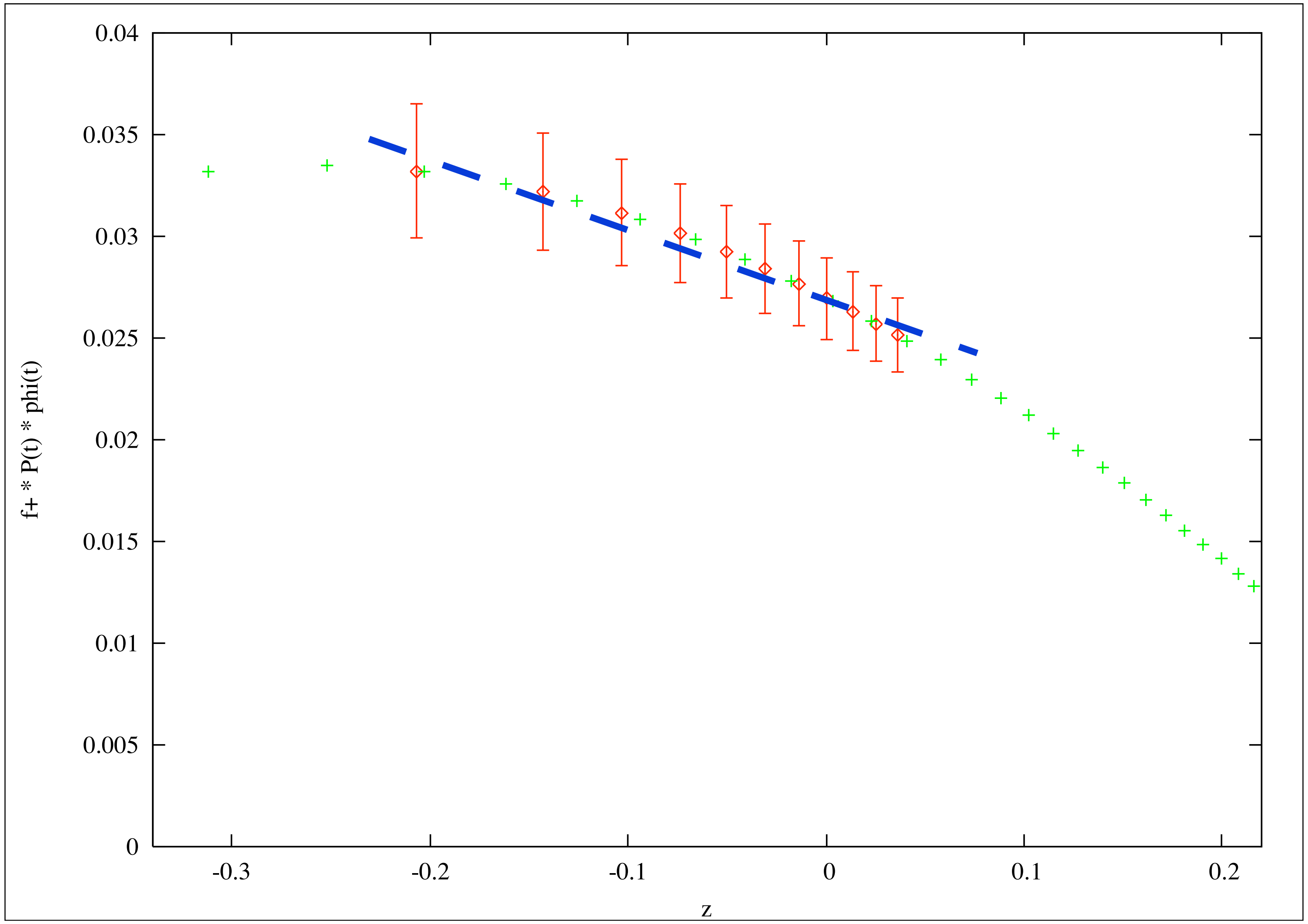


Fit our data to Z expansion:

Replot vs. z and fit ->

Our form factor data for $B \to \pi l \nu$, chirally extrapolated.





Replot of our data with P and phi removed.

$$\sum_{k=0}^{\infty} a_k(t_0)\ z(t,t_0)^k$$

Red, our lattice data; green, fit.

Parameters: a0-a4:

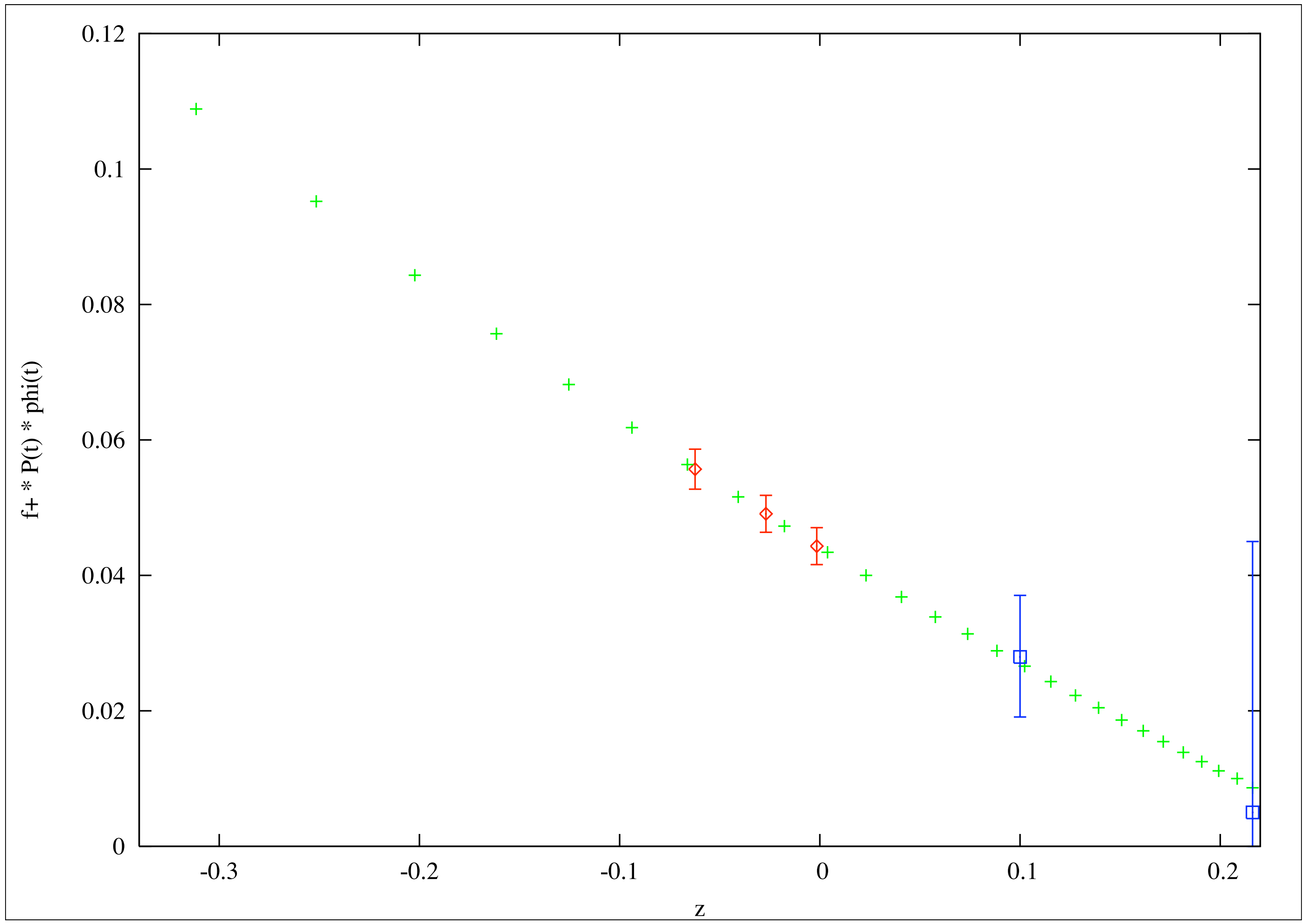
0.0270086	+- 0.00077	(-0 +-	1)
-0.0487598	+- 0.023	(-0 +-	1)
-0.0840779	+- 0.25	(-0 +-	1)
0.0263769	+- 0.9	(-0 +-	1)
-0.0062509	+- 0.99	(-0 +-	1)

Baysian priors

a0, a1, and a2 are constrained by our data,
a3,a4, ... are constrained only by unitarity.
(Fit used Gaussian priors instead of step functions given by unitarity.)

How well is the form factor constrained beyond the end of the data?

$$\sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$



This time, data at $a\, m_l$=0.02.

So, this fit gives no determination of the quadratic term, a2, and a poor limit on f+(0).

However,
f+(z=0.1)*P*phi=0.044+(-.18+/-.06) .1 +/-.64 $.1^2$ +/- $.1^3$=0.031 (0.009).
So, some constraint beyond the end of the data.

Parameters:

0.0441443	+-	0.0027	(	-0	+-	1)
-0.181346	+-	0.06	(	-0	+-	1)
0.0825966	+-	0.64	(	-0	+-	1)
-0.00715237	+-	1	(	-0	+-	1)
0.000500307	+-	1	(	-0	+-	1)

What is f+(0)?
f+(0)*P*phi=0.044+(-.18+/-.06) zm +/-.64 zm^2 +/- zm^3=0.005 (0.034)

Fitting to f_+ and f_0 simultaneously may help.

(Because constraint $f_+(q^2=0)=f_0(q^2=0)$ gives info at $q^2=0$.)

with Ruth van de Water
of Fermilab

Form factors are naturally extracted in lattice calculations in terms of the functions

$$f_\parallel(E) = \frac{\langle P|V^0|H\rangle}{\sqrt{2m_H}},$$
$$f_\perp(E) = \frac{\langle P|V^i|H\rangle}{\sqrt{2m_H}} \frac{1}{p_i}.$$

Which have the following relation to the usual form factors:

$$f_0(q^2) = \frac{\sqrt{2m_H}}{m_H^2 - m_P^2}\left[(m_H - E)f_\parallel(E) + (E^2 - m_P^2)f_\perp(E)\right],$$
$$f_+(q^2) = \frac{1}{\sqrt{2m_H}}\left[f_\parallel(E) + (m_H - E)f_\perp(E)\right].$$

-) Derive expressions for $f_\perp$ and $f_\parallel$ from the unitarity-based expressions for f_0 and f_+.
-) Fit these to lattice data to determine ak for f_0 and f_+.

Heavy quark ideas and semileptonic decay

The bounds on semileptonic amplitudes discussed so far have relied on the fact that the connection of a *B* meson with a weak current and a pion must be less than the connection with all possible decay states.

In fact, as is well known, as the mass of the decaying particle becomes larger, the branching fraction into particular exclusive channels vanishes.

Becher an Hill have calculated the power of the vanishing and commented on its importance in form factor bounds.

Becher and Hill:

The bounds discussed so far are wild overestimates of the size of the form factors.

Only a small fraction of the total branching rate of heavy particles goes into exclusive channels like *πlν*.

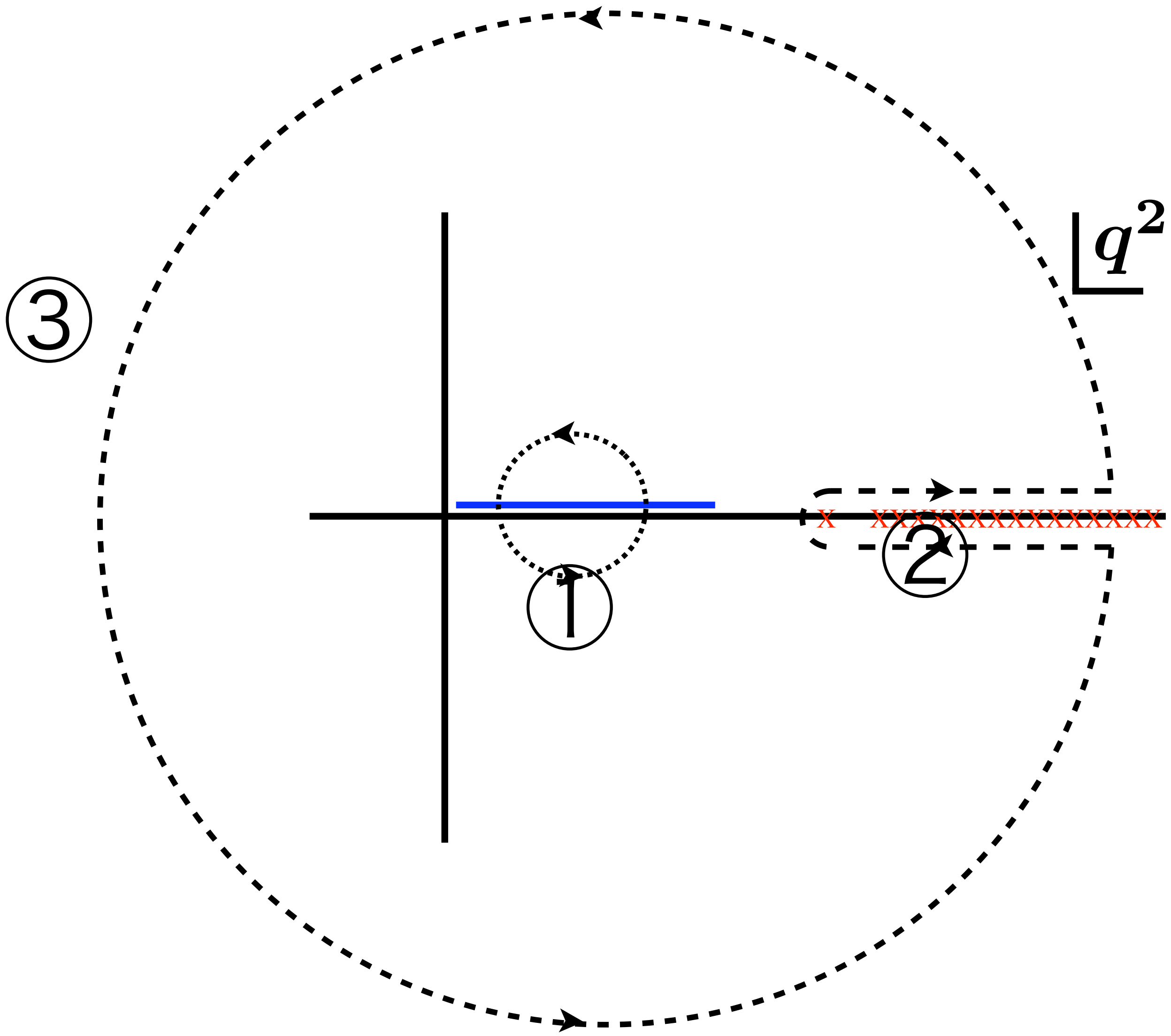
They calculate the fraction as a power of Λ_{QCD}/M_b.

Result:

$$\sum_{k=0}^{\infty} a_k^2 \qquad \text{of order } (\Lambda/m_b)^3$$

Method:

Apply standard facts about analytic functions...



Standard complex analysis:

①=②+③=②

$$F(q^2) = \frac{1}{2\pi i}\oint dt\,\frac{F(t)}{t-q^2} = \frac{1}{\pi}\int_{t_+}^{\infty} dt\,\frac{\mathrm{Im}F(t)}{t-q^2}$$

... to the function:

$$\sum_{k=0}^{\infty} a_k^2 = \frac{1}{2\pi i}\oint \frac{dz}{z}|\phi(z)P(z)F_+(z)|^2 = \frac{m_b^2}{3}\int_{t_+}^{\infty}\frac{dt}{t^5}[(t-t_+)(t-t_-)]^{3/2}|F_+(t)|^2 \equiv A\,.$$

Use HQET to evaluate size leading contributions in the important regions.

$$t - t_+ \sim m_b\Lambda,$$

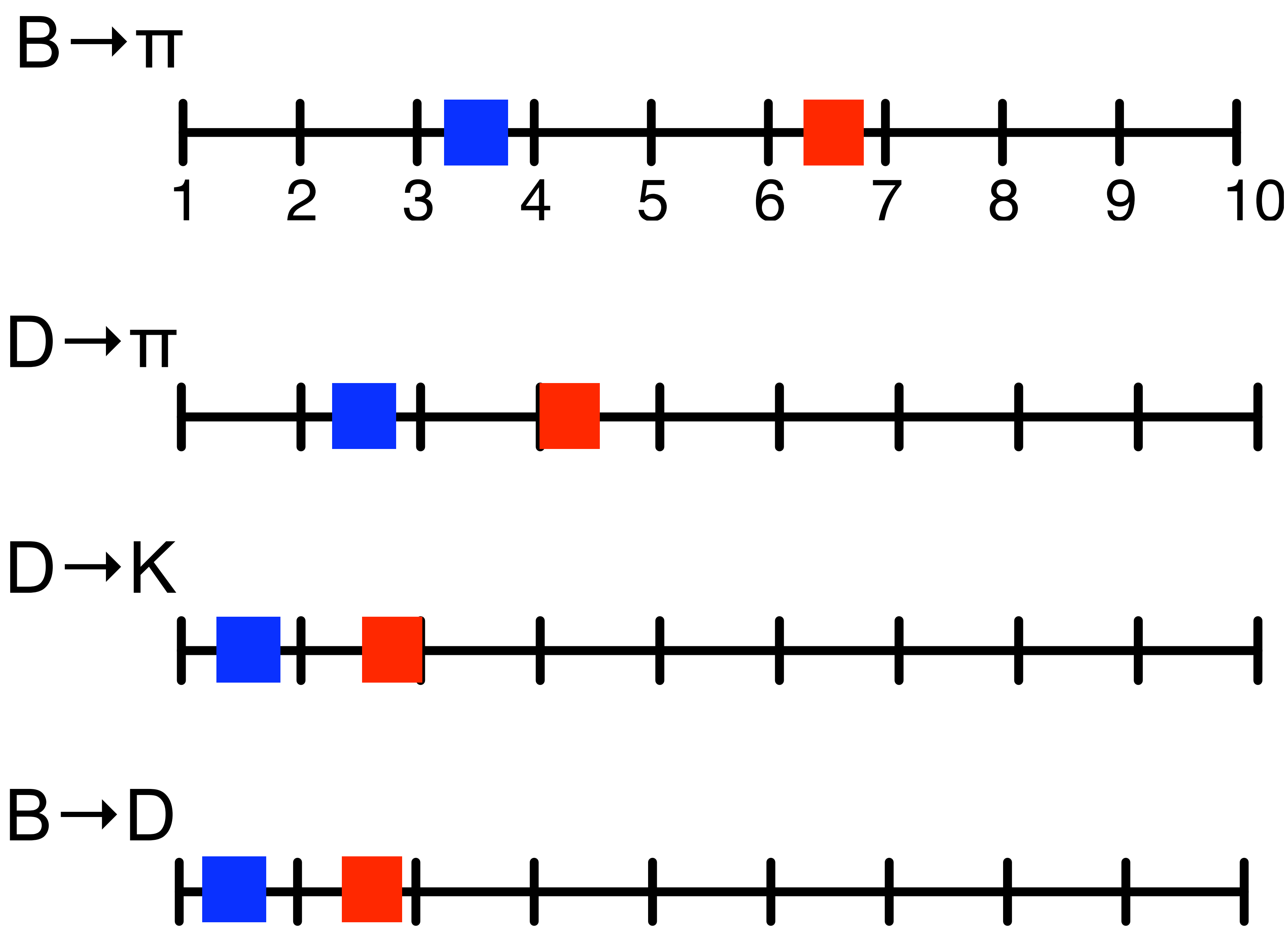
$$t - t_+ \sim m_b^2,$$

In both regions, they obtain $A \sim (\Lambda_{QCD}/M_b)^3$.

If each a_k is expected to be $\sim (\Lambda_{QCD}/M_b)^{3/2} \sim 0.1$
fewer terms are needed to described form factors accurately.

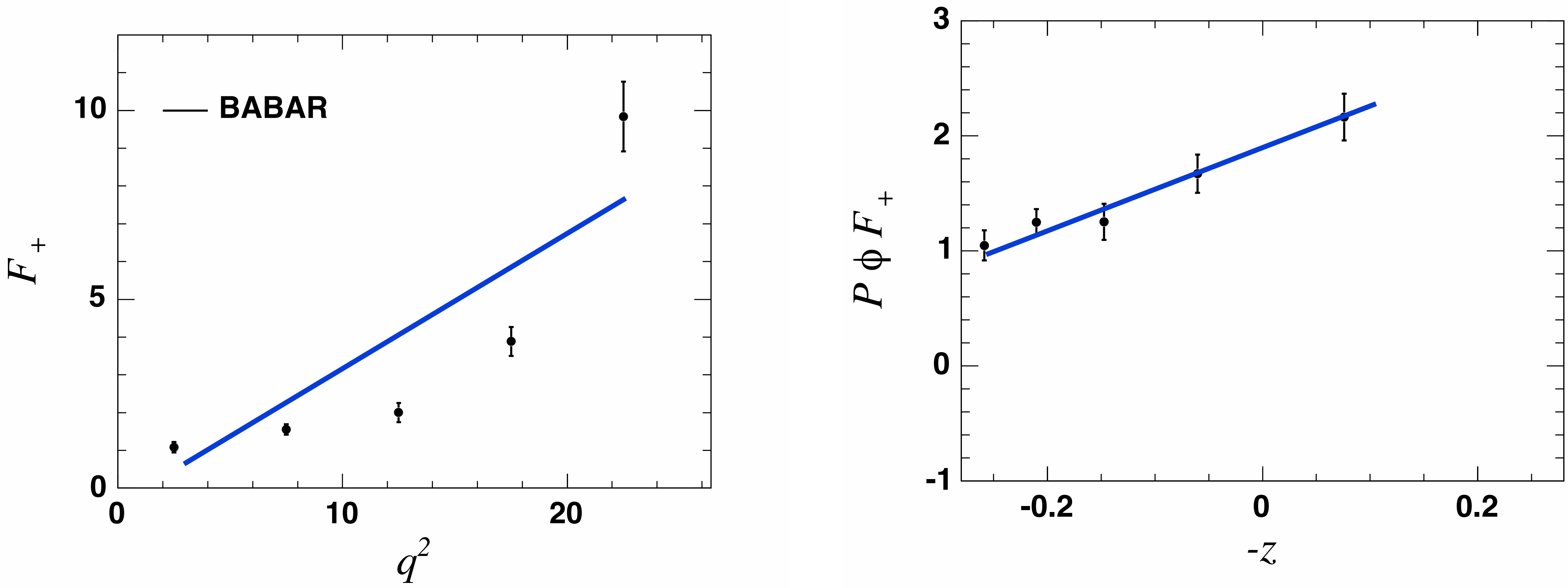
Maximum # parameters at 1% sensitivity:

(blue) $\frac{\Delta F}{F} = \frac{a_k}{a_0} z^k < z^k$ (red) $\frac{\Delta F}{F} = \frac{a_k}{a_0} z^k < \frac{1}{a_0} z^k$ (“unitarity”)



- experimental implication: N relevant parameters ⇒ need N independent measurements (bins)

BH prediction for B→π:



a_2 will turn out to be ~ 0.1, like a_0 and a_1.
a_3 and a_4 will turn out to be negligible.

Summary

- The analyticity-based *z* expansion limits the number of parameters needed to describe form factor data, without introducing model dependence.
- In terms of the *z* expansion, all semileptonic form factor data, both lattice and experiment are consistent with straight lines: normalization and slope.
  - Even $B \to \pi l \nu$.
- Heavy-quark theory arguments explain why this is so,
  - and predict that the curvature given by a2 will be measurable only in highly accurate lattice and experiment, and that the the effect of higher order terms will be negligible.